MIDDLE SCHOOL TEACHERS' USAGE OF DYNAMIC MATHEMATICS LEARNING ENVIRONMENTS AS COGNITIVE INSTRUCTIONAL TOOLS

by

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This dissertation was prepared under the direction of the candidate's dissertation advisor, Dr. Yash Bhagwanji, Department of Curriculum, Culture, and Educational Inquiry, and has been approved by the members of her supervisory committee. It was submitted to the faculty of the College of Education and was accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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ABSTRACT

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This study described, analyzed, and compared the internal and external factors that prevented or fostered the implementation of a cognitive tool, GeoGebra, in the mathematics practices of 12 middle school teachers who had completed a master's degree program in mathematics successfully. Through the application of a case study approach as a systematic method for the analysis of qualitative data, and under a social constructivist framework, the study examined different factors such as concerns of teachers; their beliefs about technology, mathematics as a subject, math teaching, and learning; external factors such as resources and school support; their TPACK development; and their instrumental orchestration approach through classroom observations.

Among the major findings, the study revealed that the personal concerns of the teacher users of GeoGebra included the desire to continue learning the new features of the software, as well as the desire to connect themselves with others in common

endeavors for the benefit of other teachers and, ultimately, the students. The external factors such as lack of working computers did not impede but restricted their use of GeoGebra in the classroom. There was a consensus among the teacher users that they had to strike a balance between their professional goals and the available resources. The users expressed feelings of accomplishment as professionals and had been recognized as such by the several awards they received. They did not over-emphasize the challenges they encountered, instead downplaying them with the result of engaging students and providing them with the best learning experiences they could.

DEDICATION

I dedicate this dissertation to José, my best friend, my love, my rock of support, my biggest fan, and my husband, who supported me each step of the way.

I also dedicate this dissertation to my good friend who pushed me to aspire professionally far beyond what I believed I was capable.

MIDDLE SCHOOL TEACHERS' USAGE OF DYNAMIC MATHEMATICS LEARNING ENVIRONMENTS AS COGNITIVE INSTRUCTIONAL TOOLS

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CHAPTER ONE: INTRODUCTION

> Nec manus, nisi intellectus, sibi permissus, multam valent: instrumentis et auxilibus res perficitur. (Human hand and intelligence, alone, are powerless: What gives them power are the tools and assistants provided by culture.)
>
> Francis Bacon, 1600 (as cited in Trouche, 2004)

Since the early 1980s, there has been a growing interest in the use of technology as a learning tool (Isiksal & Askar, 2005). The National Council of Teachers of Mathematics (NCTM) endorses the use of technology in the teaching and learning of mathematics, indicating that technology can influence the mathematics that is being taught and can enhance student learning (NCTM, 2000). However, the integration of technology in mathematics classrooms is a slow process (Guin & Trouche, 1999). One possible explanation is that the integration of technology has an effect on mathematics curriculum and on teachers' instructional practices (Mariotti, 2002). When using technology in their classrooms, teachers are faced with the possibility of altering the composition of a curriculum mostly based on paper-and-pencil activities, and of deciding on the proper mixes and sequences of skills and concepts taught in their classrooms (Heid, 1997). Another possible explanation for the slow integration of technology in the mathematics classroom is that many teachers earned their degrees when technology was not as prominent in the classroom as it is today. Since some teachers do not have the experience of learning their course content with technology, they are not prepared to implement new technologies in their teaching (Niess, 2011), and can have concerns—feelings and perceptions—about the implementation of the innovation (Hall & Hord, 2011).

The role of the teacher in the implementation of technology has been acknowledged as both critical and problematic (Artigue, Drijvers, Lagrange, Mariotti, & Ruthven, 2009; Fullan, 2008). It is critical because the way in which teachers approach the use of technology has major consequences on the effects of its use in the classroom (Kendal & Stacey, 2001). It is problematic because if teachers do not perceive the use of technology in their practice as valuable for their educational goals, they could be inclined to avoid it (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010). In order to help teachers implement technology in a way that can benefit their everyday practice, it is important to have more knowledge about their perceptions of what is difficult about its implementation, if and how their practice changes in a technology-rich environment, and how the role of technology relates to teachers' beliefs about mathematics as a subject and about the teaching and learning of mathematics. *The Stanford Encyclopedia of Philosophy* defines belief as the psychological state in which an individual holds a proposition or premise to be true (Schwitzgebel, 2011). In Chapter 2, current literature about teachers' beliefs is reviewed.

This study considered closely the use of cognitive tools in the mathematics classroom. Means (1994) classified technology in four different types according to their instructional purpose: tutorial, communication, exploratory, and cognitive tools. Cognitive tools can help students in the educational process by providing them with instruments to facilitate tasks and by enabling students to represent what they know

(Jonassen, 2000). By definition, cognitive tools can challenge students and can be used to promote deep reflection. Jonassen's (2000) definition is based on the constructivist idea that students learn more from constructing and justifying their own ideas than from studying someone else's ideas. These technologies can change the nature of teaching and learning, as they enable new forms of activity (Hmelo-Silver, 2004). GeoGebra, a collection of cognitive tools that the study examined closely, could enhance the learning environment with dynamic mathematics, and could have the potential to impact mathematics teaching and learning, instructional approaches, and problem-solving activities (Hohenwarter & Hohenwarter, 2009). Dynamic mathematics refers to a mathematics learning environment where students have the opportunity to create and manipulate figures and functions while retaining their essential characteristics and properties. For these reasons, these cognitive tools can be referred to as dynamic mathematics learning environment tools (Bu & Schoen, 2011). In the study, integration of cognitive tools was defined in terms of teachers using cognitive technology to develop students' mathematical knowledge.

Theoretical Framework

The study was based on a social constructivist approach to learning and its implications for teaching. Based on the work of Jean Piaget (1977) and John Dewey (1938), constructivism explains cognitive advancement as a process by which learners transform what they already know into new ideas as they make sense of new experiences. Constructivism can be defined as a theory, which argues that humans generate knowledge and meaning from the interactions between their experiences and their ideas. Therefore, knowledge is constructed, not transferred. Under the theory of constructivism, a person is understood to be an organizer who interprets experiences; these interpretations then shape the person's structured world (von Glasersfeld, 2002). Social constructivism considers how learning develops in social contexts. Vygostky (1978) believes that knowledge has social origins and is dependent on an individual's interaction with others.

According to von Glasersfeld (2002), constructivism carries crucial educational implications. Mathematical knowing does not exist independent and outside the minds of human beings; rather, it is afforded and constrained by one's (mental) ways of operating. Consequently, coming to know mathematics entails an active process of constructing new (to the learner) ideas—coordinated, justified mental actions and their meanings for the constructing person—via continuous interactions in one's social and physical environment; those ideas are held in continual check against newly noticed effects of mental activities and, if needed, adjusted to better fit one's experiential reality, which always includes social exchanges (Piaget, 1977; von Glasersfeld, 2002). With the integration of technology in education, social exchanges can be expanded to consider students interacting with technology. Accordingly, teaching mathematics begins with the premise that one person's knowing cannot be directly transmitted to and passively received (e.g., via lectures) by another person, nor does it amount to fostering memorization and mastery of facts and procedures. Rather, teaching requires indirect orientation of students' thought processes, via engaging them in solving problem situations (tasks) that trigger particular goals and mental operations toward those goals, orienting students' reflection to things that change and things that are anticipated to remain the same across different situations (Tzur & Simon, 2004). Students' productive participation in the learning process (engagement) is crucial in their learning. To support such productive participation, an inquisitive and risk-taking mindset is needed, including willingness to bring forth intuitive thoughts that may turn out to be wrong as well as a healthy disposition toward making and correcting mistakes as part of the learning process. Thus, teachers need to create a learning environment in which students feel safe to think, share, and critique, and are eager to explore new ideas (NCTM, 2000).

The study was influenced by the social constructivist concept of *tools*, as proposed by Vygotsky (1978) in his book *Mind in Society*. Vygotsky compared the use of tools to the use of signs (e.g. language) and concluded that they are similar, since both are used as mediators to solve problems (Figure 1).

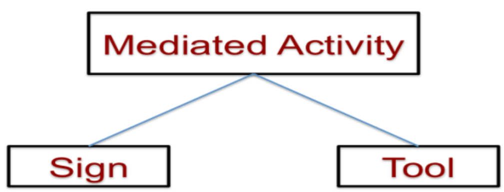

Figure 1. Diagram of Mediated Activity. Logical representation between the use of signs and tools in order to solve a mediated activity (Vygotsky, 1978).

According to Vygotsky (1978), signs and tools affect human behavior differently. He saw the function of tools as a way for humans to change objects and, therefore, to master nature. Contrastingly, signs do not change objects. Vygotsky believes that mastery of nature was linked to mastery of behavior just as “man’s alteration of nature alters man’s own nature” (p. 55). He proposed that the use of tools and signs changes all psychological operations, and the use of tools expands the range of activities within which the new psychological functions may operate. He referred to the combined use of tools and signs as a higher psychological function or behavior. The study analyzed the way in which teachers integrated technology (tools) as a way to facilitate instruction, as well as how their behavior was affected by the use of technology.

Rabardel (1995), following Vygotsky's (1978) understanding of tools, made a distinction between an artifact and an instrument. An artifact is a bare tool that is available to the user to solve a problem, but which can be useless if the user does not know how to apply it. Only when the user realizes how the artifact can be used for a specific purpose can that artifact become an instrument that mediates the activity. A bare tool, or artifact, becomes an instrument when the user establishes a relationship between the artifact and the activity to be mediated. This relationship or interaction requires a mental process from the user. Therefore, an instrument is both an artifact and the necessary mental schemata that the user develops for a particular activity (Drijvers & Trouche, 2008). Piaget (1977) defined schema as the mental representation of ideas, actions, or perceptions. He considered schemata to be the building blocks of thinking (Woolfolk & Hoy, 1990). According to Drijvers and Trouche (2008), mental schema cannot be observed directly, and researchers only can reconstruct the schema based on observation and information provided by the user.

The theoretical perspective that informed the study of teacher behavior in a technology-rich environment was the notion of instrumental orchestration. Drijvers et al. (2010) defined instrumental orchestration as "the teacher's intentional and systematic organization and use of the various artifacts available in a learning environment in a given mathematical task situation" (p. 214). Instrumental orchestration consists of three elements: pedagogical context, preparation mode, and pedagogical action (Figure 2).

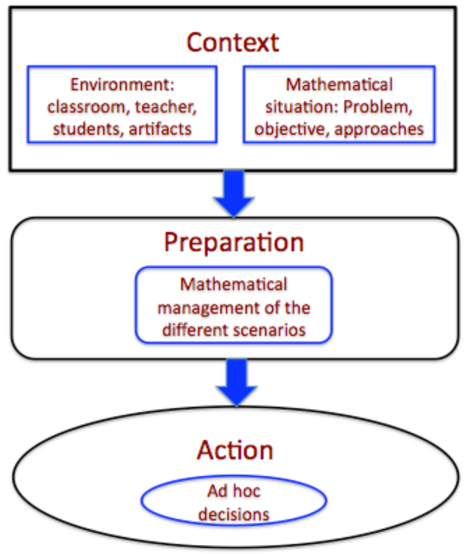

Figure 2. Instrumental Orchestration. Phases that describe the organization of artifacts in a given mathematical situation.

Pedagogical context, which refers to the arrangement of teaching settings and artifacts available in the environment, has a strong preparatory aspect, and might not be easily changed while teaching. In the mathematical classroom, this definition might be transferred to both the classroom arrangements and the topics to be covered. It also affects what artifacts will be available and what instructional approach will be used.

The preparation mode represents the ways in which teachers decide to make full use of pedagogical preparation for the benefit of instruction. It includes decisions about the way an activity is introduced and worked out, if activities will be open-ended or if specific answers are required, as well as the schema and techniques to be developed.

This mode is more flexible than the pedagogical context and can be modified while teaching.

Pedagogical action involves ad hoc decisions made during teaching about how to actually perform the chosen pedagogical configuration and preparation mode. In the mathematics classroom, this could involve a teacher deciding how to deal with unexpected aspects of technology or the activity itself, how to deal with the variations of rigor for solving a problem, as well as determining how to deal with students' questions and errors.

Drijvers et al. (2010) identified six orchestration types that they termed technical-demo, explain the screen, link-screen-board, discuss-the-screen, spot-and-show, and Sherpa-at-work. The technical-demo orchestration concerns the demonstration of tool techniques by the teacher. The explain-the-screen type refers to whole-class explanation by the teacher, guided by what happens on the computer screen. In the link-screen-board orchestration, the teacher stresses the relationship between what happens in the technological environment and how this is represented in conventional mathematics of paper, book, and blackboard. The discuss-the-screen orchestration refers to whole class discussion about what happens in the computer screen. In the spot-and-show, students' reasoning is discussed by the whole class through the identification of interesting student work while using cognitive tools. In the Sherpa-at-work orchestration, a student uses the technology to present his or her work or to carry out actions requested by the teacher. According to Drijvers et al., a teacher's views on mathematics teaching and learning and the opportunities technology offers can be seen in their justification of orchestration types.

Instrumental orchestration represents part of the teacher's style of teaching, which has to be accommodated to the specific teaching context and is deeply dependent on the teacher's pedagogical content knowledge. The study observed the instrumental orchestration of the participating teachers. Using Vygostky's (1978) words, each mediated activity will require different tools and signs. The user must decide which tools are more appropriate and why.

In summary, the study was based on social constructivist theory, which implies that learning mathematics in a technology-rich classroom involves an active process of students constructing new knowledge. Teachers facilitate that knowledge construction by continuously interacting with students in a social and physical environment, while using cognitive tools. A teacher's actions and behaviors while guiding students' construction of new ideas and knowledge constitute their instrumental orchestration (Figure 3).

Background

For the last nine years, I have been working at the Department of Mathematical Sciences at an institute of higher education on a series of National Science Foundation (NSF)-funded mathematics education projects with Dr. Heinz-Otto Peitgen as the project manager. The main project was a Mathematics Science Partnership (MSP), called *Standards Mapped Graduate Education and Mentoring* (SMGEM), with a large public school district.

The goal of the project was to eliminate possible gaps in content and pedagogy between the university-level approach to a teacher's mathematics and science preparation and the daily requirements of a diverse standards-driven classroom.

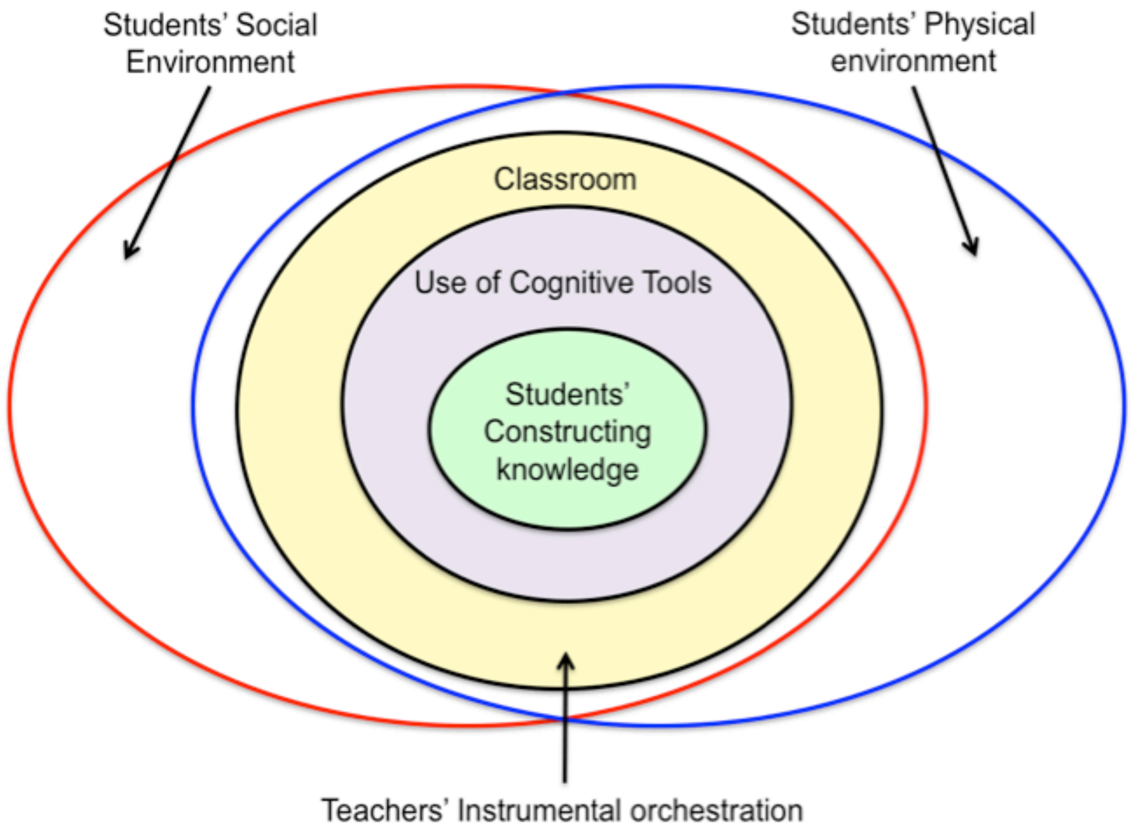

Figure 3. Construction of Knowledge in a Technology-Rich Classroom.

SMGEM created a series of eight semester-long courses that delivered a unique standards-aware and technologically literate curriculum for graduate-level middle grade mathematics teacher education. The curriculum design of all the courses was driven by the Sunshine State Standards, and enhanced the mathematical background of middle grade mathematics, infused technology in the teaching and learning of mathematics, and provided multidisciplinary connections.

The program offered a sequence of courses, intensive summer programs, biannual weekend Pedagogy Conferences, follow-up meetings, and an online community. Completion of the program took, on average, two years, with cohorts of teachers working and collaborating with each other for the duration of the program. By the end of summer 2011, six different cohorts of teachers, a total of 66 mathematics teachers, received a Master in Science in Teaching (MST) degree through this program, and more than 300 additional mathematics teachers from the district received professional development at the end-of-semester dissemination conferences.

In comparison to matched non-participating teachers, graduates from the SMGEM program: (a) saw their students make significant mathematics content gains in categories related to the Florida Sunshine State Standards, as verified by district benchmark achievement tests; (b) implemented a wider range of pedagogies in their classrooms, including more effective use of specific technologies, especially GeoGebra; (c) grew in their intentional emphasis on and mastery of the state standards; (d) increased in their mathematical self-efficacy; and (e) moved in disproportionate number into leadership positions in local school system administrations, in regional professional organizations, and in national professional presentation settings.

With respect to technology, the addition in 2006 of Dr. Markus Hohenwarter, the developer of GeoGebra, to the project's instructional leadership team dramatically enhanced the already strong modeling of pedagogical effectiveness by program personnel. GeoGebra is open source software that has achieved national and international recognition as one of the most effective (and now widely used) dynamic mathematics environments developed with and for mathematics educators. Several tools and features of GeoGebra set this software apart from other software such as Cabri and the Geometer's Sketchpad. GeoGebra allows users to operate algebraically and geometrically simultaneously in realistic problem situations, and to discover and experiment with personally meaningful models while using multiple representations and tools to construct increasingly abstract mathematical ideas (Bu & Schoen, 2011). In addition, GeoGebra allows users to integrate spreadsheet explorations while observing the geometric and algebraic representations of the explorations.

After its introduction in USA, the GeoGebra international community grew to over 140 centers worldwide, with 11 in the United States, and it supports 44 languages. In 2010, GeoGebra received the National Technology Leadership Award in Washington D.C. Advanced classes on the use of GeoGebra generated technology leaders in the district who, as part of supplemental funding, were able to provide GeoGebra workshops to other MSP projects around the nation. The Science Technology Engineering and Mathematics (STEM) faculty from the institute of higher education and master teachers from the district continue to provide professional development where GeoGebra plays a key role. Our experience of introducing the software to teachers and students indicate that they (a) swiftly become facile with the software, (b) enjoy the explorative nature of problem solving processes, (c) work cooperatively to solve and pose problems in it, and (d) learn mathematical ideas through this work (largely through back-and-forth shifts between real-world problems and the software as a representational tool).

SMGEM principal investigators, the evaluation and research teams, and the participating teachers have identified a number of important points for both the professional development of teachers and a teacher's own classroom effectiveness: (a) appropriate use of technology to engage students' interest and to clarify difficult points of comprehension; (b) the need to master multiple representations for presentation by the instructor as well as multiple constructions for the students to create; (c) a familiarity and comfort with student-centered inquiry-based instructional methods; (d) use of hands-on instruction that favors digital and concrete manipulatives; (e) adoption of a style of questioning that identifies student misconceptions and encourages exploration; (f) an openness to “risk taking” so that instructional strategies do not become stagnant; and (g) appropriate use of mathematics vocabulary.

As the SMGEM project manager from its start, I was involved in the design and delivery of high quality, innovative teacher development workshops. I worked with the participating teachers both one-on-one and in professional development settings. I also worked very closely with Dr. Hohenwarter in the development of technological materials, using GeoGebra, for teachers. I became interested in helping teachers become more comfortable in using technology, like GeoGebra, in ways that would promote students’ understanding and learning of mathematics.

While in the NSF-funded program, all participating teachers received the same mathematics content and pedagogy instruction, experienced the same enrichment materials, and encountered the same technology integration. After completion of the program, we have been able to provide the teachers with follow-up conferences to bring them up-to-date with the new GeoGebra releases, with some older cohorts participating in several follow-up conferences. However, not all teachers are equally proficient in the use of GeoGebra. Some teachers heavily integrate GeoGebra and effectively use all of its capabilities, while others only use a few or none at all.

Statement of the Problem

NCTM (2000) endorses the use of technology in the teaching and learning of mathematics, indicating that technology can influence the mathematics that is being taught. Under the social constructivist theory, mathematics is seen as the product of human activity dependent on the social and cultural context where it develops. The integration of technology in the teaching of mathematics has the potential to change the discourse in the classroom, the representations of mathematical ideas, the manipulation of symbols, the drawings, and the schemata students can develop (Artigue, 2002). New mathematical needs can emerge due to the technological tools and the representation systems involved (Balacheff, 1993). The study looked at the implementation of technology that creates dynamic mathematical learning environments and concentrated on the use of GeoGebra (developed in 2001-02). Other examples of these types of technologies are: Geometric Supposer (developed in 1985), Cabri Geometer (developed in 1988-92), and The Geometer's Sketchpad (developed in 1992). Because of the relatively recent use of these technologies in the classroom, few studies provided a comprehensive view of the possible influence they could exercise in the mathematics curriculum (Clements, Samara, Yelland, & Glass, 2008; Zbiek & Hollenbrands, 2008).

Because instrumental orchestration is a new theoretical framework in mathematics education, there many areas that still need to be researched (Drijvers et al., 2010). Drijvers et al. (2010) called for more research to investigate whether variants of their identified orchestration types can be recognized in different settings and with different types of technological tools.

When considering technology and its implementation, the teacher should be the central focus because the successful implementation of any strategy often requires teachers to change their behavior (skill, competencies) and their beliefs (knowledge, understanding) (Fullan, 2008). Although some research considered the effects of cognitive technological tools usage on in-service teachers' practices, beliefs, and concerns, most of the current research involved pre-service teachers or students (Bell, 1998; Bucher & Edwards, 2011; Scher 2005). Ball and Cohen (1999) believe that teacher learning should not be seen as something that just happens. Training of teachers should not be a one-time professional development before the innovation is implemented but a sustained support before and after implementation (Fullan, 2008). Most of the research that involves in-service mathematics teachers examined the process of teachers learning to use new technology tools (Niess, Sadri, & Lee, 2007; Ozgun-Koca, Meagher, & Edwards, 2011); very few studies were found where in-service teachers were examined years after learning how to use the technology.

Purpose of the Study

The purpose of this multiple case study was to describe, analyze, and compare information on the different ways that 12 middle school mathematics teachers, who graduated from the same 2-year long, technology-rich master's degree program in mathematics, integrate dynamic mathematics learning environments in their practice after graduating from the program two, three, or four years ago. The study also examined the different reasons for the decision whether to integrate those cognitive tools. Under the social constructivist theory, the more students interact with their social and physical environment, the more possibilities they have for constructing their own knowledge. The researcher of the study looked at teachers' behaviors and actions in the classroom while integrating technology in their practice, in an attempt to understand what they perceived as the role of technology when teachers facilitate activities that foster students' construction of knowledge.

Research Questions

The research question was: Why and how do middle school mathematics teachers integrate dynamic mathematics learning environments in their practices? The question is important because the environment teachers set up has a large influence in the learning experiences the students will encounter. The following sub-questions were addressed:

1. What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?
2. What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?
3. What do the teachers think they learned from the SMGEM program in terms of GeoGebra, and how are they using what they have learned?

Some of the internal factors that the study considered were teachers' concerns, teachers' beliefs about technology, beliefs about mathematics, beliefs about mathematics teaching and learning, and teachers' technological pedagogical content knowledge developmental stage. Some of the external factors were resources available to the teachers, school support, policies, and time since graduation from the SMGEM program. These factors are defined and described in the literature review chapter.

A qualitative approach was used for this study. Qualitative research parallels the study's social constructivist framework since qualitative research assumes that reality is socially constructed and that there is no single, observable reality (Merriam, 2009). The study consisted of an intensive, holistic description and analysis of how and why the 12 selected middle school mathematics teachers integrated cognitive tools in their practice.

Significance of the Study

Because of the relatively recent use of dynamic mathematics learning environments in the classroom and the few existing studies that provide a comprehensive view of the possible influence they can exercise in the mathematics curriculum and teachers' practices, the study attempted to understand what factors are perceived by the in-service teachers as important to the implementation of those cognitive tools in their practice. Also, with little research found on the implementation of cognitive tools in the classroom by in-service mathematics teachers years after they were exposed to the learning of those tools, this study served to lay a foundation for future research. Finally, the instrumental orchestration of the teachers while integrating cognitive tools like dynamic mathematics learning environments was examined and compared, thus giving future researchers some insight with which to perform more studies related to those theoretical frameworks. Providers of professional development also may use the results of this study to plan future sessions aimed at promoting the integration of cognitive tools in the mathematics classroom.

Summary

Although the use of technology as a learning tool in the mathematics classroom has been endorsed by professional organizations (Association of Mathematics Teacher Educators [AMTE], 2009; International Society for Technology in Education [ISTE], 2008; NCTM, 2000, 2008), its integration has been slow (Guin & Trouche, 1999). The role of the teacher in the implementation of technology has been acknowledged as both critical and problematic (Artigue et al., 2009; Fullan, 2008). In order to help teachers implement technology in a way that can benefit their everyday practice, it is important to have more knowledge about their perceptions of what is difficult about its implementation, if and how their practice changes in a technology-rich environment, and how the role of technology relates to teachers' beliefs about mathematics as a subject and about the teaching and learning of mathematics.

The literature reviewed in Chapter 2 examines the impact of cognitive technology in mathematics education, internal and external factors that may foster or prevent the integration of technology in the classroom, and possible changes that the implementation of technology can introduce in mathematics classrooms. By a review of the literature, the researcher intended to understand why and how middle school mathematics teachers integrate cognitive tools in their practices. Chapter 3 describes the methodology that was used in the study, as well as the different instruments used to collect data.

CHAPTER TWO: LITERATURE REVIEW

In an effort to improve mathematics education in the United States, professional organizations have promoted the integration of technology in the classroom with the goal of engaging students in the process of learning and understanding mathematics (AMTE, 2009; ISTE, 2008; NCTM, 2008). Researchers have recognized that effective integration of technology requires special knowledge of technology, pedagogy, and content (Mishra & Koehler, 2006; Niess, 2011). This special knowledge, called technological, pedagogical, content knowledge (TPACK), is "an understanding that emerges from interactions among content, pedagogy, and technology knowledge. Underlying truly meaningful and deeply skilled teaching with technology, TPACK is different from knowledge of all three concepts individually" (Koehler & Mishra, 2009, p. 7).

The purpose of the study was to describe, analyze, and compare information about the different ways in which middle school mathematics teachers implemented a specific cognitive technological tool: GeoGebra. Cognitive technology is a tool that helps transcend the limitations of the mind for students engaged in thinking, learning, and problem-solving activities (Pea, 1985). GeoGebra is a dynamic mathematics software that falls into the category of cognitive technology and has the potential for impacting mathematics teaching and learning, instructional approaches, and problem-solving activities. In the study, integration of technology was viewed in terms of teachers using technology to develop students' mathematical knowledge under a social constructivist learning theory, which proposes that students are responsible for their own learning, and the construction of knowledge is enhanced with peer cooperation.

The literature reviewed in this chapter examines the internal and external barriers that may foster or prevent the integration of technology in the classroom, and possible changes that the implementation of technology could introduce in mathematics classrooms. Throughout this review of the literature, the researcher intended to understand why and how middle school mathematics teachers integrated cognitive technology in their practices.

Cognitive Tools

The study focused on cognitive technologies. Cognitive tools are broadly defined as any technologies that engage and facilitate cognitive activities by enabling learners to represent what they know (Jonassen, 2000). Cognitive technologies challenge learners and can be used to promote deep reflection. Jonassen's definition, which is inclusive of many forms of technology, is based on the constructivist idea that students learn more from constructing and justifying their own ideas than from studying someone else's ideas. These technologies can change the nature of teaching and learning, as they enable new forms of activity (Hmelo-Silver, 2004).

Means (1994) classified cognitive technologies in four different types according to their instructional purpose: tutorial, communication, exploratory, and tools. Tutorials are those technologies that do the teaching and control what materials will be presented to students. Computer-assisted technologies, such as Math Blaster and Math Realm, are examples of tutorial technologies. Communication technologies, which include e-mail, videoconferencing, and the Internet, are used to send and receive messages. Exploratory technologies allow students to find information, facts, or procedures. Examples of exploratory technologies include encyclopedias, search engines, and hypermedia-based learning programs. Tools help students in the educational process by providing them with instruments to facilitate tasks. Some examples of tools include word processors, spreadsheets, Geometer's Sketchpad, Cabri Jr., and GeoGebra. These tools can enhance the learning environment with dynamic mathematics, and have the potential for impacting mathematics teaching and learning, instructional approaches, and problem-solving activities (Hohenwarter & Hohenwarter, 2009). For this reason, cognitive tools can be referred to as dynamic mathematics learning environment tools (Bu & Schoen, 2011). The study also used this classification, and specifically explored those cognitive technologies classified as tools and referred to them as dynamic mathematics learning environment tools.

Use of Technology in Today's Classrooms

Although more and more computers can be found in classrooms, their use has been scarce and disappointing (Reed, Drijvers, & Kirschner, 2010). Cuban (2001) argued that the introduction of computers in the classroom has not made a difference in teachers' instructional practices or in students' achievements. In his book *Oversold and Underused* (2001), Cuban described his qualitative study of three schools in which negligible changes in instructional strategy were observed.

Underutilization of computers can be found in other countries as well. Conlon and Simpson (2003) performed a similar study to Cuban's, but in Scotland. Their conclusions agreed with Cuban's: despite the wide availability of computers to teachers and students in and out of schools, computers generally are underutilized in classrooms.

Conlon and Simpson reported that students most often used the computers for word processing of essays and reports, as well as for Internet searches.

Becker and Ravitz (2001) suggested that with the fast evolution of technology and the widespread use of applications such as email, video editing, and the World Wide Web, Cuban's (2001) conclusions do not apply to contemporary schools. Becker and Ravitz (2001) surveyed over 4,100 teachers in over 1,100 schools across the United States. The results of the survey showed that only one out of nine of the mathematics teachers reported that students used computers on a weekly basis in their classes.

Cognitive Tools in the Mathematics Classroom

Technology is under-utilized in mathematics classrooms (Becker & Ravitz, 2001; Conlon & Simpson, 2003; Cuban, 2001); however, when it is implemented, it seems to have a positive effect on student achievement. The effect of technology is evident especially when students work in small groups and engage in student-centered activities (Becker & Ravitz, 2001; Carter & Smith, 2001; Li & Ma, 2010; Shapley, Sheehan, Maloney, & Caranikas-Walker, 2011). As demonstrated in Wenglinsky's study (1998), if technology is used for drill and practice, rather than for promoting higher-level skills, it can have a negative impact on student achievement. In the study, the researcher did not look at student achievement, but observed how technology was used in the classroom and what instructional methods the participating teachers used.

Teachers play an important role, and their attitudes toward technology influence the impact that technology has in the classroom. Teachers with a positive attitude toward the use of technology in the classroom have more organized teaching routines and develop activities that meet students' expectations; thus, they create classrooms with fewer behavior problems (Means, 2010; Shapley et al., 2011). Furthermore, teachers who demonstrate a positive attitude toward technology are more aware of their students' interactions with technology. Consequently, they tend to adjust their non-technology teaching time to address students' questions and problems during technology usage. The study examined teachers' attitudes toward technology as well as their attitudes toward mathematics teaching and learning. The researcher attempted to determine whether there was a relationship between teacher attitudes and technology usage. Studies related to teachers' attitudes and beliefs about mathematics teaching and learning were examined in the next section of the literature review.

Some of the teachers participating in the study had students with special needs in their classrooms. The literature reviewed showed that those students can benefit from technology, especially when the students work in small groups (Bottge, Heinrichs, Chan, Mehta, & Watson, 2003; Bouck, 2009; Engelhard, Fincher, & Domaleski, 2011; Li & Ma, 2010).

The impact of technology on student achievement is a complex topic, which is affected by many factors such as the activities used, the instructional methods, the attitudes of teachers, and the appropriate application of the technology. As Li and Ma (2010) indicated in their meta-analysis, good teaching cannot be replaced; in this information era, technology should be a necessary component of good teaching.

Technological Pedagogical Content Knowledge (TPACK)

The previous section indicated that there is some evidence that the use of technology impacts students' achievement in mathematics. The impact can be either positive or negative, depending not only on the affordances of the technology, but also on how it is implemented by teachers (Means, 2008). According to the National Council of Teachers of Mathematics (NCTM, 2000), "effective use of technology in the mathematics classroom depends on the teacher" (p. 25). In a technology-rich classroom, the teacher plays a crucial role in making decisions that affect students' learning in important ways. When teachers implement technological tools in their teaching practice, there might be a natural resistance due to the uncertainty that the technology might bring (Germann & Sasse, 1997). Teachers will question whether they have the appropriate knowledge required to use the technology and what that knowledge might be.

According to Koehler and Mishra (2009), the basic core components of effective teaching with technology are knowledge of content, pedagogy, and technology, along with the intersections among them. These core components and the ways they intersect with one another form what Mishra and Koehler (2006) have termed the Technological Pedagogical Content Knowledge (TPACK) Framework. The TPACK framework is an extension of the Pedagogical Content Knowledge (PCK) defined by Shulman's (1986) model. TPACK describes the knowledge necessary for teachers to teach effectively with technology. The framework also tries to explain and describe how technological knowledge is implemented and observed in practice (Koehler & Mishra, 2009).

The TPACK model consists of three main components and four areas of intersection, which constitute the seven components of the framework (see Figure 1). They are defined as: (a) content knowledge (CK): knowledge about the actual subject that is to be learned or taught; (b) pedagogical knowledge (PK): knowledge of the methods and processes of teaching, including classroom management, assessment, and student learning; (c) technological knowledge (TK): Knowledge about various technologies; (d) pedagogical content knowledge (PCK): blending of content and pedagogy into an understanding of how particular aspects of subject matter are organized, adapted, and represented for instruction; (e) technological content knowledge (TCK): knowledge of how technology can create new representations for specific content; (f) technological pedagogical knowledge (TPK): knowledge of how various technologies can be used in teaching and how it might change the way teachers teach; and (g) technological pedagogical content knowledge (TCPK): knowledge required by teachers for integrating technology into their teaching in their content area.

According to Mishra and Koehler (2006), teachers not only need to know their content and be aware of pedagogical methods, but they also need to know how the teaching and learning of the subject matter can be changed with the introduction of technology (Figure 4).

While using the TPACK framework in their study, Niess et al. (2007) observed how teachers learned about electronic spreadsheets and incorporated them as learning tools in their classrooms. Analyses of these observations led Niess et al. to propose a five-stage developmental model of TPACK to measure teachers' progress when learning to integrate technology in the teaching and learning of mathematics (Figure 5). The five stages in the model are recognition, accepting, adapting, exploring, and advancing. In the recognition (knowledge) stage, teachers consider technology another tool they can use in their practices. In the accepting (persuasion) stage, teachers need to decide if they will integrate the technology into their practices. In the adapting

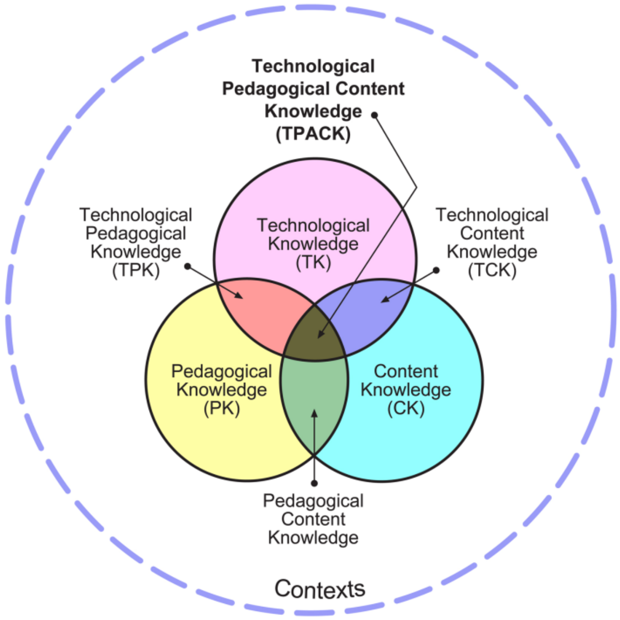

Figure 4. Technological Pedagogical Content Knowledge. TPACK model highlighting its knowledge components**.** **(**Retrieved from http://www.tpack.org/; reproduced by permission of the publisher.)

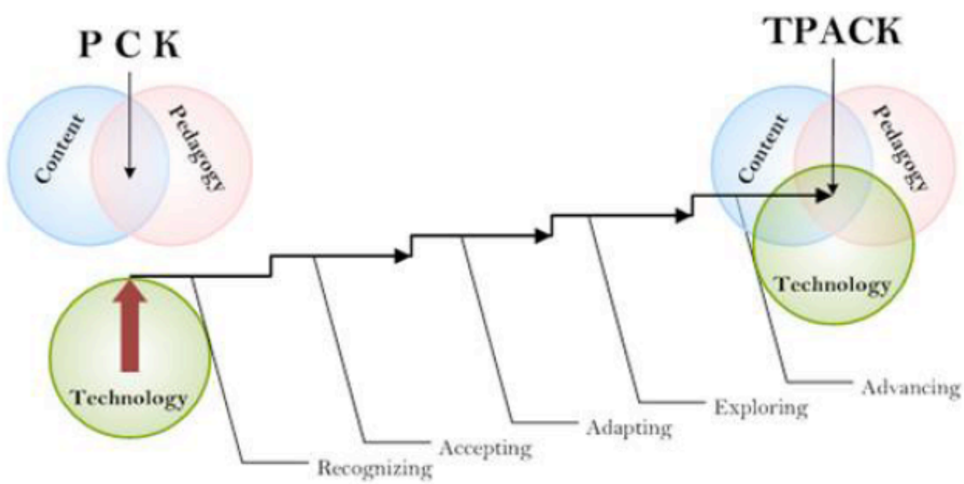

Figure 5. TPACK Development Stages. Visual description of teacher levels of development as their thinking and understanding merge toward TPACK (Niess et al., 2009).

(decision) stage, teachers begin to experiment with technology outside the classroom, without yet engaging students directly with the tools. During the exploring (implementation) stage, teachers investigate their curriculum in an effort to incorporate technology with their students. In the advancing (confirmation) stage, teachers reflect on the results of using technology in instruction.

The findings of studies assessing TPACK suggested that TPACK development seems to be directly related to PCK, since teachers with more content and pedagogical knowledge have a better understanding of how to integrate technology into their practice (Niess et al., 2007). These teachers, therefore, might develop TPACK at a faster rate than other teachers with less PCK. Development of TPACK seems to be related to specific mathematical concepts: as teachers move from one concept to another, their TPACK developmental levels might change (Ozgun-Koca et al., 2011). A solid TPACK might require a considerable amount of time (Koh & Divaharan, 2011) and factors such as support from administrators and peers can help with the advancement of this process (Ozgun-Koca et al., 2011).

The researcher of the study was interested in being able to identify the level of TPACK development of the participating teachers in order to understand how they integrate technology into their practice. To determine a teacher's level, the researcher observed and interviewed each of the teachers during two different lessons, with the goal of determining if the TPACK level is the same or if it is dependent on the mathematics activities and the teacher's content knowledge as related to those activities. Furthermore, in the study, the researcher considered the factor of external support. Interviews and observations of a teacher's lessons offered a dynamic process for assessing TPACK while offering the added benefit of contributing to the teacher's TPACK development.

Studies have suggested that effective assessment should be ongoing and repetitive, using a variety of data and more than one self-assessment survey. Angeli and Valanides (2009) used peer, self, and expert assessments on the design of instructional lessons enhanced by the use of tools in an attempt to determine teachers' TPACK. Groth, Spickler, Bergner, and Bardzell (2009) used lesson studies to assess teachers' TPACK development, relying on descriptions of teachers' classroom practices.

The interviews and observations in the study used the Mathematics Teacher TPACK Standards and Indicators (Appendix A) and the Mathematics Teacher TPACK Development Model (Appendix B) to guide interviews and observations (Niess et al., 2009). The standards and model were developed with the intention of determining the relationship between the mathematics knowledge and technology knowledge necessary for the classroom. The researchers developed standards and indicators based on four themes: (1) designing and developing digital-age learning environments and experiences; (2) teaching, learning, and the mathematics curriculum; (3) assessment and evaluation; and (4) productivity and professional practice. Niess et al. (2009) used the standards in a case study and determined that the standards alone did not provide information about how the knowledge is developed for appropriate use in the mathematics classroom. The researchers then designed a TPACK Development Model based on the five-level developmental model defined by Niess et al. (2007). The five levels (recognition, accepting, adapting, exploring, and advancing) are included in each of four themes (curriculum and assessment, learning, teaching, and access). Descriptors are included for each level within each theme, and mathematical examples are provided.

The model supported the study by including information about how technological knowledge is implemented and represented in the classroom. The suggested standards provided information on how teachers develop digital-age learning environments and experience. In addition, they illuminated the relationships between teaching, learning, and the mathematics curriculum as well as provided information about how teachers assess and evaluate using technology. Lastly, the standards indicated how technology helped teachers enhance their productivity. The development model provided information on teachers' TPACK levels with respect to curriculum and assessment, learning, teaching, and access to technology.

Concerns of Teachers

Even when teachers have the required knowledge, they still might react to the use of technology based on several other factors like their experience, self-confidence, personal preference, and teaching style. Depending on their reactions to technology, teachers may consider themselves qualified or unqualified to implement it. Consequently, the success of the implementation of technology is dependent on teachers' concerns and feelings; these concerns increasingly are seen as an important aspect of innovations in education (van den Berg & Ros, 1999). Learning to teach mathematics with technology—an innovation—is a developmental process that brings about reactions from teachers (Zbiek & Hollebrands, 2008). This section concentrates on teachers' reactions to or concerns about implementing technology in their practices.

Fuller, Parsons, and Watkins (1974) first discussed teachers' concerns in 1969 in their article "Concerns of Teachers: A Developmental Conceptualization." They indicated the need to consider the concerns of teachers, saying, "Concerns about teaching are expressions of felt need which probably possess motivation for relevant learning. If motivation is to be harnessed for learning, then curricula should consider the felt needs or concerns of teachers" (p. 2). Most of the authors' work was based on the concerns of pre-service teachers. Hall and Hord (2011) continued the work of Fuller et al. (1974) and developed a model known as the Concerns-Based Adoption Model (C-BAM), which characterizes most innovations and the change process in general. Hall and Hord described teachers' concerns as the feelings and perceptions they have about an innovation and/or a change process. They explained that these feelings and perceptions can evolve as the change process progresses. The C-BAM specifies the *Stages of Concern* (SoC) that teachers go through as they encounter innovations. The SoCs include unrelated, self, task, and impact concerns (see Table 1). Teachers who do not know or have little involvement with a particular innovation manifest unrelated concerns. Self-concerns can be informational or personal. Informational concerns arise when teachers take the initiative to learn about the innovation; personal concerns refer to those of teachers who are uncertain about the demands of the innovation, their inadequacy to meet those demands, and their role with the implementation of the innovation. Task concerns are related to management issues, which are manifested when a teacher's attention is focused on the processes and tasks of using the innovation and the best use of information and resources. The last stage of concerns, impact

Table 1

Stages of Concern about the Innovation with Typical Expressions of Concern

Areas	Stages of Concerns	Typical Expressions of Concern
Impact	Refocusing	• I have some ideas about something that would work even better.
	Collaboration	• How can I relate what I am doing to what others are doing?
	Consequence	• How is my use affecting students? How can I refine it to have more impact?
Task	Management	• I seem to be spending all my time getting materials ready.
Self	Personal	• How will using it affect me?
	Informational	• I would like to know more about it.
Unrelated	Unconcerned	• I am not concerned about it.

concerns, can be broken down into three sub-categories: consequence, collaboration, and refocusing. Consequence concerns are shown when a teacher's attention focuses on the impact of the innovation on students. Collaboration occurs when the focus is on coordination with others regarding the use of the innovation. Finally, refocusing is the category where the focus is on the exploration of more universal benefits from the innovation (Hall & Hord, 2011).

Three of the SoCs described by Hall and Hord—personal concerns, management concerns, and consequence concerns—helped organize the literature review in this section. These particular concerns were of interest to the study because the participating teachers already had professional development, which had introduced them to the innovation, GeoGebra. However, those teachers still have personal concerns that prevented them from using GeoGebra in their classrooms. Since all of the participating teachers are experienced mathematics in-service teachers, they had management concerns as well as consequence concerns. A review of the literature provided the researcher with information about teachers' perceptions of the internal and external barriers that foster or prevent the use of GeoGebra in the classroom.

Personal Concerns

Hall and Hord (2011) described a teacher in the personal concern stage as being uncertain about the demands of the technology, feeling inadequate to meet those demands, and being unsure of his or her role with the technology. Teachers at this stage might worry about having the interest and inclination to deal with the technology, as Wiske and Houde (1993) observed in their study of five high school mathematics teachers. In their qualitative study, Wiske and Houde studied five teachers from three different high schools, who were observed as they learned to use *Geometric Supposers* and implemented lessons in their classrooms. Teachers reported either being interested in the software, interested in providing more active inquiry activities to their students, or interested in engaging their visual learners. These initial interests drove their personal concerns. Teachers interested in the software were most concerned with the technology. Those teachers interested in inquiry activities were concerned with the pre-planned activities and, as a result, missed opportunities to see how the technology also could help with their lectures. The teachers interested in the visual aspect of the technology were concerned with this aspect only, and they subsequently missed some of the information about the pedagogical application of the software.

Teachers have many concerns about integrating technology when they are in the later stages of a successful teaching career. In a case study conducted by Mitchell, Bailey, and Monroe (2008), an experienced, mature mathematics teacher was observed during the process of making a paradigm shift to a technology-integrated pedagogy. The teacher expressed personal concerns about his ability to make the shift, and as the study progressed, the researchers could see the teacher's struggle. The teacher indicated that teaching with technology caused him stress since he was uncertain about the quality of the mathematics teaching he was providing. The teacher was not convinced that this new form of pedagogy would help the students learn the new concepts, and he believes that the time he and the students spent learning the technology would have a negative impact on the overall learning experience and on students' attitudes towards the subject.

Germann and Sasse (1997) conducted a study that monitored the changes in concerns of elementary and secondary teachers involved in a two-year program designed to integrate the use of computers in science teaching. During the first year of the program, teachers in cadre 1 were learning about the different technologies. During the second year of the program, teachers in cadre 1 served as mentors for teachers in cadre 2. The concern profile of teachers in cadre 1 indicated a decrease in personal concerns during the first year; however, the same teachers' profiles reverted to high personal concerns during the second year. The researchers explained that the mentoring of other teachers in the use of a large bundle of technologies may have caused the increase in personal concerns in the second year. As teachers were mentoring others, they were talking about the technologies; this created more demands for information, which increased their personal concerns.

When information about technology continually changes, personal concerns might remain high for an extended period of time. Wedman, Heller, and Strathe (1986) conducted a pre- and post-test of K-12 teachers taking a college course on computers in education. The teachers consisted mostly of non-users and, consequently, all exhibited high personal concerns. Although the teachers' concerns moved somewhat toward impact concerns, their self-concerns remained relatively high. One explanation given by the researchers for the teachers' high self-concerns was that it might have resulted from the nature of the technology, which constantly is changing and challenging the information demands of the user.

The literature showed that personal concerns vary according to the teachers, their experience working with technology, and their interest in using technology (Wiske & Houde, 1993). Experienced teachers may have stronger personal concerns about using technology in their practices than less experienced teachers, since experienced teachers have developed non-technology approaches over the years, which help students learn the concepts (Mitchell et al., 2008). The amount of time spent learning the technology, the personal effort necessary to integrate the technology, the amount of information that must be learned, and the rapidly changing technologies are all factors that may cause teachers to have strong personal concerns for a long period of time (Germann & Sasse, 1997; Mitchell et al., 2008; Wedman et al., 1986).

Management Concerns

Hall and Hord (2011) explained that teachers are in the management concern stage when their attention is focused on the processes and tasks of using the technology.

Teachers' concerns at this stage are centered on the best use of information, efficiency, organization, managing, scheduling, and time demands.

Mitchell et al. (2008) observed a mature teacher struggle while trying to change his teaching approach to include technology. The teacher was concerned about the slow nature of the material production, which resulted in his covering less content in the class with technology than he did when teaching the same class without the use of technology. The teacher had questions about which content to cover and how much of it he should cover, as well as how to assess the content with technology. In addition, the teacher questioned why it was necessary to make changes. It took time for the teacher's students to explore, develop new ideas, and become familiar with the technology.

Wiske and Houde (1993) observed five high school mathematics teachers dealing with management concerns as they faced decisions about lesson designs, managing interactions with students during the lesson, and mapping the overall structure of the course. When designing lessons for the students, the teachers did not provide enough structure for students to understand how to collect data, analyze it, and make conjectures. Some students did not have the prior knowledge necessary for the exploration of the activities designed by their teachers. The teachers in the study had difficulty leading discussions, which resulted from the use of technology with the students; as a result, they missed other teaching opportunities. This left the researchers to wonder whether the teachers were too preoccupied with the demands of the technology and the new approach to notice teaching opportunities. The teachers in the study also had difficulty giving the students control of class discussions and class time.

Teachers in Wiske and Houde's (1993) study had problems deciding on a teaching format when planning their lessons. The teachers had to make decisions about whether it was best to address the whole class or just circulate among the students as they worked together in small groups. The software prompted teachers to change the sequence in which they presented topics. For example, when investigating triangles, the teachers had to decide either to include the concept of area and provide related activities at that particular time or follow the book sequence and present the concept months later.

The way in which teachers use technology seems to be related to teachers' management concerns. Tharp, Fitzsimmons, and Brown Ayers (1997) investigated teachers using graphing calculators in pre-calculus classes and found that a teacher's use of calculators is related to their teaching methods. Teachers who perceived the graphing calculator as a computational tool stressed teacher-centered activities, while teachers who viewed the graphing calculator as an instructional tool used student-centered approaches.

The ways in which teachers use technology in the classroom do not seem to be related to how they used technology as learners. Zbiek (1998) observed an algebra teacher who recalled enjoying and valuing open-ended explorations with technology as a learner. However, that same teacher did not feel comfortable using those types of activities in her own classroom; in order to fulfill her need for an orderly classroom, she provided detailed directions for technology use to her students.

In summary, teachers at the management concern stage struggle with designing lessons, pacing course content, managing interactions with students during lessons, and mapping the overall structure of their courses (Mitchell et al., 2008; Wiske & Houde, 1993). The way teachers perceive the usefulness of technology as an instructional or computational tool has an impact on classroom activities and overall classroom management (Tharp et al., 1997). Some teachers, however, do not feel comfortable introducing student-centered activities when using technology, even if they enjoyed learning the concepts with the use of technology when they were students (Zbiek, 1998). Wiske and Houde (1993) argued that when technology is used in the classroom, the approach is more student-centered, and teachers have to make a series of decisions that are not present in teacher-centered approaches. For example, teachers must know how to develop students' abilities to understand inductive reasoning, foster students' confidence and skills as members of a group of investigators, and help students express their own ideas.

Consequence Concerns

Hall and Hord (2011) described teachers in the consequence concern stage as being focused on the technology and its impact on their students' learning outcomes. At this stage, teachers might revise their use of technology according to the reactions of the students as well as to concrete results after using the technology, such as test scores of students' work.

Van den Berg and Ros (1999) performed a longitudinal study, which observed ten elementary schools, one high school, and one vocational school implementing adaptive teaching. Adaptive teaching is a form of independent learning, in which students are taught at their individual levels as they work independently. Adaptive learning includes two major elements: a change from teacher-centered to student-centered activities and a change to more active learning. In this study, the general distinctions among the personal, management, and consequence concerns were measured through questionnaires and interviews performed at the beginning of the study and then at the end of the study, two years later. The researchers expected that teachers in the high school would be at the consequence concern stage, since they had implemented the innovation before the beginning of the study. These particular teachers showed a low level of personal and management concerns; their concerns, as expected, were in the consequence category. Teachers in the high school studied indicated that they were concerned about the evaluation methods used in this innovation and about their students' learning processes. In addition, the teachers voiced concerns about other colleagues not implementing the innovation as they were supposed to, and they indicated that other schools were using the innovation with better results than they were observing (van den Berg & Ros, 1999).

Wiske and Houde (1993) found that as teachers cope with their personal and management concerns, they slowly move to the consequence concern stage. The teachers studied by Wiske and Houde began worrying about students' learning of the traditional course materials as measured by standard achievement tests. They also expressed concerns about how well they had covered the traditional curriculum and how adequately students could reason inductively in order to understand and apply the new process of learning mathematics. In general, the teachers in Wiske and Houde's study who were using the *Supposers* software expressed three main concerns: time, assessment, and authority.

With regard to time, teachers were worried about finding time to prepare for the innovation by developing a new set of materials and activities and including time after class discussions for students to reflect on what they had learned. Teachers felt that the structured schedule of secondary schools is better suited for teacher-controlled lessons than for student-centered, inquiry-oriented ones, since once students start working on a problem, teachers lose control over the focus and pace of their thinking. Teachers had to alter the pre-determined number of weeks that were supposed to be devoted to certain topics, and they also had to alter the sequence in which topics were presented.

Assessment was another worry expressed by the teachers in Wiske and Houde's (1993) study. Teachers wondered what the students were learning and how they could assess student learning in a way that reinforced the purpose of the course. When giving computer-based assignments, teachers needed to define criteria for assessing students' work. As the teachers began to clarify the skills and knowledge they wanted their students to master, they developed the ability to design tests and assignments to match and support the mastery of those skills and knowledge.

Finally, the worries expressed by the teachers in Wiske and Houde's (1993) study revealed to the researchers that the teachers were experiencing a shift in authority in their geometry classes, which was manifested in three different ways. First, teachers exercised more authority over the course content when they used their own judgment in deciding how geometry should be taught, sometimes disregarding the textbook and curriculum guides. Second, students exercised more authority as makers of their own knowledge rather than passive receivers of knowledge transmitted by others. Third, a shift in authority originated from the structure of the *Supposers* software. For example, the software menu included "inversion," which is a topic rarely taught in traditional geometry courses. As teachers and students became interested in this topic, they challenged the textbook as the curriculum authority of the course.

In the case study conducted by Mitchell et al. (2008), the mature teacher was having difficulties changing from a traditional lecture-based teaching approach to a technology-integrated approach, since he had taught in the traditional way with satisfactory results for many years. The researchers reported that while a shift in teaching approaches is necessary, it also is the biggest challenge to teachers introducing alternative teaching strategies. The perception of the teacher in the case study was that the time spent learning the technology would reduce students' overall learning. The researchers provided the teacher with a technology-literate student teacher who could assist him with integrating the technology. The teacher and student teacher shared teaching responsibilities and collaborated on the planning, implementation, and discussion related to the change process. At the end of the project, the mature teacher became the faculty technology integration advisor for his school. Another teacher in the same school became interested in the project and followed the project path. Mitchell et al. (2008) pointed out that the new teacher was more computer literate than the mature teacher and had more knowledge in alternative teaching methods. With this difference in skills and attitudes, the new teacher required less support in the integration of technology than the mature teacher.

Summary

The literature review revealed that when implementing technology in their practice, teachers experience concerns before, during, and after implementation. Hall and Hord (2011) classified teachers' concerns into different stages as defined in their

Concerns-Based Adoption Model (C-BAM). For the study, personal, management, and consequence concerns were of particular interest, since those were the concerns that the researcher expected the participating teachers would express.

At the personal concern stage, teachers worry about not having enough information about the technology and feel uncertain about the changes they will have to implement into their practice with the use of that technology (Wiske & Houde, 1993). Teachers feel inadequately prepared to use technology in their classrooms, and they worry about spending time to learn how to use it (Mitchell et al., 2008). When the technology changes, teachers need to learn new information; this process creates new personal concerns, which might limit the use of technology in the classroom (Germann & Sasse, 1997; Wedman et al., 1986).

At the management concern stage, teachers most consistently indicate that time to prepare and the availability of computers and software represent the most serious barriers to their teaching (Germann & Sasse, 1997; Mitchell et al., 2008). Teachers also are concerned about their lesson designs and new course structure, since technology might offer opportunities to cover material in a sequence that differs from what is recommended by the textbook (Wiske & Houde, 1993). Technology usage in the classroom promotes discussion among students; teachers need to learn how to promote and guide the discussions, while letting students control what is discussed. Some teachers feel uncomfortable having these discussions or implementing student-centered activities, even if they liked those approaches as learners (Wiske & Houde, 1993; Tharp et al., 1997; Zbiek, 1998).

At the consequence concern stage, a teacher contemplating technology integration may experience a fear of instructional change, which relates directly to the time and effort needed and the potential benefits that could be obtained if these changes are to be sustained over time. Teachers exhibit concerns about changing their instructional approaches from a traditional format to a more student-centered approach (Mitchell et al., 2008; van den Berg & Ros, 1999; Wiske & Houde, 1993). Teachers also are concerned about the time they spend using technology in the classroom and how much longer it might take to cover concepts than it previously took with traditional approaches. Teachers also express concerns about their assessment of student learning (van den Berg & Ros, 1999; Wiske & Houde, 1993), as well as their authority and control over the curriculum, lesson design, and classroom discussions (Wiske & Houde, 1993).

Participating teachers in the study already had information about the dynamic mathematics learning environment that this study focused on: GeoGebra. The researcher investigated whether the teachers felt they had enough information to implement the technology and whether they still had concerns about the possible changes it may bring in their practices. The researcher took note of the amount of computers available to the teachers, the time it took for the teachers to prepare a technology activity, and the time it took for the students to start working on the activity. After observing a technology lesson, the researcher interviewed the participating teachers and investigated whether they felt the use of technology forced them to change their instructional approaches in a way that made them feel uncomfortable with the new teaching situation.

For an innovation to be successful, the needs and worries of the teachers should be the upmost priority. Teachers are likely to struggle with paradigm shifts, and careful consideration needs to be given to the actual problems and worries teachers may have. Teachers' concerns determine whether the technology is implemented in the classroom and how effective the implementation is with students' achievement. The researcher of the study used interviews to determine teachers' concerns using the *Stages of Concern* (SoC) survey developed by Hall and Hord (2011).

Teachers' Beliefs about Technology, Mathematics, and Teaching and Learning Mathematics

The Common Core State Standards Initiative (2011) address the practical need for students to use technological tools in order to explore and deepen their understanding of mathematics concepts. As more and more schools implement technology in their programs, it is becoming clear that the teacher is the critical component in the successful implementation of technology in the classroom (Sugar, Crawley, & Fine, 2004). Fullan (2008) argued that any educational change, such as the implementation of technology, is highly dependent on what teachers do and think; teachers must be convinced of the advantages of using technology before they implement it. Fullan suggested that implementation of any innovation is multidimensional, consisting of materials, skills, and beliefs. Beliefs, according to Fullan, are at the heart of implementation. Bitner and Bitner (2002) also agreed that a key factor in the successful implementation of technology is the understanding of the teachers' beliefs toward technology in their classrooms.

Although computer tools have become more popular in schools, the actual benefits of using such tools have been disappointing (Reed et al., 2010). The use of technology is believed to enhance the learning experience, increase students' understanding, and foster critical thinking skills; however, when technology is used mostly for drill-and-practice, these positive effects are not generated (Hennessy, Ruthven, & Brindley, 2005). In 2009, the U.S. Department of Education published a report titled "Evaluation of the Enhancing Education Through Technology Program: Final Report." The report used data collected from 4,934 teachers who were surveyed in the 2004-2005 school year and 1,515 teachers surveyed in the 2006-2007 school year. One of the key findings in the report was that only 35% of the surveyed teachers reported using technology with their students on a weekly basis with the purpose of practicing or reviewing mathematics concepts (U.S. Department of Education, 2009).

This section of the literature review examined teachers' beliefs when incorporating technology into their practices. The goal was to understand what the literature says about teachers' beliefs about teaching mathematics with technology, about mathematics, and about the teaching and learning of mathematics. *The Stanford Encyclopedia of Philosophy* defines belief as the psychological state in which an individual holds a proposition or premise to be true, while an attitude is a hypothetical construct that represents an individual's degree of like or dislike for something based on the individual beliefs (Schwitzgebel, 2011). The researcher of the study investigated the participating teachers' beliefs about the use of GeoGebra in their practice with respect to technology, mathematics, and the teaching and learning process.

Teachers' Beliefs about Technology

Teachers' beliefs about technology influence the types of activities they create for their mathematics students. In a qualitative study, Noss, Hoyles, and Sutherland (1990) observed three high school teachers who participated in the same yearlong course on using computers in the classroom. Researchers collected data through observations, interviews, and analysis of project work and observed that there was a relationship between teachers' attitudes and the attitudes they projected to their students. For example, teachers who initially experienced anxiety with technology believe their students would experience the same feelings; subsequently, they proposed activities that would help reduce the students' anxiety. Teachers who liked structured activities and step-by-step directions argued that the students would benefit from a similar approach.

Noss et al. (1990) classified teachers as proactive or reactive, according to their motivation for using technology in the classroom. If a teacher's motivation for using technology is to find new ways to infuse their practice with innovative approaches that motivate and engage their students, they are considered proactive. Reactive teachers are those who are interested in learning how to use the technology because the school or district requires them to implement the innovation. The proactive teachers see technology as an instructional tool, while the reactive teachers see it as a computational tool. This distinction regarding the use of technology was of interest to the study. Since GeoGebra is a cognitive tool intended to be used as an instructional tool, the researcher was interested in finding if the participating teachers were reactive or proactive.

Other researchers have suggested that there is a relationship between the role of the teacher in the classroom and the use of technology. Bottino and Furingetti (1996) observed five secondary school teachers learning how to use technology in their classroom. After observing the teachers in the classroom and conducting interviews with them, the researches indicated that teachers who believe they are the authority in the classroom tend to treat technology as a computational tool, provide their students with detailed instructions about the use of technology, and believe students must understand the mathematical concepts before exploring them with technology. The researchers wondered if that was the reason why the teachers believe technology is better suited for upper-level high school mathematics.

Although the two studies analyzed in this section had sample sizes of only three and five mathematics teachers, their observations indicated that there might be a connection between teachers' beliefs about technology and the ways they implement it in the classroom. If teachers believe that technology is a computational tool, they are more likely to provide very structured and detailed activities for their students. These teachers would prefer to use technology after students have learned the mathematical concepts. If teachers believe technology is an instructional tool, their activities are more likely to be less structured and more student-centered. In the study, the researcher surveyed and interviewed teachers with the intention of identifying their beliefs about technology use in the classroom, and compared their beliefs to their practices.

Teachers' Beliefs about Mathematics

Some teachers believe that mathematics is a body of knowledge that must be transmitted from the teacher to the students, while others think mathematics should be explored and discovered. Noss et al. (1990) found in their study that for teachers who believed mathematics was everywhere (ethnomathematics), technology offered the opportunity to extend their activities and created the possibility of discovering more mathematics. The teachers who were more curriculum-focused saw technology as another vehicle for introducing curriculum content.

Curriculum-focused teachers, or rule-based teachers, discourage engaging in exploration and discovery in their lessons and prefer instead to give lectures. Tharp et al. (1997) worked with secondary teachers in a telecourse on calculators in mathematics and science. Their analysis of questionnaires and journals revealed that rule-based teachers believe that calculators would hinder instruction rather than help it. These teachers expressed a need to control all activities in the classroom, and they discouraged any inquiry-based lessons. The researchers of this study argued that teachers prefer control over the classroom in order to avoid the embarrassment of having students find uses of the calculator that are beyond their own scope of knowledge.

Akujobi (1995) found a similar pattern while examining teachers' knowledge and beliefs about the use of computers in high school mathematics. Teachers who held a conceptual view of mathematics supported activities that integrated technology in the classroom. Teachers with rule-based beliefs, or computational beliefs, tended to avoid the use of technology and assigned it exclusively for drill-and-practice or remediation purposes.

It seems that teachers' views of school mathematics influence how they use technology in the classroom. As suggested by Noss et al. (1990), Tharp et al. (1997), and Akujobi (1995), teachers who view mathematics within the parameters of what is in the curriculum tend to be ruled-based and limit the use of technology in their classrooms. As a result, students in those classrooms are less likely to encounter exploratory activities outside of what is found in the textbooks. It would seem that technology could have a negative effect on teachers and students in those classrooms in which both the teachers and the students struggle to use technology to its fullest potential to achieve the mathematical goals that teachers believe can be obtained without technology. In the study, the researcher tried to identify the views of mathematics held by the participating teachers, since those views seem to influence the use of technology in the classroom.

Teacher Beliefs about Mathematics Teaching and Learning

Teacher beliefs about how learning occurs and how they foresee teaching to achieve learning influence how they use technology to teach mathematics. Three teachers in Heid, Blume, Zbiek, and Edwards' (1999) study described themselves as facilitators of students' learning; however, their actions did not agree with this description. Heid et al. described first-year algebra teachers' experiences using Computer Algebra Systems (CAS) for the first time. Teachers in the study were more interested in correct answers than in the process of students' construction of understanding. The researchers showed video clips of classroom instruction to the teachers in the study and later interviewed them about how students' responses were targeted during the lesson. The teachers in the study confused the role of facilitating with that of questioning, as they praised the teachers in the video clip for leading students to the correct answer rather than having students articulate their strategies and reasons for giving an answer. Teachers who believe that the goal of learning is for students to reach correct answers prefer to use technology as a computational tool.

Teachers also can be influenced by their estimation of students' potential, without realizing that unintended learning can occur when using technology. In Heid et al.'s (1999) study, teachers could predict the success of their students on tests by looking at their different representations and CAS use, because they believe students could know only what teachers had taught them. However, teachers were surprised when students demonstrated knowledge of concepts not explicitly taught in class.

Some teachers modify their use of technology to match their perceptions of students' abilities and avoid things they perceive to be beyond those abilities. The teacher referred to as *Teacher B* in Kendal and Stacey's (2001) study believed his calculus students were less capable than students in other classes and therefore they would be confused by the variety of CAS symbolic representations. Teacher B saw the use of CAS with this particular group of students as an intrusion to learning; therefore, he only used it to show graphs. Kendal and Stacey observed three high school teachers for two years while implementing CAS activities with their students. The strategies that these teachers favored remained consistent over time, and the methods used by their students to solve mathematical problems reflected the strategies expressed by the teachers.

The activities teachers create for their students may be related to their experience with using technology and teaching mathematics. Laborde (2001) examined the activities created by teachers who had different experiences teaching mathematics and using Cabri, a dynamic geometry program, during a three-year project. The group of teachers included one novice teacher who had experience with Cabri, two experienced teachers who were familiar with Cabri but had never incorporated it into their practice, and one experienced teacher who had no knowledge of the software. Based on how Cabri was used, Laborde classified the activities generated into four categories: (1) to produce more quickly a drawing that could be used to solve a problem, (2) to assist students in generating conjectures, (3) to increase the level of thinking that is needed to complete a task using technology instead of paper and pencil, and (4) to create a problem that is only meaningful in the technological environment and can be solved with technology.

At the beginning of the project, only the novice teacher created activities of the first type; all of the teachers created activities of the second type. Toward the end of the project, only the experienced teachers created activities of the third and fourth type. Laborde (2001) suggested that it is easier to adapt paper-and-pencil activities to technology use than it is to create novel activities that are different in nature from what one might do with paper and pencil.

The studies reviewed in this section suggested that what teachers do in their classrooms is influenced by their beliefs about learning, their students' abilities, and their own teaching experiences. In the study, the researcher observed the teachers, attending to their questioning strategies, to determine whether they were looking for correct answers or processes. The researcher inquired about the teachers' perceptions of their students' abilities and about the technological activities used with those students.

Summary

This brief review of the literature provided insight into teachers' beliefs about technology, mathematics, and the teaching and learning of mathematics. Teachers see technology as either an instructional tool or a computational tool, and this view is related to their teaching style. Teachers who favor structured activities tend to see technology as a computational tool and provide students with detailed instructions for its use. These teachers prefer to use technology after students master the concepts, and they elect to use it for remediation and drill-and-practice activities (Bottino & Furingetti, 1996; Noss et al., 1990).

Teachers' beliefs about mathematics also influence their choice of technological activities. If teachers believe mathematics is everywhere and must be discovered and explored, they prefer inquiry-based activities and see technology as an instructional tool that can motivate and engage students. If teachers believe mathematics is a fixed body of knowledge that teachers transmit to students by covering the curriculum, they favor structured activities and use technology mostly for drill-and-practice activities (Akujobi, 1995; Noss et al., 1990; Tharp et al., 1997).

Beliefs about the teaching and learning of mathematics also influence the use of technology in the classroom. If teachers believe students construct their own knowledge and see themselves as facilitators of learning, they will use inquiry-based activities (Heid et al., 1999). Teachers accommodate the activities according to their conception of their students' abilities, limiting the use of technology for less capable students (Heid et al., 1999; Kendal & Stacey, 2001). Teachers' experiences with technology and with teaching mathematics also influence the activities they use. Experienced teachers—even those who are not too familiar with the software—can provide students with challenging activities better than novice teachers can. However, even for experienced teachers, creating new and innovative activities that are different from the typical paper-and-pencil activities can be a challenge (Laborde, 2001).

In the study, the participants were experienced teachers who have been experimenting with GeoGebra for two, three, or four years. Through surveys and interviews, the researcher inquired about the teachers' beliefs regarding technology, in order to determine whether they see GeoGebra as a computational tool or as an instructional tool. Through observations, the researcher determined whether teachers believe mathematics should be explored by students or transmitted. Additionally, the researcher determined, through observations, whether teachers believe they are facilitators of learning, and also explored what types of technological activities they provided to their students. Analysis of these interviews and observations helped the researcher understand if existing teachers' beliefs influence the adoption of technology in their classrooms.

The review of the literature in the areas of teachers' knowledge, concerns, and beliefs helped answer the questions in the study about why and how middle school teachers integrate GeoGebra in their practices. More specifically, it gave the researcher information on what internal barriers teachers perceived as fostering or preventing the use of GeoGebra in their classrooms and whether the teachers' teaching philosophies were related to their use of technology.

Teachers' Implementation of Cognitive Technology in the Mathematics Classroom

The previous section of this literature review identified the types of knowledge teachers need to have before implementing technology in their classrooms and the concerns and beliefs they have about using technology. Once teachers decide to implement technology in their classrooms, the potential for change is introduced. This section of the literature review examined the possible changes that technology can bring with regard to the mathematics curriculum, the nature of the classroom environment and tasks, and the new roles and responsibilities assumed by teachers and students when using technology. This review of the literature helped the researcher of the study to determine how the participating teachers integrate GeoGebra—the technological cognitive tool of interest in the study—in their practices. Additionally, the researcher was able to identify the similarities and differences in how the teachers use GeoGebra in their classrooms.

Changes in Curriculum

The use of technology in mathematics classrooms can change the instructional activities in two ways: it can increase the number and range of examples that students can come into contact with, or it can alter the arrangement of the mathematics content. The first type of technology is called an amplifier (Pea, 1985); it is used mostly for drill-and-practice or remediation. The second type of technology, called a reorganizer (Kilpatrick & Davis, 1993), is used for investigation and exploration. Kilpatrick and Davis (1993) alleged that technology organizers can change fundamental questions for those considering mathematics curricula, such as what is the mathematical knowledge that tomorrow's society will demand, and what mathematics students should learn in order to be wise citizens of that society.

When using reorganizers in their classrooms, teachers are faced with a series of new issues (Heid, 1997). One of those issues deals with the composition of the curriculum and the teachers deciding on the proper mixes and sequences of skills and concepts taught in their classrooms. Teachers worry that the valuable activity of experimenting with mathematics and making conjectures will replace the need for deductive mathematics (Kilpatrick & Davis, 1993). There are some valid concerns about what students will not learn if technology is used in the teaching of mathematics. Some fear that calculators will replace students' mental computational abilities and basic skills (Schmidt, 1998); others fear that those untaught skills will be needed in future courses (Kysh, 1991).

GeoGebra can be considered a reorganizer, since it can change the mathematics content, combining different areas of mathematics in one simple activity. For example, when using GeoGebra to find the perimeter of a polygon, an invaluable discussion on rounding irrational numbers can be brought to the surface, therefore combining notions of number theory and geometry in one activity in a dynamic and interactive way. For this reason, this section of the literature review looked at how the implementation of technology reorganizers can change the mathematics curriculum.

Potential for Impact on Curriculum

Fey, Hollenbeck, and Wray (2010) believe that when considering technological opportunities, a reevaluation of the content of mathematics curriculum is necessary. The reasons given by the authors were that technology can bring many changes such as: (a) altering the order in which things are taught, (b) generating real mathematical experiences, and (c) making the study of algorithms more important.

There can be fundamental changes in the order in which concepts are presented, with concepts and applications being taught before skills (Heid, 1997). Heid (1997) examined the effects of initially focusing on concepts and applications in a calculus course using Computer Algebra Systems (CAS). Interviews, classroom interactions, and test results showed that students using CAS understood mathematical concepts more robustly than students in a traditional calculus course. After 12 weeks of using CAS, the students had three weeks of traditional calculus skills instruction. Both groups of students—the CAS users and those in the traditional course—had comparable results on the final calculus examination. Heid concluded that attending to concepts and applications first does not translate into a loss in manipulative skills.

Mathematical topics—such as matrices, sequences, and functions—that are presented to students in their later schooling years now can be presented to the students earlier. Sutherland and Rojano (1993) worked with 10- and 11-year-old Mexican and British students who had no previous formal algebra instruction. At the end of 12 hours of hands-on spreadsheet time (one hour per week for 12 weeks), the students were able to move to an algebraic way of thinking. The researchers worked with students in groups of two or four, encouraging them to work independently while using a range of numbers as input to express their relationship symbolically. A comparison of pre- and post-treatment interviews showed that students improved in their understanding of functions and algebraic symbolism.

The second reason for curriculum reevaluation is that technology provides students with the opportunity to collect data and process it more easily, allowing them to engage in real mathematics modeling experiences. Boyd and Rubin (1996) worked with two sixth graders as they experimented with motion through interactive video. The researchers interviewed the students three times after they watched QuickTime digitized videos and created graphs representing what they had seen. The students were able to find the connection between the everyday world shown in the video and the world of mathematical representations; they also found the relationship between relative and absolute measurements.

The third reason for curriculum reevaluation is that technology makes the study of algorithms more important. An algorithm is a finite, step-by-step procedure for accomplishing a task (Usiskin, 1995). Usiskin (1995) recalled that when paper was introduced in Europe in the 10^{th} and 11^{th} centuries, it was considered the advanced technology of the time. People feared that a loss of mental power would result from using paper-and-pencil algorithms rather than mental arithmetic. Even though paper-and-pencil algorithms ultimately became popular and were adopted by almost all mathematicians, it still was expected that formulas and facts be memorized. Usiskin argued that although calculators and technology eventually will overtake paper-and-pencil algorithms, some still will remain in the curriculum—not because they train the mind, but because they are good algorithms.

The use of technology in the classroom can reorder the topics covered in mathematics classrooms (Heid, 1997), introduce new concepts to students at a younger age (Sutherland & Rojano, 1993), give students opportunities to experiment with real world information (Boyd & Rubin, 1996), and allow teachers to rethink which algorithms are important to continue doing with paper and pencil (Usiskin, 1995). These changes can cause a radical rethinking of the mathematics curriculum (Heid, 1997). In the study, the researcher interviewed the teachers to determine whether they believe that the use of technological activities altered the curriculum and the content they were required to cover. The researcher also observed the teachers in their classrooms while they used technology, to identify whether any alterations to the curriculum were made.

Specific Impact on Current Mathematics Courses

As demonstrated in the previous section, technology can bring about changes in the mathematics curriculum, and those changes already can be seen in the strands of algebra, geometry, and calculus. This section will explore some of those changes, especially in the areas of algebra and geometry, since the teachers in the study teach in middle schools and calculus is not part of the middle school mathematics curriculum.

Algebra examples. The practitioner journals published by the National Council of Teachers of Mathematics (NCTM) provide teachers with articles that focus on algebraic thinking characterized by the use of technology in the exploration of patterns and relationships between the real world and the world of mathematic representation. However, the algebra curriculum is filled with symbolic manipulation. There are several types of technology available for the teaching of algebra, the most popular of which are computer algebra systems, graphing tools, and multiple representation tools (Heid, 1997).

Computer Algebra Systems (CAS) are computing tools that perform most graphical, numerical, and symbolic routines (Heid, 1997). There is ongoing debate about the advantages and disadvantages of students using computing tools. Some feel that students who use computing tools will not know how to do symbolic manipulation by hand, a skill required for success on standardized tests (Waits & Demana, 1992). Waits and Demana (1992) argued that students will miss the concepts behind the symbolic manipulations when they perform them by hand. In contrast, Dick (1992) explained that with the time students save by not doing tedious manipulations with calculators, they can spend more time understanding the meaning of the symbols and the notations they use. Waits and Demana (1992) also argued that the exact answers produced by calculators are of no real use, to which Dick (1992) replied that precision in arithmetic can be more relevant than approximations, and students can focus on studying the patterns of those answers.

CAS technology can play a role in the conceptualization of problems, since it is more than just a tool to aid with the solution to the problem. Geiger, Faragher, and Goos (2010) performed a one-year study of three algebra teachers who were implementing CAS in their classrooms. The researchers observed that the solution given by the technology prompted students to re-evaluate fundamental assumptions they had made, while also allowing them to reformulate, solve, interpret, and evaluate the problem with the revised assumption. Teachers were able to generate a list of error messages given by the technology, which made them aware of students' misconceptions. With that awareness, teachers were more informed about how to prepare their next lessons; however, none of the three teachers believed that the opportunity for error analysis could have been created ahead of time since they believed the errors were accidental.

Graphing tools and multiple representation tools are the second type of technology used in algebra courses; of those, graphing calculators are the most commonly used. Graphing calculators can display the graphs, equations, and value tables of functions dynamically, making them a tool that generates a mathematically rich environment for learning about functions.

Dewey, Singletary, and Kinzel (2009) surveyed 109 Algebra I and Algebra II teachers to determine the status of the teachers' graphing calculator use, the changes being made to the algebra curriculum due to the use of graphing calculators, and the characteristics of those teachers who were using the technology. The researchers reported that while 78% of the respondents had access to the technology, only 28% were using it regularly. The Algebra II teachers in the study used calculators more frequently than the Algebra I teachers, and it was found that older teachers with more years of teaching experience more commonly incorporated calculators in their teaching. The teachers in this study believed that the graphical solutions to problems were secondary to the symbolic solutions. In addition, the teachers believed that graphing calculators supplemented their instruction but did not expand the curriculum.

In some programs, the algebra curriculum has been changed due to the influence of technology. Star, Herbel-Eisenmann, and Smith (2000) compared one school's Algebra I curriculum with its eighth grade algebra curriculum, based on the Connected Mathematics Project (CMP), a project that includes many interactive activities for the students. The researchers pointed out some differences between the traditional conception of teaching algebra and the one introduced by CMP. For example, in the regular algebra, the fundamental objects in the curriculum are equations and the symbolic representation while, in the CMP, the fundamental objects are fundamental relationships represented in tables, graphs, and equations. The elements of a typical lesson in algebra are review homework, present new content, and allow time for work on the next assignment. The elements in a typical lesson in the CMP are small group work and whole class discussions, with some mix of teacher presentations.

In summary, the most commonly used technologies in algebra courses are CAS and graphing calculators. Debate still exists about the costs and benefits of using technologies like CAS in the classroom, with some people believing it can deter the learning of concepts (Waits & Demana, 1992) and others believing it can enhance it (Dick, 1992). Even though teachers see the benefits of using CAS-based technologies, they still think that their experiences with helping students analyze their errors were accidental and could not have been planned ahead of time (Geiger et al., 2010). Graphing calculators are used mostly by Algebra II teachers; however, teachers believe this technology only supplements instruction and does not change the curriculum (Dewey et al., 2009). Mathematics programs like the Connected Mathematics Project have created changes in the algebra curriculum, with technology being used to support students' work (Star et al., 2000).

Some participating teachers in the study were Algebra I teachers. GeoGebra is a form of software that can operate as a graphing calculator, providing graphs, equations, and tables of functions. The new version of GeoGebra (4.2) also includes a CAS application. The capabilities of the GeoGebra software make it a valuable tool in the teaching of algebra concepts. The researcher of the study observed whether the teachers use the GeoGebra software when teaching algebra concepts, while also attending to how the software was used by the students.

Geometry examples. Geometry construction tools such as Geometric Supposer (developed in 1985), Cabri Geometer (developed in 1988-92), the Geometer's Sketchpad (developed in 1992) and GeoGebra (developed in 2001-02), can turn classrooms into laboratories for the discovery of geometric relationships by providing students with opportunities to create and manipulate geometric figures while retaining the essential characteristics of those figures. The three tools previously mentioned have the capability to drag the figures, which is the property that allows users to move, dilate, translate, or rotate figures without changing their significant properties.

The tool's dragging capability allows users to differentiate between constructions and drawings. Drawings look like the real figures, but when they are dragged, they do not retain the essential elements that proper constructions do. Scher (2005) conducted two interviews with 11 middle school students who had no previous interactive geometry experience. In one of the interviews, the students were asked to construct a square using the tools provided by the Geometer's Sketchpad. Some of the students' work did not display construction standards for a square to remain a square when dragged, but it still contained some geometric properties. The activity allowed the teachers to study students' work and determine which students needed more help with the properties of a square. Based on the interviews, the researcher recommends that teachers have class discussions about the merits of each construction, based on the belief that student-led critiques may be more effective than teacher-imposed definitions.

The use of technology in the classroom creates a new challenge for teachers who must find the balance between formal proofs and the role of evidence in a technology-rich classroom. Butcher and Edwards (2011) explored rigid motions using GeoGebra, through the application of 'what if not' questions that can be explored only through the use of technology. The authors contended that there is a connection between transformational and Euclidean geometry that deepens students' understanding. Even though the article is not a study, the authors commented on the use of this approach with students, arguing that with technology, proof is no longer an activity reserved for Euclidean geometry; rather, it is a means to make connections within the content throughout the school year. The use of technology helps students test hypotheses and visualize different scenarios for their proofs in a rigorous, mathematical way.

The van Hiele (1986) model is a well-known learning theory that specifically addresses the learning of geometry. Researchers ground their research on technology use in geometry by using the van Hiele model. Bell (1998) focused her dissertation on the investigation of a dynamic, geometry-enhanced environment that promoted an inquiry-based approach to conjecturing. Bell examined the relationship between the effects of this environment and the van Hiele levels by using scores on achievement tests as well as surveys and interviews with students from five different classes—two classes in the experimental group and three in the control group. The experimental group used the Geometer's Sketchpad at least twice a week, with the purpose of conjecturing in an inquiry-based setting. The results of the study indicated that there was a significant relationship between the use of technology for an inductive approach to conjecturing and improvement in the van Hiele levels as well as in basic geometry knowledge. The study supplied evidence that students' levels of geometric thinking, as described in the van Hiele model, are related to achievement.

The studies reviewed in this section indicated that the use of geometry software can provide teachers with new methods of assessing students' knowledge and understanding by the use of the drag mode (Scher, 2005). They also revealed that the use of technology does not deter formal proofs, but can extend the need for proofs in more areas in geometry other than Euclidean Geometry (Bucher & Edwards, 2011). Furthermore, the studies indicated that geometry construction tools can help students advance through the van Hiele levels of geometry thinking (Bell, 1998). One of the most frequently used features of GeoGebra is its graphical representation of geometric figures and relationships. The study investigated how teachers use GeoGebra while teaching geometry concepts as well as how the drag mode was used and integrated with the writing of formal deductive proofs. While the study did not attempt to find a relationship between the van Hiele levels of the students and the usage of GeoGebra, the researcher interviewed the teachers and inquired about what they believe to be the benefits of using the software for the advancement of students through the van Hiele levels.

Summary

The studies reviewed in this section indicate that the use of technology in algebra and geometry courses could transform the curriculum by altering the order in which concepts are presented, by providing students with real mathematics experiences, and by allowing teachers to pay more attention to what algorithms are important to continue to perform by paper and pencil. Technology can be more than a reorganizer of priorities, activities, and topics; it also can cause teachers to rethink what is really necessary for students to learn and, subsequently, introduce change in the curriculum. Technology creates the possibility for students' intellectual capabilities to be expanded (Heid, 1997); but this opportunity is highly dependent on how teachers choose to integrate technology in their crowded curricula. For this reason, the study focused on the teachers, rather than on the students.

Changes in the Classroom

As was exemplified in the previous section, technology is creating some changes in the mathematics curriculum. Technology also is creating other changes in the classroom, namely the ways that teachers and students work together (Heid, 1997). The researcher of the study observed classrooms in which technology had been implemented; therefore, it is important to understand what may be involved in the changes introduced by technology. The NCTM standards (1991) describe four important dimensions of a mathematics classroom in which the changes created by technology can be seen: (1) nature of the environment, (2) nature of the tasks, (3) nature of teachers' actions, and (4) nature of students' actions. The descriptions of these dimensions, as well as what current researchers say about them, will organize this section of the literature review.

Nature of the Environment

"Student's learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematics ideas" (NCTM, 1991, p. 58). According to Duarte, Young, and DeFranco (2000), technology enhances the learning environment by providing opportunities for students to investigate ideas, verify their thinking, construct graphs and diagrams, and discuss their ideas with peers and adults. Sheets and Heid (1990) believe that even if teachers do not plan for group work, technology fosters the development of collaboration in small groups as a result of the public character of the computer screen, the need for interaction with computer programs, and the need for discussion when students share computers.

Vygotsky (1978) asserted that knowledge is built within a community through the social interactions of its peers. Social interaction has been shown to have a positive impact on learning, social behavior, and motivation (Zurita & Nussbaum, 2007). Hoyles and Noss (1992) identified four benefits of computer-enhanced discussions aimed at the promotion of learning: distancing, conflict, scaffolding, and monitoring. Distancing occurs when students discuss their computer work with each other, representing their thoughts and raising them to a conscious level. Conflict occurs when the computer or the group discussions provokes a reconsideration of initial perceptions. The group provides scaffolding, or a way of reasoning that individuals would not be able to construct on their own. Groups monitor their discussions, thus facilitating metacognition.

Group work with computers is ideal for individuals learning mathematics, and the benefits can be long term (Healy, Pozzi, & Hoyles, 1995). Healy et al. (1995) conducted a three-year multisite case study that investigated students working in groups with computers in a variety of curricular settings. The researchers trained the teachers of the students in the study to organize group work prior to the study. The researchers' goals were to establish a relationship between tasks, content, software, group processes, and students' mathematical learning using quantitative and qualitative methods. Healy et al.'s results indicated that highly structured group work with computers does not appear to disadvantage students across gender or ability level. They found that a group's organizational style and the patterns of interaction among students were likely to influence the success of the work, and the influence depended on the task and the type of learning taking place. When the tasks were conceptually based, learning seemed more likely if the students worked in an integrated style, with the students participating actively in constructions at the computer and having group discussions about the constructions as well as their mathematical implications. Students working in an integrated style had reflected on their own constructions alongside the constructions of the others in the group. When conflicting strategies were identified through discussions and computer work, the students were able to reach a consensus. The researchers believe that the use of technology aided in the students' mathematical learning because without the technology, the students could continue to engage in their own ways of interpreting the problem. As students do their constructions with technology, they make sense of the mathematics involved in forms that require them to clarify and formalize their ideas. The sharing of ideas with others helps them synthesize their ideas and accept the ideas of others.

The availability of several types of technology appears to influence collaboration. Loch, Galligan, Hobohm, and McDonald (2011) conducted a semester-long multi-case study to investigate the impact of tablet technology on students' learning. A video recording of the observed class showed that students were showing their work on the tablets to each other as well as displaying their work on the projector screen. The video showed that students were able to identify, discuss, and correct mistakes shown on the projections. Students were able to make mistakes in a safe environment without losing confidence, and they could correct their errors with the help of their peers before presenting their final work for grading.

The integration of technology and the use of small group work increases opportunities for student learning. Roschelle et al. (2010) performed a randomized experiment with the goal of investigating whether group-level feedback increases student engagement in explaining mathematics to each other and consequently increases students' learning. The researchers studied fourth grade students from two different classes. During the first half of the instruction time, the teacher explained how to work with fractions. During the second half of class time, the researchers randomly selected half of the students from the two classes and combined them in the experimental group; the remaining students were assigned to the control group. The experimental group was given a Peer-Assisted Learning (TechPALS), which used wireless handheld technology to structure feedback in the group as they solved fractions problems. Students in the control group were given a popular desktop product that provided feedback to the individual students as they solved fractions problems individually. After analyzing and comparing test results from both groups, Roschelle et al. concluded that the students in the experimental group, who used technology in groups, learned more than the students in the control group, who worked independently. Using observations as data, the researchers confirmed that the students working in small groups participated socially in questioning, explaining, and discussing as they solved the problems, while the students in the control group did not.

Technology can foster collaboration among students, and small group work seems to help with student learning (Healy et al., 1995; Hoyles & Noss, 1992; Loch et al., 2011; Roschelle et al., 2010; Zurita & Nussbaum, 2007). In the study, the researcher observed technological lessons to determine if there was any small group work being done, if there was cooperation among students, and how that cooperation evolved throughout the technological lesson.

Nature of the Tasks

"Tasks are the projects, questions, problems, constructions, applications, and exercises in which students engage. They provide the intellectual context for student's mathematical development" (NCTM, 1991, p. 20). According to Duarte et al. (2000), meaningful mathematical tasks that involve technology promote significant mathematical learning, which is based on the interest of the students. These tasks draw on the students' backgrounds, are embedded in a realistic context, and are focused on problem solving.

Mathematical modeling can be defined as an approach that includes conjecturing, modifying, and adapting mathematical theories to real-world problems (Ferrucci & Carter, 2003). According to Ferrucci and Carter (2003), these problems require a particular way of thinking and behaving that helps students construct knowledge by putting together abstractions and formalizations and relating them to real situations. Swingle and Pachnowski (2003) discussed the real-world problem experienced in a Calculator Based Laboratory (CBL) when a teacher was preparing a lesson about the bounce of ping-pong balls. The teacher accidentally moved the motion detector away from the bouncing balls and then placed the detector back to where it had been. Some of the information was missing; however, instead of discarding the data, the teacher asked the students to interpolate the missing data. The researchers observed great interest from the students in working with the motion detector and graphing calculators being used in the classroom. The students in the study expressed appreciation for using what they thought of as static algebraic equations to obtain their results. The researchers concluded that students demonstrated a better understanding of the connections of physics, mathematics, and technology after engaging in the activity.

Technology allows students to enhance their ability to shift among different representations and solution approaches of the same real-world problems, which can impact their achievement (Ferrucci & Carter, 2003). Van Streun (2010) was interested in the effects of students' use of graphing calculators for problem-solving approaches to real-world problems. The yearlong study included two groups of pre-calculus students: one group, the experimental group, used the graphing calculators constantly during the year; the second group, the control group, used the graphing calculators occasionally. After analyzing pre- and post-test results, the researchers observed that the students in the experimental group used a wider range of solution methods and solved more problems graphically than the students in the control group. Students in the experimental group continued solving problems using analytical and numerical approaches but extended their solution methods by adding graphical approaches. Students in the experimental group with low scores on the pre-test showed better results in the post-test than their counterparts in the control group.

The context of the problems makes a difference in students' engagement. One of the reasons given as to why students find word problems difficult is that the context of those problems is unfamiliar to the students. Word problems differ from real-world problems (Choi & Hannafin, 1997) in that they can be more abstract and unrelated to students' experiences. Choi and Hannafin (1997) suggested providing word problems with more meaningful contexts and making them more relevant to students. Choi and Hannafin's study involved 101 fifth grade students who came from a low-to-middle income background and had similar mathematics abilities. The researchers adapted 10 word problems found in age-appropriate textbooks and changed the context and reasoning complexity of each problem. Contextualized problems provided familiar, relevant context, while decontextualized problems provided minimal, non-meaningful information. Simple problems isolated the key data, while complex problems required that necessary data be extracted from the information given in the problem.

Students were randomly selected to solve problems that were simple and decontextualized, simple and contextualized, complex and decontextualized, or complex and contextualized. After analyzing the students' post-test results, the researchers indicated that students who studied simple and decontextualized problems performed well on simple, one-step questions, but were less able to apply their knowledge to solve increasingly complex questions. Students who studied complex, conceptualized problems were able to solve one-step and multi-step problems. The researchers argued that rich mathematical instructional contexts best support mathematical problem solving.

Familiarity of contextualized problems can bring pleasure to students, and technology can add elements of comfort. Pierce and Stacey (2011) participated in a project that investigated the use of real world context problems with the assistance of technology to enhance middle school students' engagement and achievement in mathematics. Through the project, Pierce and Stacey observed teachers emphasizing the use of technology to students for aesthetic reasons: to add color and clarity to the activity and, ultimately, brighten the classroom. The analysis of this phenomenon led the researchers to conclude that one of the motivations for teachers to use real world problems was to associate mathematics with pleasurable aspects of students' lives. The teachers hoped that by using pleasant context, colorful images, and objects in their lessons, the students, by association, would feel more inclined to learn mathematics. The researchers concluded that even if the teachers were using technology for superficial reasons, it was an important part of the lesson.

In summary, NCTM (1991) recommended that teachers develop mathematical tasks that engage students. Real-life problems give students opportunities to apply mathematics to real situations, and the combination of real-life problems with the use of technology fosters a particular way of thinking, which allows making conjectures and then modifying and adapting those conjectures (Duarte et al., 2000; Ferrucci & Carter, 2003; Swingle & Pachnowski, 2003). Technology helps students increase their repertoire of solution approaches and the variety of representations to the real-world problems (van Streun, 2010). Word problems should not be confused with real-world problems since they are different with regard to the familiar context they provide to the students (Choi & Hannafin, 1997). Real-world problems not only engage students, but they also utilize a familiar context and include attractive visual enhancement through technology, which can bring pleasure to students (Pierce & Stacey, 2011). In the study, the researcher observed teachers conducting a mathematics lesson and noted the type of tasks given to the students, the context of those tasks, and how engaging those tasks were.

Nature of Teachers' Actions

Teachers facilitate the learning process by providing rich learning environments, assigning meaningful tasks, incorporating appropriate tools, and stimulating interaction and communication in the classroom. Teachers need to listen to their students' responses and ask students to clarify and justify their thinking (NCTM, 1991). Cornell University's project, *GoodQuestions,* defines questioning as "a pedagogical strategy that aims to raise the visibility of the key concepts and to promote a more active learning environment" (Cornell University, n.d.). The Cornell team identifies questions as good ones if they stimulate students' interest and curiosity in mathematics, help students monitor their own understanding, and support teachers' efforts to foster an active learning environment.

Questioning can be used to suggest to students that they analyze the results obtained from technology in mathematics activities. In their observational case study of two pre-calculus classes using mathematical modeling and graphing calculators, Doerr and Zangor (2000) documented a teacher's use of questioning. The teacher had 20 years of experience and was skilled in the use of graphing calculators. She would ask her students if the calculator always told the truth and to what extent it was justifiable to believe the calculator. Her questions removed the students' attention from the tool computation and focused it on the interpretation and justification of the tool's results.

Teachers' questions and interactions with students in the classroom can obscure the mathematics behind the task and make students focus only on the correct answers.

Heid et al. (1999) documented the questioning techniques of three inexperienced teachers in an Algebra I course. The teachers used interviews to verify whether students' thinking and solutions matched their own but did not bring forth the students' understanding and strategies used. The researchers described the questions of a sample teacher who began with one question and then asked increasingly more guiding questions until students gave the correct answer. Teachers questioned the students' technological skills and their understanding of the terminology. According to the researchers, this type of questioning limited the students' technology approaches and mathematical reasoning.

Technology helps teachers observe and reflect on their own classroom practices. Piliero (1994) videotaped an experienced teacher learning how to implement Function Probe in the classroom. As the students were learning to use the technology, the teacher's questioning improved as she balanced the need to give students too many directions to giving too little. While watching a video of herself working with a student, the teacher recognized she was generating good questions, but she needed to see the video to realize how much she had improved.

The art of developing good questions is crucial for effective teaching, and technology can help teachers create questions that fit students' needs and assess the progress of the class as a whole as well as the progress of individual students. Novak, Fahlberg-Stojanovska, and Di Renzo (2011) used electronic surveys in their classes as a way to assess how students were learning. The researchers used Google Forms to email the survey to their students. The survey allowed students to reflect on their work and the mathematics involved. The students' responses were compiled into one spreadsheet, giving the teacher an overall picture of the class's understanding as a whole.

Teachers' actions contribute to the learning environment; one of those actions is communicating by questioning the students. Questioning can promote active learning by allowing students to reflect on their own work and scaffold their understanding of the concepts (Cornell University, n.d.; NCTM, 1991). Teachers' questions can help students focus on the interpretation and understanding of the results (Doerr & Zangor, 2000), or it can narrow students' thinking by focusing only on correct answers (Heid et al., 1999). Teachers can analyze their own questioning skills with the help of videos (Piliero, 1994) and can assess students' understanding individually or as a whole by using electronic surveys (Novak et al., 2011). The researcher of the study noted the questions used by the participating teachers as they taught a technology-enhanced mathematics class, since their questions can be about the technical use of GeoGebra or about the interpretation and understanding of the results obtained from GeoGebra.

Nature of Student Actions

Students should feel confident about using technology as tools, and they should be willing to take risks that may bring them into new situations or allow them to interpret an idea with better understanding (Duarte et al., 2000). If students are confident about their ability to use technology, they may investigate ideas beyond their grade level and explore deeper mathematical problems (NCTM, 1991).

What students learn when using technology appears to be related to their attitude towards mathematics as a subject, their attitude toward the technology used, and the behaviors displayed when using the technology (Reed et al., 2010). Galbraith and

Haines (1998) devised a set of targeted attitude measures designed to allow a better understanding of the impact of technology in teaching and learning based on student characteristics. These attitudes relate to students' confidence, motivation, and engagement with respect to mathematics, computers, and the interaction between them. Students display technology confidence when they feel self-assured operating the technology, believe they can master the technical procedures required of them, and feel more secure of their answers when supported by technology. Students display technology motivation when they feel that technology makes learning more enjoyable.

Galbraith and Haines (1998) used their scales to measure the attitudes of 156 mathematics students; they concluded that technology confidence and motivation are strongly associated with mathematics, but they are less strongly associated outside of mathematics. The researchers reported that mathematics engagement is strongly associated with motivation, and computer attitudes are more influential than mathematical attitudes in facilitating the active engagement of technology activities in the mathematics classroom.

Following the same definition of technology confidence used by Galbraith and Haines (1998), Pierce, Stacey, and Barkatsas (2007) developed the Mathematics and Technology Attitudes Scale (MTAS) questionnaire consisting of 20 items. After using the scale with students from six different schools, the researchers reported that students with positive attitudes towards mathematics and mathematical technology tools overcame initial difficulties with the technology and progressed to more effective behaviors such as using the technology to explore mathematics and develop strong conceptual understandings. Students with negative attitudes avoided the technology and therefore did not experience success improving their mathematical understanding. Pierce et al. concluded that mathematics confidence, technology confidence, attitude to learning mathematics with technology, and behavioral engagement contribute to the effectiveness of the learning experience.

Barkatsas, Kasimatis, and Gialamas (2009) used Pierce et al.'s (2007) scale with 1,068 junior high school students from Greece. They reported that boys express more positive attitudes than girls towards mathematics and the use of technology in mathematics. Additionally, they found that high achievement in mathematics (as reported by the teachers) was positively related to high levels of mathematics confidence, high levels of affective and behavioral engagement, high confidence in using technology, and overall learning of mathematics with technology. Low levels of mathematics achievement were associated with low levels of mathematics confidence and were strongly related to negative levels of affective and behavioral engagement. Students who showed low levels of mathematics achievement also demonstrated low confidence in using technology and a negative attitude towards leaning mathematics with technology.

Teachers who promote learning with mathematical technology need to take several factors into account: student attitudes, student learning behaviors, and meaningful mathematical discourse. Reed et al. (2010) designed a mixed-method study that investigated student attitudes and behaviors on the outcomes of learning functions while using electronic workbooks with embedded applets in realistic contexts. The learning behaviors investigated by the researchers were termed purposeful, investigative, reflective, and communicative. In the whole sample (n = 521), student attitudes could account for a 3.4 point difference in test scores between individuals on a 10-point scale. Detailed observation of a small number of students (n = 8) revealed that positive attitudes towards mathematics and technology increased learning behaviors, and a positive attitude toward technology and learning behaviors benefited technology mastery. The researchers concluded that the relationship between technology mastery and test scores seemed to be most affected by students who displayed reflective behavior while working on the problems.

Students' confidence in the use of technology is strongly related to their confidence in mathematics, and this can be reflected in their motivation to use technology and engage in mathematics activities (Barkatsas et al., 2009; Galbraith & Haines, 1998; Pierce et al., 2007). Students' self-reflection seems to be an important behavior that can have the most impact on learning (Reed et al., 2010). In the study, the researcher administered the Mathematics and Technology Attitudes Scale to the students of the participating teachers with the goal of determining the confidence of the students and their attitudes toward technology, mathematics and the learning of mathematics with technology. Their levels of confidence and their attitudes might be a barrier for the teachers to implement technology in the classroom.

Summary

The study used the steps in instrumental orchestration defined by Drijvers et al. (2010) as the basis for understanding teachers' intentional and systematic organization of the various artifacts available in their classrooms. As described in Chapter 1, instrumental orchestration consists of three elements: pedagogical context, preparation mode, and pedagogical action. Pedagogical context refers to the classroom arrangements, the topics to be covered, what artifacts will be available, as well as what instructional approach will be used. The preparation mode represents the ways in which teachers decide to make full use of pedagogical preparation for the benefit of instruction. It includes decisions about the way an activity is introduced and worked out as well as the schema and techniques to be developed. Pedagogical action involves ad hoc decisions made during teaching about how to perform the chosen pedagogical configuration and preparation mode. In the mathematics classroom, this could involve a teacher deciding how to deal with unexpected aspects of technology or with the activity itself, as well as determining how to deal with students' questions and actions.

This section of the literature review about the changes in the classroom due to the implementation of technology helped the researcher better understand all the elements that form teachers' instrumental orchestration when implementing technological activities. Implementation of technology may cause a break in a teacher's habitual instrumental orchestration because of the many changes it can introduce into the classroom environment. Some of the changes brought upon by the implementation of technology can be seen in a classroom environment that relies on cooperation and small group work; implements tasks that are more compatible with real-world problems embedded in a context of personal interest to the students; incorporates a teacher's questions and communication with the students; and reflects students' confidence, attitude, and engagement toward technology, mathematics, and learning technology with mathematics. The researcher took abundant notes during the classroom observations and used that data to help answer the research question of what some of the differences and similarities are with respect to how and why teachers use GeoGebra in their practice.

Changes in Roles and Responsibilities

Vygotsky (1978) suggested, "Learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and when in cooperation with his peers" (p. 90). If we consider the classroom to be the learning environment, the kind of interactions that take place are either human interactions (student-teacher, student-student) or interactions between humans and learning technology. When interaction occurs in a technology-rich classroom, teachers and students assume different roles. This section of the literature highlights some of the differences and similarities in teachers' roles when they use technology in their classrooms.

Researchers have made a list of the different roles assumed by teachers when using technology. Farrell (1996) used videotapes of six precalculus classes to determine the roles that teachers assumed while using calculators. He considered the roles that manifested themselves when using technologies and how those roles compared to those of the same teachers in non-technology classrooms. Farrell used teachers' roles that were adapted from the Systemic Classroom Analysis Notation (SCAN) developed by Fraser et al. (1987).

In 1990, Heid, Sheets, and Matras developed their own list of roles and responsibilities from observations of teachers using a combination of technologies in Algebra I classrooms. They observed that when students worked mathematics problems with technology, their work could take them into areas not referred to in the textbook;

as a result of this, the teachers became collaborators in the pursuit of solutions and explanations. The researchers observed other changes in the dynamics of the classroom. Evaluation of students' learning also changed with the presence of technology since students' learning became more visible through their different representations of the same problem. Allocation of time changed due to the nature of the open-ended investigations that led to class discussions of unpredictable length and content. Heid et al. (1990) pointed out that the teachers were not used to these discussions and did not know how to bring them to an end. Pacing students' work at the computer was another challenge for teachers, especially since some students needed to finish the work out of the classroom and some of them did not have the technology at home. The researchers noticed that the teachers made changes in their lesson planning due to the new challenge of considering a range of applicable problem-solving strategies. This made the lesson itself somewhat unpredictable, since the teachers then had to deal with on-the-spot mathematical analysis.

Zbiek and Hollebrands (2008) developed a list of roles that combined the roles identified by Farrell (1996) and Heid et al. (1990), along with a few descriptions to include newer technologies used in the mathematics classroom. The list displayed in Table 2 includes Zbiek and Hollebrands' (2008) list of roles, presented in alphabetical order.

Farrell (1996) studied the roles teachers assumed when technology was used and compared them with their roles when technology was not used. The researcher videotaped lessons and coded each five-minute segment according to whether a particular role was present. Her results indicated that teachers function as *managers*

Table 2

List of Roles and Descriptions in a Technology Classroom

Role	Researcher	Description
Allocator of Time	Heid et al., 1990	The teacher is working with the time requirements of the school as well as orchestrating time for accommodating the needs of individual students.
Catalyst and Facilitator	Heid et al., 1990	The teacher facilitates the introduction of a new problem or real-world context and the discussions of various solutions so that the lesson reaches an appropriate close.
Collaborator or Fellow Investigator	Heid et al., 1990 Farrell, 1996	The teacher is initially unfamiliar with both the problem and the solution and therefore is a true participant in mathematical learning.
Counselor	Farrell, 1996	The teacher is familiar with the problem and is able to advise and assist students when they ask for teacher input. It includes playing the devil's advocate as well as providing encouragement or serving as a stimulator or diagnostician.
Evaluator	Heid et al., 1990	The teacher uses informal and formal assessments of different types to describe individual students' emerging understanding with and without the technology.
Explainer	Farrell, 1996	The teacher demonstrates, establishes the context, focuses the classroom direction, and serves as a rule giver and knowledge source.
Manager	Farrell, 1996	The teacher serves as tactical manager, director, and authoritarian.
Planner and Conductor	Heid et al., 1990	The teacher plans and implements with technology and without technology activities, and chooses among whole class, small group, or individual settings as needed. This includes selection, chronology, and creation of curriculum materials and technology tools.
Resource	Farrell, 1996	The teacher presents a system to be explored and functions as a giver of factual information
Task Setter	Farrell, 1996	The teacher is a questioner and decision maker who also sets the examples and strategies.
Technical Assistant	Heid et al., 1990	The teacher helps students with hardware and software difficulties.

almost 100% of the time with or without the use of technology. The role of *counselor* occurred 49% during technology-use segments compared to only 19% during nontechnology-use segments. The roles of *resource* and *fellow investigator* were reported only 5% and 17%, respectively, in technology-use segments, and those roles also had a low incidence of 3% and 6% in nontechnology-use segments.

When Dynamic Geometry software is used in the classroom, teachers need to create an environment in which students' thinking is valued and their reasoning about the properties of the figures—not just the appearance of the drawing—is encouraged (Hoyles & Jones, 1998). In a study with middle school students using Cabri, Hoyles, and Jones (1998) found that teachers assumed a combination of roles, especially counselor, catalyst, and facilitator; this further indicated that those roles started prior to the teaching of the lesson in the classroom. This is consistent with Zbiek's (1998) observations. She observed 13 preservice secondary mathematics teachers using curve fitters, graphing utilities, and other computing tools during modeling activities. The preservice teachers assumed the role of counselor when students were sharing their models with their classmates, with the intent of encouraging the students to challenge each other's understanding and conclusions.

Students' roles and responsibilities also change when technology is used in the classroom. Heid (1997) observed that students' roles are related to the nature of the problem. Students needed to learn how to engage in cooperative group work and how to use other students as resources. During group work, students had to communicate orally and in written form more frequently than when technology was not in use. Students needed to assume greater responsibility for their own learning, assessing themselves at times in this new environment, deciding how to complete the open-ended challenges, and developing new strategies for studying for the teacher's assessment.

Changes in student-teacher interaction also have been observed. Relying on several teachers' observations, Schofield (1995) noticed a decrease in teachers lecturing and an increase in teachers serving as coaches or guides with the expectation that students would construct their own knowledge as they worked with technology. The researcher pointed out that the teachers had less control of the classroom activities and, therefore, they were less authoritarian and more likely to serve as collaborators. Teachers spent more time working with students who requested help and less time conducting routine checks of students' progress. Teachers' help was more individualized; as a result, students tended to help one another more often, and they independently decided how and when they needed help from the teacher.

When technology is used in the classroom, teachers and students tend to change their roles and responsibilities. The general direction suggests that teachers give more control of the lesson to the students as they replace the role of lecturer with the role of coach and counselor (Farrell, 1996; Heid et al., 1990; Hoyles & Jones, 1998). Students, in turn, take more control of their own learning as they decide how to tackle the problems and how and when to ask for help (Heid, 1997; Schofield, 1995). The researcher of the study made note of the different roles assumed by teachers when teaching a lesson with technology. The researcher used the list of roles provided by Zbiek and Hollebrands (2008), but did not look at the frequency of each role used in the classroom, only noting the more prevalent ones. After the observation, the researcher interviewed the teachers, asking them about whether they believe the different roles they assume also are assumed during nontechnology lessons.

Summary

The studies reviewed in this section suggest that when technology is used in the classroom, changes can be seen in the curriculum presented, the classroom dynamics, and the roles and responsibilities of teachers and students. Technology can reorganize the topics, activities, and priorities that take place in the classroom by altering the order in which topics are presented and possibly by having teachers rethink what is really necessary for students to learn. Technology can change the nature of the classroom environment by introducing cooperative activities, changing the nature of tasks by analyzing and modeling real-world problems, influencing the types of questions teachers ask, and improving students' confidence when solving mathematics problems with the help of technology. All of this has the potential to cause changes in the roles and responsibilities adopted in the classroom; teachers could replace the roles of lecturer and authoritarian with those of collaborator and counselor, and students could take more responsibility for their own learning.

The study used the steps in instrumental orchestration defined by Drijvers et al. (2010) as the basis for understanding the teachers' intentional and systematic organization of the various artifacts available in their classrooms. The use of technology in the mathematics classroom brings new challenges to the teachers, who need to decide how to deal with unexpected aspects of the technology or classroom activities as well as determine how to deal with students' questions and actions. Since teaches are the ones responsible for many of those decisions, the study focused on teachers rather than students.

The studies reviewed in this section have focused mainly on the use of graphing tools; very few of the studies examined the use of dynamic mathematics software like GeoGebra. The researcher of the study encountered only a few studies that have examined the changes that Cabri or the Geometer's Sketchpad can bring, but none of those studies examined the implementation of GeoGebra. The study looked at the changes that can be seen in the classroom due to the use of GeoGebra as well as the changes perceived by the participating teachers. The researcher took abundant notes during the classroom observations and interviews and used them to address the following research questions: how do teachers integrate GeoGebra in their practices, what are the differences and similarities in teachers' practices when teaching lessons with and without the use of technology, and what are the differences and similarities among the participating teachers when using GeoGebra.

Impact of External Factors on Teacher Use of Technology

Integration of technology in the classroom is not only dependent on a teacher's internal factors like knowledge, concerns, and beliefs, but also is influenced by factors external to the teacher (Zbiek & Hollebrands, 2008). Hew and Brush (2007) reviewed 48 empirical studies that examined barriers affecting the use of technology in K-12 schools for instructional purposes and found over 100 barriers or factors. After reviewing some of the existing literature, the researcher of the study made a selection of factors affecting teacher use of technology in the classroom that seem to be the most frequently mentioned by teachers in the studies reviewed, and that might have a direct impact on the study. The factors were organized in three categories: resources, school, and policy. The goal of reviewing literature on external factors affecting teacher use of technology is to understand possible answers to the research question about a teacher's perceptions of the external barriers that foster or prevent the use of GeoGebra in the classroom.

Resources

Lack of resources may include the following: technology, access to technology, time, and technical support. Pelgrum's (2001) paper reported on the results of the Second Information Technology in Education Study (SITES) that surveyed representative samples of schools from 26 countries from 1997 to 1999. The main focus of Pelgrum's paper was on school principals' and technology experts' views of what are the main obstacles to the integration of technology in schools. The most frequently obstacle mentioned was the insufficient number of computers available, of software copies, and of computers with Internet access. The second most mentioned obstacle was that teachers did not have sufficient knowledge of and skills in technology. Some of the top ten obstacles mentioned were insufficient computer time scheduled for students, insufficient teacher time, and lack of technical staff.

A common frustration for teachers when implementing technology is the amount of time spent on technical issues rather than instructional ones. Sandholtz and Reilly (2004) conducted a 4-year research study in a K-8 public school district in California. This particular district was selected because it implemented a new technology program and provided technology training for all 260 teachers over a 5-year period. Data were collected over 4-years using documents, surveys, teacher journals, interviews and observations with the intent to see the progress, adaptation, and sustainability of the new technology program. The researchers reported that the resource most often requested by teachers was time; they wanted time to learn, to prepare lessons, to experiment, and to attend technology in-service sessions. After time, teachers requested additional collaboration with colleagues since they considered colleagues as an important source of knowledge and support. Sandholtz and Reilly noticed that the district, without realizing it, provided sufficient technical support and this reduced technical issues for teachers, allowing them to move quickly past the entry stage of technology use and concentrate on curriculum and instruction. The researchers concluded that with limited or no technical support, even teachers with a solid plan for integration technology may reduce or even abandon the plans.

Access to technology is a logistic problem in many schools. Becker (2000) found that teachers who have five to eight computers in their classroom are more likely to give students more computer opportunities than teachers who have computers in a centralized location, like in computer laboratories. Becker indicated that teachers who have to use computers in a centralized location need to schedule laboratory time for their students, making it more difficult for the teachers to integrate technology as analytic and exploratory tools. The use of laptop carts can eliminate some of the logistic problems of having students meet in a different room; however, the carts need to be scheduled and shared by several teachers.

School

School barriers may include leadership, school-scheduling structure, and opportunities for professional development. Fullan (2008) stated that one of the fundamental problems with educational reform is that people, including school administrators, do not have a clear understanding of the reasons for educational change—what it is and how to proceed. Sandholtz and Reilly (2004) found that, in response to the needs of their communities, some school leaders are making changes in their institutions, such as including parents as technical support staff for the teachers. Fox and Henri (2005) found that if school principals do not understand the benefits of technology in education, teacher use of technology in their classrooms is restricted. Sandholtz and Reilly (2004) reported that leaders who have a clear vision of the benefits of technology are able to take several steps toward securing successful implementation of technology in school classrooms. As a result, those leaders are able to accomplish tasks such as securing funds for equipment and training for teachers, planning a training rotation for all the teachers in the district, and giving teachers time to plan lessons and collaborate with other teachers.

Inflexible timetables also can act as a barrier to the implementation of technology in classrooms. In a survey of 4,000 teachers in over 1,000 schools, Becker (2000) found that most secondary school students have less than a continuous one-hour duration of class time to do work. This time constraint discourages teachers from planning technological activities on a regular basis, especially if the computers are not available in their own classrooms. Becker found that secondary school teachers who have longer teaching times (e.g., 90-120 minutes) were more likely to report frequent use of technology during class compared to teachers who taught in 50-minute blocks of time.

Professional development can influence teachers' attitudes and beliefs toward technology. In a study on the effects of different characteristics of professional development on a sample of over 1,000 teachers, Garet, Porter, Desimone, Birman, and

Yoon (2001) found that effective professional development focuses on content and skills, gives teachers opportunities for hands-on work, and is highly consistent with teachers' needs. The researchers reported that focusing on technology skills is important because teachers will not integrate technology unless they have mastered some basic technology skills. Teachers also need to have the necessary technology-supported pedagogy knowledge and skills in order to integrate technology for instructional purposes. Garet et al. found that teachers can develop technological activities only after they have seen the value of technology for instruction and learning.

When leaders see the value of technology, they implement continuous professional support for teachers. In their 4-year study of technology implementation in schools, Sandholtz and Reilly (2004) reported that the district leaders had established professional growth plans for the teachers, with the first-year focus being on technology skills for all teachers and the following years' focus centering on instructional issues. The district professional development program incorporated a pedagogic training approach that provided teachers with a constructivist environment in which they could learn how to explore, reflect, collaborate with peers, work on authentic learning tasks, and engage in active learning. The district program had four main components: classroom visits, hands-on technology training, group discussions, and participant collaboration. Teachers in this study also had to develop their own personal professional growth plan. In describing their plans, teachers had to identify areas that they planned to focus on during the upcoming year. In addition, they were given the opportunity to request additional support and opportunities related to their plans, making the support highly related to their needs.

Professional development can influence teachers' willingness to integrate technology in their classrooms. Van Braak (2001) surveyed secondary teachers in Belgium and reported that teachers' resistance to change or adopt technology can be influenced not only by the introductory professional development they receive, but also by on-going professional development that provides them with support when they need it. Van Braak concluded that teachers' willingness to implement technology is more significant than personal factors such as age, gender, computer attitudes, and computer experience.

The support teachers experience in their schools is a major factor in their willingness to implement technology in their practices. Sheingold and Hadley (1990) used survey data to identify that the schools with wider success in technology implementation were those that provided sufficient technology, support, and time for teachers to learn the technology, as well as an academic and cultural structure to encourage teachers to take an experimental approach to their practice.

Teachers' professional engagement in professional activities outside their classrooms can have a direct impact on their teaching practices. In their study, Becker and Riel (2000) measured professional engagement by observing how frequently teachers had informal but substantive communication with other teachers at their school and in their district. Furthermore, the researchers observed how often teachers engaged in peer leadership activities like mentoring, workshops, and conference presentations. The study found that teachers who participated in professional interactions beyond their classrooms taught in different ways than teachers who had minimal contact with other peers. The researchers saw a relationship between involvement and the use of technology: the more involved teachers were in professional activities, the more likely they were to have teaching philosophies compatible with constructivist learning theory and to use technology in exemplary ways. These teachers used technology to engage their students in cognitively challenging tasks, promoting communication among the students. According to the survey results in Becker and Riel's study, teachers who do not participate in professional activities outside of their schools are more likely to focus on traditional methods of information delivery in their classrooms; those teachers do not place a high value on collaborative knowledge construction in their classroom or for themselves in the educational community.

Policy

Policy barriers may include assessments and textbooks. Hew and Brush (2007) defined high-stakes testing as assessments with serious consequences, such as promotion or graduation for students or rewards versus sanctions for schools and teachers. The pressure of such tests can be a major barrier to technology integration. Butzin (2001) noted that the pressure to score high on standardized tests, along with the need to cover vast amounts of materials in a short amount of time, creates a challenge to teachers. Consequently, teachers feel they can cover more material if they use traditional instructional approaches, like lectures. Teachers believe that using traditional methods ensures that all students receive the same information and are working on the same tasks, rather than using technology that requires more planning time on their part and more time in the classroom with students having to log in and set up for the activities. The "No Child Left Behind" act has placed great emphasis on testing and on comparing schools' test results (Hew & Brush, 2007). This act has resulted in a shift:

schools have moved from using technology for teaching and learning to using it to facilitate computer-based testing and the warehousing of assessment results (Bichelmeyer, 2005). There is also a tension between the use of technology and the need to conform to the requirements of traditional testing (Hew & Brush, 2007). For example, there is a concern among teachers about the use of graphing calculators, since those instruments are prohibited in national examinations. This suggests a need to align assessment with instruction and to have high-stakes assessments that accommodate these innovations (Zbiek & Hollenbrands, 2008).

Teachers may find it difficult to match technology use not only to their assessments, but also to the class textbooks. Monaghan (2000) studied 13 teachers who were incorporating technology in their mathematics classrooms. At the beginning of the study, 11 of the teachers incorporated technology while following the mathematics textbook. After several months, 3 of those 11 teachers found the textbook inappropriate, and the 5 teachers who did not follow the textbook had difficulties finding materials that would match what they wanted to do with technology. The teachers struggled with creating original materials to supplement their technology lessons, which increased their planning time considerably. The researcher concluded that the availability of written curriculum materials aligned to the technology seems to be a limiting factor for teachers implementing technology in their classrooms.

Lumb, Monaghan, and Mulligan (2000) reported that some teachers believe that textbook mathematics is different from technology mathematics; as a result, they had to create their own worksheets. Some of the teacher-generated worksheets focused more on the technology than on the mathematics content, and they gave directions to the students that emphasized what technological procedures to follow in order to solve the various mathematical tasks. One of the teachers in Lumb et al.'s study used two different books—one traditional and one investigational—but rejected both of them because he felt the tasks did not match his goals for the lesson.

Summary

This review brought to light that there are many external factors that can impact the implementation of technology in the classroom (Hew & Brush, 2007). The barriers most often identified by teachers relate to lack of resources (Pelgrum, 2001), lack of technical support (Sandholtz & Reilly, 2004), time required for learning, and time restrictions in the school schedule (Becker, 2000). A school's vision for technology implementation and the principal's support of it also are important factors in the successful incorporation of technology in mathematics classrooms (Fox & Henri, 2005). District and school administrators can provide support by establishing a well-defined professional development plan (Garet et al., 2001) that encourages teachers to continue growing their technological knowledge and to collaborate with other teachers within their schools and in their communities. High-stakes testing can discourage teachers from implementing technology, since they have to cover all the material in a shorter amount of time (Butzin, 2001), leaving them no time to implement technological activities. The lack of written materials to supplement technology lessons and the lack of alignment with the textbooks is another barrier for teachers, since they feel they may have to create their own materials to supplement the lessons (Monaghan, 2000).

The researcher of the study interviewed teachers with the intent to learn about some of the external barriers that teachers perceive to prevent their use of GeoGebra in the classroom. The participating teachers teach in the same district but in different schools. The researcher of the study suspects that some of the external barriers will be district-imposed and, therefore, common to the teachers, while others may be school related.

Conclusions

As seen in this literature review, technology is under-utilized in mathematics classrooms (Becker & Ravitz, 2001; Conlon & Simpson, 2003; Cuban, 2001); however, when it is implemented, it seems to have a positive effect on students' achievement. Teachers play an important role, and their attitudes toward technology influence the impact that technology has in the classroom (Means, 2010; Shapley et al., 2011). Effective implementation of technology by teachers can be influenced by their Technological Pedagogical Content Knowledge (Niess et al., 2007); their personal, management, and consequence concerns (Hall & Hord, 2011); and their beliefs about technology, mathematics, and mathematics teaching and learning (Heid et al., 1999; Noss et al., 1990). When technology is used in the classroom, changes can be seen in the curriculum presented, the classroom dynamics, and the roles and responsibilities of teachers and students (Farrell, 1996; Heid et al., 1990; Hoyles & Jones, 1998). At present, the researchers of the study had not come across any reports of research that described, analyzed, and compared information on the different implementation forms and the possible factors impacting the implementation of dynamic mathematics technology, such as GeoGebra, by middle school teachers in their practices. Chapter 3 outlines the methodology proposed for this study.

CHAPTER THREE: METHODOLOGY

Integrating cognitive tools into teaching takes teachers time, because they first need to understand why and how learning might occur in a technology-rich environment and then be able to create appropriate learning situations (Faggiano & Ronchi, 2011). The literature reviewed revealed that teachers' Technological Pedagogical Content Knowledge (Niess et al., 2007); their personal, management, and consequence concerns (Hall & Hord, 2011); and their beliefs about technology, mathematics, and mathematics teaching and learning (Heid et al., 1999; Noss et al., 1990) can influence the way they implement cognitive tools in their practice.

The multiple case study had two goals: (1) to describe, analyze, and compare information on the practices of technology integration by a selected group of 12 middle school mathematics teachers, and (2) to examine the different reasons for the decision whether to integrate those cognitive tools.

Research Questions

The study addressed the following research question: Why and how do middle school mathematics teachers integrate dynamic mathematics learning environments in their practices? The following sub-questions were addressed:

1. What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?

2. What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?
3. What do the teachers think they learned from the SMGEM program, and how are they using what they have learned?

The 12 participating middle school teachers were selected from a group of 53 teachers who received their master's degree in teaching mathematics after participation in a 2-year program funded by a grant from the National Science Foundation (NSF). The program was described fully in Chapter 1.

This chapter describes the methodology was used in the study. First, the research design will be described, followed by an explanation of the sampling plan. The instruments used will be described in relation to how they served to answer the research questions. Finally, limitations, delimitations, and the role of the researcher of the study will be discussed.

Research Design

A qualitative research is an appropriate method for the study since the researcher is interested in the constructed meaning of the participants. A qualitative approach for this study parallels its social constructivist framework because qualitative research assumes that reality is socially constructed and that there is no single, observable reality (Merriam, 2009). The main goals of qualitative research are to understand how people interpret their experiences, and what meaning they attribute to those experiences. In particular, the study used a multiple case study approach. According to Merriam, a case study is a detailed investigation of individuals, groups, or any social units bounded by place and time. In a case study, the researcher attempts to analyze the variables relevant to the subject under study. The focus of a case study is not on making generalizations, but on understanding the particulars of the case and its complexity.

The study consisted of an intensive, holistic description and analysis of how and why the 12 selected middle school mathematics teachers integrate—or do not integrate—cognitive tools in their practice. The unit of analysis was the individual participants in the study. Data were gathered by a variety of methods, including surveys, pre- and post-interviews, and observations. To facilitate validation of data, triangulation was done by comparing and cross-checking data collected by the different methods. The researcher utilized repeated observations to increase the reliability of the case studies (Gall, Gall, & Borg, 2007). Member checking added to the validity of the proposed study.

Sampling Plan

Site

The study took place in a large public school district that has 36 middle schools with approximately 500 middle school mathematics teachers and over 52,000 middle school students. The district serves a diverse student population. There are students from 173 different countries speaking 53 different languages. The student demographics include 51.41% White, 39.06% Black, 28.32% Hispanic, 3.58% Asian, 2.20% Native American/Native Alaskan, 0.11% Native Hawaiian/Pacific Islander, and 3.65% multi-racial. There are over 40,000 exceptional student education children, consisting of approximately 30,000 with special needs and 10,000 gifted students. This district was selected because it is the district partnered with the institute of higher education as part of the grant from the National Science Foundation (NSF). All the participants in the

NSF funded master's degree program who received the cognitive tools training were teachers from this district. The district is representative of the large, diverse, rapidly changing school systems throughout the country.

Prior to the actual collection of data, the researcher completed the Institutional Review Board (IRB) requirement at Florida Atlantic University (Appendix C). In addition to the FAU review board, the school board's IRB process also was completed prior to communicating with the school principals (Appendix D).

Participants

The sample was purposeful since the researcher selected 12 in-service middle school mathematics teachers from a group of 53 mathematics teachers who have taken the professional development of mathematics and technology in the classroom as described in Chapter 1. Creswell (2009) warned researchers that the more cases an individual studies, the greater the risk of losing depth. The rationale for investigating 12 cases was not to generalize the findings but to have a deeper perspective of the teachers' perceptions of what is difficult about technology implementation, if and how their practice changes in a technology-rich environment, and how the role of technology relates to teachers' beliefs about mathematics as a subject as well as about the teaching and learning of mathematics.

Sixty-six mathematics teachers graduated from the Standards Mapped Graduate Education and Mentoring program. The first cohort of teachers consisted of 13 handpicked teachers who volunteered to participate in the pilot test of the program. Because those preselected, handpicked teachers already were technologically motivated and were considered master teachers by the district mathematics curriculum specialists, the researcher of the study did not include them in any part of the research, since they are not representative of the rest of the teachers who participated in the program. The researcher surveyed the remaining 53 teachers. From the respondents to this survey, 12 teachers were selected.

Since the researcher knows all 53 surveyed teachers personally, and in order to eliminate bias, a researcher's colleague who did not know the teachers did the selection of the participating teachers following the outlined criteria. The first criterion for selection was whether they currently are teaching mathematics in a middle school, since some participants were likely be teaching in high school. Only teachers who teach middle school mathematics were considered for this study. The second criterion for selection was their current use of the technology. This information was asked in item number 12 in the Cognitive Tool Survey. The researcher's colleague divided the respondents into two categories: currently using GeoGebra and currently seldom using GeoGebra. From each category, the colleague further divided the respondents according to the cohort of which they were a part while participating in the program. The reason for selecting teachers from different cohorts who had graduated at different times was to try to understand if some of the factors that affect the use or nonuse of GeoGebra were related to the length of time since they were introduced to the cognitive tool.

From the group of middle school teachers who currently were seldom using GeoGebra, the colleague selected at most two teachers who graduated one year ago, two teachers who graduated two years ago, and two who graduated three years ago. If there were more than two teachers in a cohort, the colleague used the answer to item 13 in the survey to select those who initially used GeoGebra after graduation but had stopped, instead of those teachers who never used the tool. From the group of middle school teachers who currently were using GeoGebra, the researcher's colleague selected at the most two teachers who graduated one year ago, two teachers who graduated two years ago, and two who graduated three years ago. If there were more than two teachers in a cohort, the colleague used the responses to question 14 in the survey, which asked the teachers to indicate approximately how often they use GeoGebra in different instructional activities, giving preference to the teachers who used it in a more variety of instructional activities on a daily or weekly basis. The final criterion was that the participants were willing to participate in the study and to have the researcher observe and interview them.

Data Sources

Surveys

An initial online survey, the Cognitive Tools Use Survey (Appendix E), was given to all 53 teachers who graduated from the program. The survey questions have been modified from the survey, Teachers and Technology Survey: Assessing the Present, Planning for the Future (Buckenmeyer & Freitas, 2005). The survey was modified to make specific reference to GeoGebra. The focus of the survey was to inquire about the teachers' personal information, technology inventory in their classroom, current use of GeoGebra, their technology needs, their expertise in using GeoGebra, and their perceived level of technology use. The levels of use were adapted from the levels of use defined by Hall and Hord (2011). The survey had a Likert scale where teachers specified their level of agreement, ranging from strongly disagree to strongly agree, to a series of statements. The survey took no more than 15 minutes to complete. The results of the survey informed the researcher about the teachers who currently were using GeoGebra frequently, teacher who seldom were using GeoGebra, how long ago they graduated from the master's degree program that introduced them to the usage of the cognitive tool GeoGebra in mathematics, and for what instructional activities they used it. The results of the survey were used to determine the teachers who participated in the balance of the study using the criteria described in the previous section.

A second online survey was given to the selected middle school teachers (Appendix F). The second survey gathered information on the beliefs of teachers regarding the use of technology in education, the nature of mathematics as a subject, the teaching of mathematics, and the learning of mathematics. The survey has been used in the 2008 Teacher Education and Development Study in Mathematics (TED-M) (Tatto, et al., 2008). The researcher of this study has modified some items from the Beliefs about Teaching Mathematics section. The original survey was intended for pre-service teachers and asked questions about their education program; those items were eliminated. The survey has the same Likert scale as the first survey. The survey was given to the participating teachers before any interviews or observations were done, and it took no more than 20 minutes to complete. The researcher examined the individual results of the survey and used the results to guide the interviews. The intention of this survey was to determine whether teachers believe mathematics should be explored by students or should be transmitted, and if they believe they are facilitators of learning. The survey also informed the researcher about the teachers' beliefs about technology, because the reviewed literature showed that this belief might influence the types of activities they create for their mathematics students (Noss et al., 1990). In the case of the teacher users, analysis of these surveys were compared to information gathered from interviews and observations and helped the researcher understand if existing teachers' beliefs influence the adoption of technology in their classrooms.

Interviews

The researcher of the proposed study interviewed each of the six participating teachers who currently were using GeoGebra in their classrooms seldom (Appendix G). Since the researcher intended to understand the teachers' perceptions and understandings, a semi-structured interview was used. Roulston (2010) defined a semi-structured interview as one that consists of a series of open-ended questions that the interviewer follows up with probes, seeking further detail and description about what has been said by the interviewees. Although the interview protocol provided the same starting point, each interview was varied according to how the interviewee responded, which determined the follow up questions to elicit further information.

Teachers described their mathematical and technological experience in context by giving an account of their personal history with mathematics and technology: how were they as math students in school, what mathematics classes they took in college, why did they decide to become mathematics teachers, what experience they had in the NSF program as learners of technology, what is their view about technology in the classroom, and what factors most influenced their decision to not use GeoGebra as an instructional tool for mathematics. Results from this interview added more information on their beliefs, and were combined with the information given in the beliefs survey. In this interview, the researcher intended to understand the concerns of teachers with respect to the technology. Some interviews were performed over the telephone to assure convenience for the participants, while others were done in person. The interview lasted no more than approximately 30 minutes. All interviews were audiotaped with permission of the teachers in order to reduce any loss of content or detail, and later were transcribed verbatim for analysis. Transcribed interviews were given to the interviewed teacher for verification and correction before analysis was done. If teachers wanted to add more information at this time, they were allowed to do so.

For teachers who currently are using GeoGebra, in-depth, phenomenological interviews, as described by Siedman (1991), were one of the main sources of data collection. According to Siedman, "people's behavior becomes meaningful and understandable when placed in the context of their lives and the lives of those around them. Without context there is little possibility of exploring the meaning of an experience" (p. 10). Siedman described a series of three separate interviews with each participant. The first interview establishes the context of the participants' experience. The second allows participants to reconstruct the experience within the context in which it occurs. The third encourages participants to reflect on the meaning their experience holds for them. All interviews were semi-structured and consisted of open-ended questions. The interviews were audiotaped with permission of the teachers and transcribed verbatim for analysis. Transcribed interviews were given to the interviewed teacher for verification and correction before analysis is done.

The first interview protocol (Appendix H) was very similar to the interview protocol for teachers who seldom use GeoGebra since the intention of these two interviews were similar. The interview lasted no more than 30 minutes, and was done either over the phone or face-to-face, at the convenience of the teacher.

The second interview concentrated on the teachers' present experience with the teaching of mathematics and the goal was to gain information about their experience as teachers. The second interview consisted of two parts: before observation and after observation. The first part began with the teacher describing the instruction that recently has taken place in the classroom and his or her specific plans for the lesson that was observed, which includes GeoGebra as a demonstration or investigation tool (Appendix I). The interview focused on the teacher's rationale for the lesson, including his or her understanding of mathematics and why the chosen activities (including GeoGebra) are likely to bring about students' learning. Then the lesson was observed while the researcher took field notes. The pre-observation interview took no more than 10 minutes, and was done either face-to-face or over the phone, at the convenience of the teacher. The pre-observation interview was done on the same day of the observation, or the day before.

The observation was followed by a second interview in order to probe the teacher's impressions on the observed lesson, their reasons for particular decisions or changes in the original plan, and if the teacher thought the students accomplished the learning goals. Their opinion was solicited about the use of GeoGebra, about how students reacted to the experience, and if they feel the use of technology forced them to change their instructional approaches in a way that makes them feel uncomfortable with the new teaching situation. This post-observation interview took about 10 minutes and was done over the phone at the end of the school day or face-to-face immediately after the observation. The second interview, with its pre- and post-observation parts, was done twice because the researcher was interested in observing two different technology lessons in order to gain a better understanding of the teacher's usage of GeoGebra as an instructional tool.

The last interview from Siedman's (1991) set was a focus group with the six teachers present, who all know each other. The goal was to engage the teachers in conversation regarding the use of GeoGebra in mathematics teaching and learning. The focus group addressed the future plans of the teachers and their practice with regard to using cognitive tools. The researcher introduced the topic by posing a question and encouraging discussion. Roulston (2010) suggested having two to five questions, and allowing 10-20 minutes to discuss each one. The focus group took place at the end of the data collection cycle of the proposed study and lasted about 90 minutes (Appendix J). The focus group took take place in one of the teacher's classroom, and the researcher provided food and beverage as a token of appreciation for their participation in the study.

Observations

Observations were conducted in order to get an understanding of the dynamic of the class, the interaction between the students and the teachers, the lesson designed by the teachers that includes GeoGebra, and the teachers' Technological Pedagogical Content Knowledge (TPACK) level of development. Each of the six teachers who currently are using GeoGebra in their classroom were observed twice in order to gain a better perspective of how the teachers use GeoGebra as an instructional tool. The researcher observed the lesson for 50 or 90 minutes and took notes (Appendix K). The researcher did not interact with the students and did not interfere in the activities.

The observation protocol in the study used the Mathematics Teacher TPACK Standards and Indicators and the Mathematics Teacher TPACK Development Model (Niess et al., 2009). The model supported the study by including information about how technological knowledge was implemented and represented in the classroom. The suggested standards were used to gather information on how teachers designed and developed digital-age learning environments and experience. In addition, they illuminated the relationships between teaching, learning, and the mathematics curriculum, as well as provided information about how teachers assessed and evaluated using technology. Lastly, the standards helped to indicate how technology helped teachers enhance their productivity. The development model helped to provide information on the teachers' TPACK levels with respect to curriculum and assessment, learning, teaching, and access to technology.

While observing the technology-rich lessons, the researcher attended to the teachers' instrumental orchestration. In Chapter 1, instrumental orchestration was defined as a teacher's intentional and systematic organization and use of the various artifacts available (technological tools in this case) in the learning environment in a given mathematical task situation.

Researcher Log

The researcher kept a log with comments and observations from the results of the surveys, interviews, and field observations. The log also served as an audit trail.

Data Analysis

While collecting data, the researcher took notes in the researcher log. These notes included possible codes and themes that the researcher noticed from the observations and interviews, and potential links with the theoretical framework of the study. In the first analysis of the data, the researcher focused on the major themes that are described below. In the second analysis of the data, the researcher used the possible specified codes or any others that seemed important to the study using an axial coding approach. During this approach connections are made between the themes and the categories of each theme (Strauss & Corbin, 1998).

In order to answer sub-questions 1 and 3, which dealt with the perceived internal and external factors as well as teachers' beliefs, the researcher used information from the surveys, interviews, and observations. The themes used for analyzing their perceived internal factors that foster or prevent the use of cognitive tools in the classroom were teachers' concerns and beliefs, and each theme was subdivided into categories. For concerns, the categories were personal, management, and consequence. The categories for beliefs were technology, mathematics, mathematics teaching, and mathematics learning. The codes and categories for teacher's concerns have been adapted from Hall and Horde's (2011) Stages of Concerns Questionnaire. The possible codes for each category are shown in Table 3.

The technological pedagogical content knowledge (TPACK) development of teachers was considered as a possible barrier for the implementation of technology lessons. From observations and interviews, the TPACK development was determined using the themes of curriculum and assessment, learning, teaching, and access. The codes and indicators were taken from Mathematics Teacher TPACK Development Model by Niess et al. (2007), and are specified in Table 4.

Table 3

Theme, Categories, and Possible Codes for Internal Factors

Theme	Categories	Possible Codes
Concerns	Personal	• Change in teaching styles; decisions on curriculum; time and energy commitments; changes in classroom roles.
	Management	• Time for planning; conflict between interest and responsibilities; coordination of tasks and people take too much time; inability to manage technical problems.
	Consequence	• Students' attitudes; effects on students; evaluation of students; students' motivation.
Beliefs	Technology	• Anxiety of technology; experience using technology; general classroom structure of activities; structure of GeoGebra activities.
	Mathematics	• Computational focus; conceptual focus; instructional methods used; incorporation of students' prior knowledge, understanding, and thinking.
	Math teaching	• Emphasis on answers; questioning skills; teacher responsibilities; assessment.
	Math learning	• Student understanding; content understanding; student responsibilities.

Table 4

Codes and Indicators of TPACK Development

Theme	Code	Indicator
Curriculum & Assessment	Recognizing	• Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum
	Accepting	• Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.
	Adapting	• Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.
	Exploring	• Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.
	Advancing	• Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.
Learning	Recognizing	• Views mathematics as being learned in specific ways and that technology often gets in the way of learning.
	Accepting	• Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.
	Adapting	• Begins to explore, experiment and practice integrating technologies as mathematics learning tools.
	Exploring	• Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.
	Advancing	• Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.

Table 4
Continued

Theme	Code	Indicator
Teaching	Recognizing	• Concerned that the need to teach about the technology will take away time from teaching mathematics.
	Accepting	• Uses technology activities at the end of units, for “days off,” or for activities peripheral to classroom instruction.
	Adapting	• Uses technology to enhance or reinforce mathematics ideas that students have learned previously.
	Exploring	• Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.
	Advancing	• Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.
Access	Recognizing	• Permits students to use technology ‘only’ after mastering certain concepts.
	Accepting	• Students use technology in limited ways during regular instructional periods.
	Adapting	• Permits students to use technology in specifically designed units.
	Exploring	• Permits students to use technology for exploring specific mathematical topics.
	Advancing	• Permits students to use technology in every aspect of mathematics class.

The themes for the perceived external factors were resources, school, and policy. The codes that were used for each theme are specified in Table 5.

Table 5

Themes and Possible Codes for External Factors

Theme	Possible Codes
Resources	Technology available; access to technology; time; and technical support.
School	Leadership support; school-scheduling structure; opportunities for professional development.
Policy	Assessment; textbooks.

The question about the differences and similarities on a teacher's use of technology, sub-question 2, was answered using the information from the surveys, interviews, and observations. The theoretical framework of instrumental orchestration defined the themes of pedagogical context, preparation, and pedagogical action. Pedagogical context refers to the arrangement of teaching settings and artifacts available in the environment. The mentioned instruments provided the information necessary to be able to describe the pedagogical context of the observed lessons. The preparation mode represented the ways in which teachers decide to make full use of pedagogical preparation for the benefit of instruction. The pre-observation interview was used to get the teachers perception of the preparation that took place. The observations verified how the plans were carried out. Pedagogical action involves ad hoc decisions made during teaching about how to perform the chosen pedagogical configuration and preparation mode. Observations and post-observation interviews provided the information necessary to interpret the actions that took place in the lesson.

During the observation, the orchestration types defined by Drijvers et al. (2010) were used as codes (Table 6).

Table 6

Themes and Codes for Instrumental Orchestration

Theme	Code
Pedagogical context	Technology inventory, classroom arrangement, students in class, topic to be covered.
Preparation	Decisions about the way an activity is introduced and worked out, open-ended activities, specific answers required, questions to be asked, schema and techniques to be developed, consideration of needs of students, equitable access to technology for all students.
Pedagogical action	Technical-demo, explain-the-screen, link-screen-board, discuss-screen, spot-and-show, Sherpa-at-work, reflection, others.

A summary of the data sources that were used to answer the research questions can be seen in Table 7.

The information obtained from surveys, interviews, and observations was cross-referenced in order to answer the main research question, which refers to why and how middles school teachers integrate cognitive technology in their classrooms. All research questions were answered based on the information obtained from the teachers who currently are using GeoGebra in their practices and from the teachers who are not using it.

Table 7

Research Questions and Data Collected

Question	Data Collected
Why and how do middle school mathematics teachers integrate dynamic mathematics learning environments in their practices?	Surveys 1 & 2 Interviews Observations
What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?	Surveys 1 & 2 Interviews
What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?	Surveys 1 & 2 Interviews Observations
What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?	Survey 2 Interviews Observations

Validity

Since the researcher of the study knew the participants and had a direct connection to them, the research could be categorized as "backyard research" (Creswell, 2009). This type of research can introduce new sources of bias and threats to validity; therefore, the researcher utilized several strategies to strengthen the validity of the data analysis. The researcher used researcher reflection, peer examination, and member checking to verify findings. Researcher reflection refers to a sensitivity of the researcher when relating to the situation being studied. As part of this researcher reflection, the researcher clearly defined and documented the role, relationships, and assumptions done during the study (Gall et al., 2007). The researcher kept a researcher log for these notes. Peer examination refers to asking "colleagues to comment on the findings as they emerge and to review a draft of the case study report" (Gall et al., 2007, p. 476). When the researcher thought that bias could have entered in the data analysis or that the effectiveness of the GeoGebra integration was not clear, the researcher sought peer review from colleagues who worked with in-service mathematics teachers and who were knowledgeable in the implementation of GeoGebra. The researcher used the principal investigators of the NSF grant as peer reviewers since they knew the participants, were knowledgeable in the use of GeoGebra, had solid mathematics knowledge since they were mathematicians, and were highly involved in the design of the master's degree program from which the participants graduated. The researcher consulted with these individuals during the data analysis process. Member checking refers to asking participants to review statements in a report for accuracy and completeness (Gall et al., 2007). The researcher asked participants to review the transcripts of their interviews for accuracy.

Reliability

According to Wolcott (as cited in Merriam, 2009), it is not appropriate to consider reliability when studying human behavior. In order to achieve reliability, the researcher must manipulate conditions so replicability can be achieved. To make sure that the results of this study were consistent with the data collected, analysis of individual teacher interviews, observations, and surveys, as well triangulation of the data, was done.

Transferability

This study used a small sample size of only 12 teachers. The sample chosen was representative of the teachers who graduated from the master's degree program, but not equivalent to the general middle school mathematics teachers since the participating teachers in the study volunteered to participate in the master's degree program. The study offered descriptive data so that it will be the responsibility of another researcher to determine if transferability can be done. This study did not intend to be transferable but intended to add knowledge to the current studies of how teachers integrate technology into their practices.

Limitations and Delimitations

The purposeful selection of the participating teachers presents limitations to the study. Participants were the teachers enrolled in the master's degree program. This group of teachers might not represent the general population of middle school mathematics teachers. Some instruments of this study present potential limitations as well. The interview protocols were researcher-designed and had not been used in any other studies. Although the surveys were gathered from other researchers, some of the questions were modified for the study, and none of these instruments had been used previously as such. There is, therefore, no information regarding the reliability of these instruments, which could pose a limitation to the study. Triangulation of the data helped establish the reliability of the surveys, however. Additionally, data from the surveys and interviews were self-reported by the participants. Research indicates that self-reported data often are biased. Classroom observations, however, provided practice-related data, which may balance possible self-report biases.

A delimitation of the study is that the participating teachers were chosen from a very particular group of teachers who have graduated from the same master's degree program offered at the higher education institution. Another delimitation is that the study concentrated on the use of the dynamic mathematics learning environment called GeoGebra and not on any other cognitive tool that teachers might use in their classrooms. Due to the qualitative nature of this study and the small number of participants, the findings are not generalizable. Although not generalizable to larger populations, this study serves to share the experiences of the participants, to inform future implementation of cognitive tools research, and to address questions previously posed by other researchers.

Role of Researcher

The researcher is an experienced mathematics teacher who enjoys teaching, solving mathematics problems, and using dynamic mathematics learning environments. The researcher took the role of an observer and sat in the teachers' classrooms without interacting with the students, making judgments, or providing any evaluations. The researcher anticipated that the established relationship with the participating teachers not only provided entry to the research site, but also promoted a trust that allowed openness and honesty throughout the study. The researcher believes that all teachers are capable of learning new technologies and integrating them in their practice.

Chapter 4 presents the findings of the two groups of teachers who participated in the study – the seldom or non-users and the users of GeoGebra as an instructional tool.

CHAPTER FOUR: FINDINGS

This chapter presents the research findings of the case studies, grouped to address the two groups of mathematics teachers who participated in the study: the ones currently using GeoGebra and the seldom or non-users of GeoGebra.

The Cognitive Tool Use survey was sent to just 52 of the 53 math teachers who graduated from the NSF sponsored master's program called Standards Mapped Graduate Education and Mentoring (SMGEM), since one teacher had moved to another state. Out of those 52 teachers, 25, or about 48%, answered the survey. A colleague of the researcher used the responses to the Cognitive Tool Survey to identify the possible candidates according to the specifications outlined in the previous chapter. Of the 25 respondents, 4 were teaching in high school, and 1 was teaching science in middle school. Table 8 classifies the remaining 20 according to their year of graduation from the program and their use of GeoGebra as an instructional tool.

Table 8

Distribution of Respondents to Cognitive Tool Survey

Graduation Year	Users	Non-Users
2008	3	0
2009	3	5
2010	2	4
2011	0	3

From the group of middle school teachers who currently were seldom or not using GeoGebra, selected at most were two teachers who graduated in 2011, two teachers who graduated in 2010, and two who graduated in 2009. Since there were more than two teachers in each cohort, the answer to item 13 in the survey was used to select those who used GeoGebra after graduation but then stopped using the tool, rather than those teachers who never had used the tool. As a result, the two teachers from the 2009 cohort were identified. At this point, none of the 2010 cohort used GeoGebra in the classroom and only one teacher from the 2011 cohort used it more after graduation than at present. In order to continue the selection from those groups, the answer to item 16, "I need more time to learn to use GeoGebra," was used to select the teachers who answered disagree or strongly disagree to that item. The rationale was that those teachers must have other reasons for not using GeoGebra in the classroom since it was not their belief that they lacked the necessary knowledge about the software.

From the group of middle school teachers who currently are using the software in their math classrooms, none were from the 2011 cohort; on the other hand, the three teachers who responded to the survey from the 2008 cohort are all users. At this point, the researcher decided that it was best to observe six teachers who use GeoGebra from three different cohorts, even if those cohorts were different from the cohorts of the teachers who do not use it. The reason for selecting teachers who graduated at different times from different cohorts was to try to understand if some of the factors that affect the use or nonuse of GeoGebra were related to the length of time since they were introduced to the cognitive tool.

Cohorts 2008 and 2009 had three teacher users in each. The responses to survey question 14, which asked the teachers to indicate approximately how often they use GeoGebra in different instructional activities, were used to give preference to the teachers who used it in a greater variety of instructional activities on a daily or weekly basis. In both cohorts, there were two teachers, or 33%, who responded using GeoGebra every other day or weekly, and one teacher who used it weekly or monthly.

When the researcher sent the second survey to the selected teachers, a teacher user from 2010 cohort moved from the area and stopped teaching math, and a teacher user from the 2008 cohort did not wish to participate in the study. The researcher decided to ask the third teacher from cohort 2009 as well as the third teacher from the 2008 cohort. In summary, the final selection of teachers is described in Table 9.

Table 9

Final Selection of Participants According to Graduation Year

Graduation Year	Users	Non-Users
2008	2	0
2009	3	2
2010	1	2
2011	0	2

The analytic progression used in this study was suggested by Rein and Schon (as cited in Miles & Huberman, 1994). They recommended a natural progression from telling a first “story” about a specified situation or person, to constructing a “map” that locates the key variables or themes, to building a model that shows how the themes influence each other. Carney (as cited in Miles & Huberman, 1994) referred to this analytic progression as a ladder of analytical abstraction consisting of three levels: summarizing and packaging the data, repackaging and aggregating the data, and developing and constructing an explanatory framework. A version of this ladder, with the steps taken in this study, is represented in Figure 6.

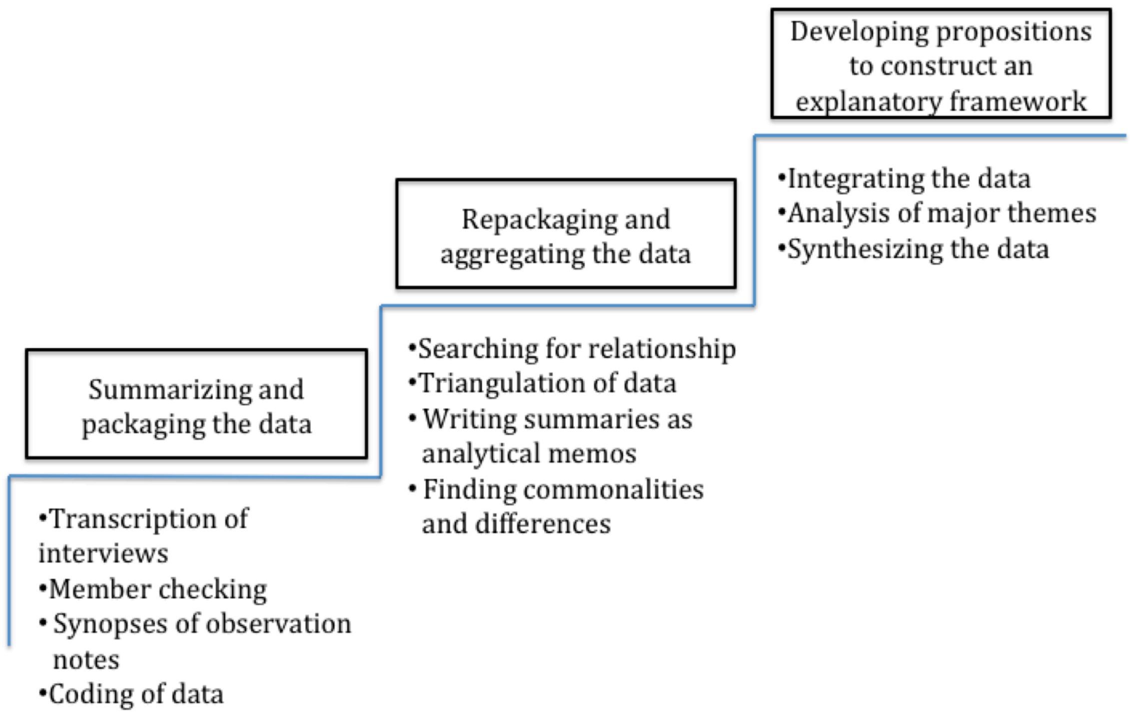

Figure 6. Ladder of Analytical Abstraction (Carney as cited in Miles & Huberman, 1994)

For the first level of the analytical progression, the transcribed interviews were sent back to the teachers for their final approval. Survey responses, approved transcribed interviews, and observation notes, when available, were read and coded using *a priori* themes related to the teachers' perceived internal and external factors that foster or prevent the use of cognitive tools in the classroom: their perceived knowledge of GeoGebra, their concerns and beliefs, their Technological Pedagogical Content

Knowledge (TPACK) development, their experiences in the SMGEM program, and external factors.

For the second level of the analytical ladder, after the data for each teacher was triangulated, it then was used to write in-depth summaries of each participating teacher. Examples of these summaries are in Appendix L. After summaries were done, they were read and coded again according to each theme, with themes subdivided into categories. For GeoGebra knowledge, the categories were basic and implementation. For concerns, the categories were personal, management, and consequence. The categories for beliefs were technology, mathematics, mathematics teaching, and mathematics learning. The TPACK development areas were curriculum, learning, teaching, and access. Experiences in the SMGEM were broken down into math content and technology content. The external factors were resources, school, and policy. Figure 7 shows the breakdown of the themes into categories.

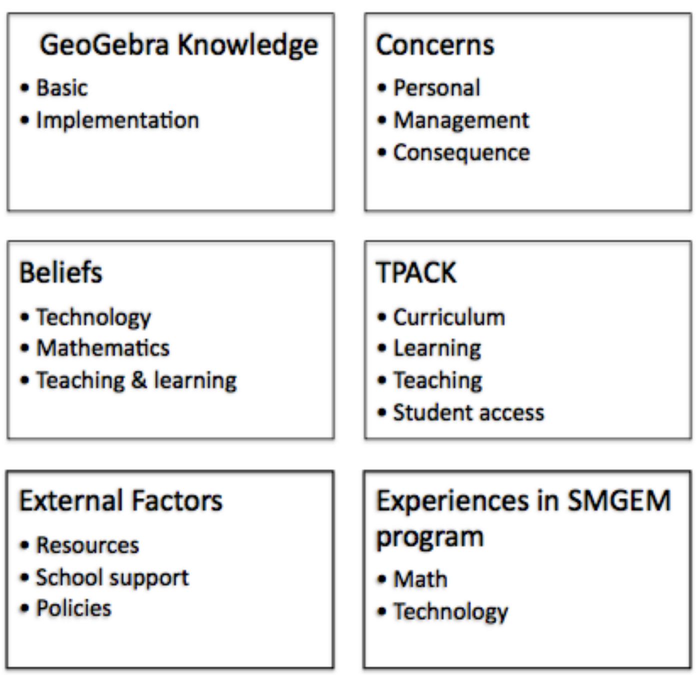

Figure 7. Themes and Categories For Non-Users.

For the third and final level of the ladder of analytical abstraction, the researcher combined the summaries of the teachers in two different groups: the seldom or non-users of GeoGebra, and the teachers who were using GeoGebra as an instructional tool. The data for each group were clustered according to themes, and categories were developed with the intention of answering the research questions of the study.

The findings that follow describe the two groups according to the themes in the data clusters.

Seldom or Non-Users of GeoGebra

Six math middle school teachers were selected for this group from the 25 teachers who answered the survey. These six teachers answered two surveys and were interviewed by the researcher once using a semi-structured interview protocol. The interview was audio taped and then transcribed by the researcher. A copy of the transcribed interviews was sent back to each of the teachers for member checking. The pseudonyms assigned followed this pattern: NU (non-user), year of graduation from the SMGEM program, and either A or B since two teachers were selected from each cohort of graduates from the program. These six teachers are female. Table 10 summarizes their basic educational and professional background information.

Table 10

Basic Background Information of Non-Users

Pseudo-nym	Bachelor Degree	Gender	NSF Workshops Attended after Graduation	Years of Teaching Experience	Number Of Years Using Technology in Classroom	Current Teaching Assignments
NU-09A	Elementary Education	F	1	16	16	6th grade GEM and regular math
NU-09B	Accounting	F	3	10	10	8th grade advanced and regular math
NU-10A	Mathematics & Computer Science	F	1	10	10	6th and 7th grade regular and remedial math
NU-10B	Business Education & Psychology	F	1	8	8	8th grade GEM, advanced, and regular math
NU-11A	Mathematics Education	F	0	8	8	6th grade GEM and advanced math
NU-11B	Biology	F	0	10	10	6th grade regular and remedial math

The teachers had a variety of technology tools in their own classroom available to their students, who average 22 per class. Table 11 reflects those instructional tools.

The rationale for investigating these six teachers was not to generalize the findings but to have a deeper perspective of the teachers' perceptions of what is difficult about technology implementation. The gathered data were used to answer the following research sub-questions:

Table 11

Technology Available to Non-Users

Teacher	Number of Computers Available for Instruction	Location of Computers Available for Instruction	Instructional Technology Available	Average Number of Students Per Class
NU-09A	0	N/A	Portable tablet Clickers	22
NU-09B	1	Classroom	Document Camera Reader LCD Projector	22
NU-10A	1	Classroom	Smart Board Document Camera Reader LCD Projector	22
NU-10B	1	Computer cart	Promethean Board LCD Projector Gizmo	22
NU-11A	19	Computer cart	Promethean Board LCD Projector Students tablets or smart phones Math game: DimensionU	22
NU-11B	1	Classroom	Promethean Board	22

1. What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?
3. What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?

Findings for each theme were used to develop an explanatory proposition for each of these questions. Figure 8 shows the themes used.

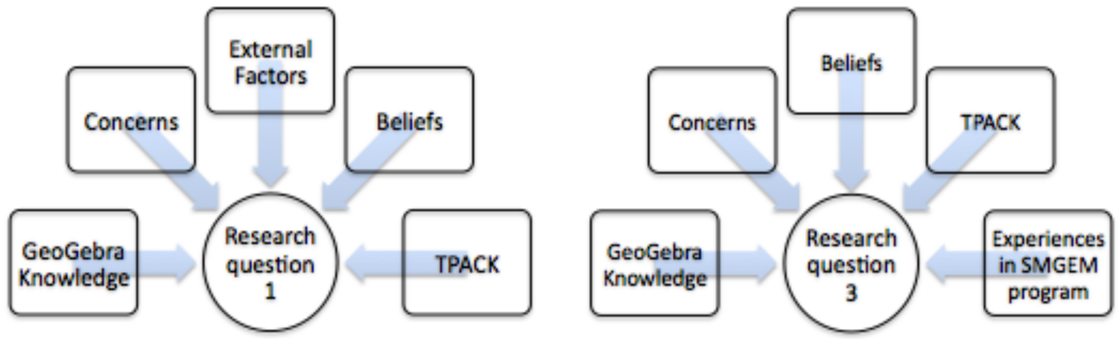

Figure 8. Themes for Each Non-User Research Question.

Teacher Knowledge about GeoGebra

The researcher used the answers to the Cognitive Tool Survey and the transcribed interviews to gather information about the teachers' knowledge of the basic functions of GeoGebra and their ability to use it in their classrooms. Using a Likert scale, ranging from strongly agree to strongly disagree, the teachers were asked to indicate their needs with respect to the use of GeoGebra. The six participants indicated feeling comfortable with their knowledge of the use of GeoGebra. They all agreed to knowing how to navigate through GeoGebra's menus and to use its tools and commands. The seldom or non-user teachers believed they knew when and how GeoGebra (GGb) could enhance their classroom activities, and indicted they could use it for problem solving, investigation, and exploration. However, some of their responses differed in what they felt they could do with GGb in their classrooms. Table 12 summarizes the responses from the Cognitive Tool Survey and from the interviews.

Table 12

Non-User Expressed Capability for Designing GeoGebra Activities

Teacher	Expressed Capability		
	I can design student learning activities that integrate GGb into the daily life of my classroom	I need more examples in the use of GGb in my classroom	I need good reasons why I should incorporate GGb in my classroom
NU-09A	Agree "Whenever I wanted to explore a concept, I would pop up the computers. I used them for my algebra kids, my general kids, I used them for everybody."	Strongly Disagree "I know what to do with GGB, I just don't have computers."	Disagree "I think GeoGebra helped me, but I was also using it to enhance what I was doing, so it worked both ways."
NU-09B	Agree "When trying to find how to use GeoGebra in the classroom, I made an HTML file involving adding and subtracting integers."	Agree "I wanted to bring it into my classroom and I couldn't find a way to link it to the curriculum I had to teach."	Disagree "I think the program is an amazing tool if the kids had access to it at school."
NU-10A	Disagree "My students were having such a difficulty grabbing the basic concepts that introducing that extra…to me it would confuse them more."	Agree "I search the websites for different activities for fractions and things like that, for the visual representations. Until they get the basics, I can't do GeoGebra with them."	Agree "I just found that it would be more confusing to them and I run those ideas passed a couple of other teachers, they all agree with me."
NU-10B	Disagree "I have never used it in my classroom but I can see that it is really good for demonstration."	Strongly Agree "It would be nice if the textbooks use GGb as a resource so we would know how to use it properly to target the standards."	Disagree "I am all for it and I see why it is important."
NU-11A	Agree "The benefit of GeoGebra is that there are a lot of files created by teachers, so I just simply took one, modify it so it goes along with what I was teaching at the moment and use it."	Agree "What I teach now does not cover much geometry, so there is not a lot that I can do with GGB, or that I know."	Disagree "When I use GeoGebra to demonstrate a concept, the students always experience that "Wow" moment."
NU-11B	Disagree "To get them to construct in GeoGebra at the level that I enjoyed it, I am not there yet."	Agree "I do not know if it was me or their immaturity. Maybe I am not using it right."	Agree "I do not know if it was me or their immaturity. Maybe I am not using it right."

Three teachers felt they could not design student-learning activities that integrate GGb. NU-10B had never used GeoGebra in the classroom, and she indicated her desire to have more training on it and to be able to practice designing GeoGebra activities. Teachers NU-10A and NU-11B mentioned that the reasons for not knowing how to design GGb enriched activities were the performance and development level of their students. NU-10A described her students:

> Most of the students in my sixth grade classes are in the lower 30 percentile, 1's and 2's, and in the ESE population. So this current year, out of 22 students in the room, 10 of them in every single class has an ESE disability as well as the students whose IQ prevented them from being in an SVE (Supported Varying Exceptionalities) room, all the way to having 3 autistic kids in one class.

NU-10A felt that her students would not benefit from the use of GeoGebra since the students still were struggling with math basics. She viewed the use of GeoGebra as an extra, fun activity for which she did not have the time to implement in her classroom:

> Their attention span and their capabilities are so low that I thought about it and I searched the websites for different activities for fractions and things like that, for the visual representations. They are just so off base that when they ask questions, they don't have anything to do with the topic we are covering. Also the pacing guide does not give me time to do the fun activities, to take an extra day once we had gone through it, and show them anything. The pacing guide doesn't allow any time extra for these low performing students.

Teacher NU-11B expressed having difficulties with the discipline and immaturity of their students. She also felt the district pacing guide limited the activities she could do in her classroom. She described one of her first experiences using GeoGebra in her classroom:

> I made my first GeoGebra file for exploring the sum of the angles in a triangle. I made a worksheet and got them to explore moving around the triangle. It was an enlightening experience for 10% of my students. The rest of them saw it as a game and saw no connections whatsoever. This is because of their immaturity, and they saw no connection between the measure of the angles and the sum. And others were: wow I can do this! They were changing it and inverting it and seeing that it always was the same sum, and others were just making dumb designs with the triangles. So my experience was good with those higher motivated students who were willing to read the directions and explore the concepts, and the others I had to go and tell them: wait a minute, you have to answer the questions, what do you see here? Stop playing around. It was probably immaturity, and my fault too because I would say: oh this is so much fun, this is a cool thing, you are going to see a concept. So maybe I set it up wrong. You can't force a kid to get out of it what you want them to get out of it. Also, the instructional focus calendar for Broward County is so intense, I pretty much have a topic to do every day and I need to be able to move on to the next topic and I don't want to just cover things, I want to teach it, I want them to understand it.

Even though these two teachers indicated that GeoGebra was beneficial as an instructional tool, they did not see reasons for using GeoGebra with the students they had. In their interviews, teacher NU-10A said: "I think GeoGebra is a great tool, but not for these low kids." Teacher NU-11B mentioned: "This is such a great tool for the kids, but I do not have the time." Teachers NU-10A and NU-11B were the only participating teachers in this group of non-users who were teaching remedial or recovery courses.

Teacher NU-09A was the only teacher who expressed not needing any more training with GeoGebra, and also was the only teacher who did not feel the need to work with colleagues to become more proficient in the use of GeoGebra.

From this analysis of the findings, a first conclusion to emerge would suggest that even though the teachers felt they knew the basics of GeoGebra and how it can enhance their classroom activities, its implementation in the classroom was not so apparent for some of them. The teachers who found it difficult to implement GeoGebra activities were teaching low-level students. They saw the educational value of the software; nevertheless, some had difficulties formulating reasons for using it with their current students. Five of them would welcome the opportunity to attend professional development that would show them specific examples on how to use it in their classrooms. A summary of the main findings in this theme is shown in Table 13.

Teacher Concerns

Responses to items in the Cognitive Tool Survey and the transcribed interviews helped gather information about their concerns and external factors that influenced their decision to not use GeoGebra as an instructional tool. Since none of the teachers indicated needing more information about GeoGebra or needing more time to learn how to use it, personal concerns were not addressed. Table 14 shows their responses on the Cognitive Tool Survey on items related to concerns in the area of management.

Table 13

Non-User Perceived Knowledge of GeoGebra

Teacher	Basic Knowledge	Implementation Knowledge
NU-09A	• Comfortable	• Comfortable
NU-09B	• Comfortable • Would like more training • Would like to work with colleagues	• Needs more examples in the use of GGB with students
NU-10A	• Comfortable • Would like more training • Would like to work with colleagues	• Cannot design GGb activities for students • Needs more examples in the use of GGb with students • Needs reasons for implementing GGb in classroom
NU-10B	• Comfortable • Would like more training • Would like to work with colleagues	• Cannot design GGb activities for students • Needs more examples in the use of GGb with students
NU-11A	• Comfortable • Would like more training • Would like to work with colleagues	• Needs more examples in the use of GGb with students
NU-11B	• Comfortable • Would like more training • Would like to work with colleagues	• Cannot design GGb activities for students • Needs more examples in the use of GGb with students • Needs reasons for implementing GGb in classroom

Table 14

Management Concerns of Non-Users

Teacher	Management Concerns		
	I need more time to plan activities	I need to try pre-made GGb activities before trying in the classroom	I need more instructional time to incorporate GGb activities
NU-09A	Strongly Agree	Disagree	Disagree
NU-09B	Agree	Agree	Disagree
NU-10A	Disagree	Agree	Strongly Agree
NU-10B	Strongly Agree	Strongly Agree	Agree
NU-11A	Agree	Agree	Agree
NU-11B	Agree	Agree	Agree

The teachers' management concerns were very similar. Five, or 83% of them, felt they need more time to plan activities that incorporate GeoGebra, and four of them felt they need more instructional time to incorporate GeoGebra activities in their classroom. Five teachers felt the need to be able to try out pre-designed GeoGebra activities before they were comfortable using them in their classrooms.

In their interviews, NU-09A and NU-11B pointed out management concerns related to classroom management. Teacher NU-11B indicated that her students would act up when she used GeoGebra to demonstrate concepts. NU-09A saw the fact that the computers were working poorly as the reason for problems with her classroom management:

> When you have 20 students in the classroom, you might have 25 computers but only 12 of them work properly, one of them is not starting up, the other one has a crazy screen saver that somebody put on it that keeps running…It becomes a

management issue because you hear the kids: mine does not work, mine is not showing that. Honestly I just got discouraged.

The teachers expressed consequence concerns when they talked about the implications of using GeoGebra with their students. Four of them pointed out the benefits they saw in their students when they were using GeoGebra as an instructional tool. NU-09A remembered:

> My students were engaged, they were excited, and they were picking up things that had not pick up before. One of the things I want to start doing next semester, because I am feeling guilty now, is to take my students to the lab at least twice a month so they get the opportunity to use it.

NU-09B had a similar recollection:

> The students got more enjoyment out of it when they were able to type things up and try things on their own, where now it is much more difficult to do anything using any type of technology, not just GeoGebra, but any type of technology in the classroom.

Teacher NU-10B, who has never used it in her classroom, knew she would like to do projects with her students if she had the software available. NU-11A remembered how much it helped her teach the unit on sum of integers:

> I never thought of that visual way and how it could benefit me and my students to really comprehend the concept of integers, which is a concept that the students struggle with a lot. I used GeoGebra to teach that skill in a different way and they all got it right away.

Two of the teachers did not see the use of GeoGebra with their current students as a helpful tool. These were the teachers who were teaching remedial courses. NU-10A thought it would confuse her students even more. NU-11B felt the use of GeoGebra with her students was distracting and created discipline problems in her classroom, "I can't waste 10 minutes on just one person coming up to the front and acting up." She recalled using it with older students, but still not being a helpful tool for their needs:

> I had a set of computers for three years but I could only use them with the 6th graders and their level of maturity was not there. The tech person uploaded the program in all the computers for me and it was accessible and easy but it was not what I needed to bring my students where they needed to be. It was good for the 7th and 8th graders but they had other work they needed to do.

Hall and Hord (2011) defined several stages of concerns of individuals about any innovation. The stages that were investigated in the study were personal, management, and consequence. In the personal stage, teachers would be concerned about the basics of GeoGebra and their inadequacy manipulating those basic commands and tools. These six teachers felt comfortable with their knowledge on the basics of GeoGebra, and none expressed any concerns at this stage.

In the management stage, attention is focused on the uses of the innovation and its consequences in terms of organization, efficiency, time demands, and management. The participating teachers expressed concerns related to this stage. The concerns in the consequence stage relate to the impact of GeoGebra on the students. Only two teachers expressed concerns about the usefulness of GeoGebra with their low-performing students.

Table 15 summarizes their concerns in the management stage.

Table 15

Concerns of Non-Users

Teacher	Concerns		
	Personal	Management	Consequence
NU-09A	• None	• Classroom management	• None
NU-09B	• None	• Need time to plan	• None
NU-10A	• None	• Need more instructional time • Efficiency of GeoGebra	• Efficiency of GeoGebra
NU-10B	• None	• Need time to plan • Need more instructional time	• None
NU-11A	• None	• Need time to plan • Need more instructional time	• None
NU-11B	• None	• Need more instructional time • Classroom management	• Efficiency of GeoGebra

External Factors

The external factors that prevented this group of teachers from using GeoGebra, as indicated in the Cognitive Tool Survey, are depicted in Table 16.

Table 16

External Factors Expressed by Non-Users in Cognitive Tool Survey

Teacher	External Factors				
	Access to more computers	More technical support	Opportunities to work with colleagues	Administration to provide PD	Administration to provide resources
NU-09A	Strongly Agree	Strongly Agree	Disagree	Disagree	Agree
NU-09B	Strongly Agree	Agree	Agree	Agree	Agree
NU-10A	Agree	Disagree	Agree	Agree	Agree
NU-10B	Strongly Agree	Strongly Agree	Strongly Agree	Agree	Agree
NU-11A	Strongly Agree	Strongly Agree	Agree	Agree	Agree
NU-11B	Strongly Agree	Strongly Agree	Agree	Agree	Agree

NU-09B teacher stated, “Sadly the computers that we have in our school are so outdated that many of them don’t even work.” The ones that work have old operating systems, and do not have the updated software versions. According to NU-11A, “Our laptops are very old, and sometimes they do not run Java properly.” In some schools, the working computers are reserved for FCAT testing. NU-09A pointed out:

> We had a laptop cart for each department that was designated specifically for that department, and that was when I was using GeoGebra. This was 2008. What I noticed is that the laptop carts have not changed since then. Think about our smart phones, they are constantly changing and the technology is improving. The computers are slow, and the students have damaged many of them. The few

> computers that work properly are being used for standardized testing now. We have computerized testing now for 6th, 7th, and 8th grade, so we literally have carts of computers that are pretty much put in a storage room until FCAT because they don't want the computers to get messed up before FCAT.

Consequently, five teachers agreed that technical support in the school is needed. As NU-09B said:

> I will say that there is a definitely need for tech support as well. You need to have somebody to call for help in case you cannot get online or you can't download, or you can't do something with the technology. There should be someone available to help the teachers that are not technically prepared to surpass those problems.

According to teacher NU-09B, her school is a well-off school and yet the money to repair computers is not there:

> My school is not a title I school; it is very affluent. However only a few teachers have technology in the classroom; most of us don't. There were a few grants that came out in prior years, and I think about 10 teachers got smart boards placed in their rooms, but I wasn't one of them. Through the years money has been cut and cut, and unfortunately the computers have slowly died away through the years. I don't know why there is no money to get them back in service, it seems like they find money for other things. I actually have watched the technology disappear from the classrooms before my eyes. It is sad.

Teacher NU-11A mentioned how the GeoGebra keeps improving and newer versions have been released. However, the computers in her school remained the same and the operating systems do not support new releases of the necessary supporting software, like Java, for GeoGebra.

Five of the teachers would like the administration to provide professional development in GeoGebra. Teacher NU-10B pointed out that administrators should become familiar with GeoGebra in order for her school to download the software in their computers.

> I am doing a lot more this year to get the administration to upgrade my system but I think it is going to take time and it is frustrating. I think we need to expose to the administrators and the principal that GeoGebra is an effective tool in mathematics. I think they are not knowledgeable about it, so when we tell them, and we request it, it just falls in deaf ears. I have been requesting it for the last two and one-half years, seriously. They just give more importance to the reading, and math tends to take a second place in their agendas.

In the interview, the researcher asked the teachers what would need to change in their schools or classrooms for them to consider using GeoGebra as an instructional tool. Their responses are shown in Table 17.

Four of their responses relate to their external concerns about the lack of resources. The other two responses exemplify their concerns about consequences on the impact of the use of GeoGebra with their current students.

The external factors were classified as resources available, school support, and school policies. None of the teachers mentioned any factors in terms of school policies.

Table 17

Changes Non-Users Needed to Consider Using GeoGebra in the Classroom

Teacher	Changes Needed
NU-09A	Have my own set of computers for my classroom, but that is not possible.
NU-09B	Computers. We need to have the technology there and we need working computers.
NU-10A	We do have the technology that I can use, and I know how to use it. Put me back with the gifted kids, where I honestly feel I belong. I think I should be back in the GEM classes, or move me to high school where I can teach the algebra and the calculus, that is my ultimate goal and my principal knows that.
NU-10B	Download the GeoGebra in the computers, and provide an in-house workshop.
NU-11A	I would say it is a time factor. Time to prepare the lesson, and make sure I have the GGb file that I really need to teach my students a lesson. Maybe have more updated computers also, that is another problem.
NU-11B	First of all, I would like to have older, more mature and motivated students. But given that I have the class that I have, I think we need a much more strict discipline policy in the school because the kids think they can play and it's all right because nothing is done about it.

Teacher NU-09A had indicated that the working laptops were reserved for the FCAT testing. Five teachers had mentioned the pressure they experienced in covering the curriculum and having the students ready for the FCAT, especially the teachers with low-achieving students. Teachers NU-10A and NU-11B complained about the fast pacing guide they had to follow. NU-09B wanted more instructional time to do everything she had to do to get her students ready for FCAT:

> The problem is that I have the FCAT coming up and my students need to be ready for it. I feel I don’t even have time to cover the material in the textbooks properly. You wouldn’t believe how much time the warm-up activities take and we have to do them every day. Also we lose days when we take the benchmark testing from the district. We need to comply with the requirements and do our best so the students do well on the test, or our paychecks suffer!

The summary of the mentioned external factors can be seen in Table 18.

Beliefs about Technology, Mathematics, Math Learning, and Math Teaching

Responses from both surveys and the interviews were used to gather information on this theme. Since observations were not done with these teachers, it was not possible to compare their beliefs with their practice.

The six teachers believe they are proficient and skillful in the use of productivity tools for professional use like Word, Excel, Power Point, and Flipping Charts, which were tools used frequently in the SMGEM program. In general, they feel comfortable with the use of technology in the classrooms.

As previously mentioned, these teachers either have a Promethean board in their classroom or portable tablets with software similar to the Promethean board. They have laptops, LCD projectors, and document cameras in their rooms. One of them is using electronic clickers, and another is allowing their students to use their own devices and some of the applications like QR Codes readers.

Table 18

Summary of External Factors Found by Non-Users

Teacher	Resources	School support	Policies
NU-09A	• More computers • Technical support	• N/A	• FCAT
NU-09B	• More computers • Technical support	• Professional development	• FCAT • Pacing guide
NU-10A	• More computers	• Professional development • Leadership support	• FCAT • Pacing guide
NU-10B	• More computers • Technical support	• Professional development • Leadership support	• FCAT
NU-11A	• More computers • Technical support • Time	• Professional development	• FCAT • Pacing guide
NU-11B	• More computers • Technical support	• Professional development • Leadership support	• FCAT • Pacing guide

Note: the column heading "External Factors" spans Resources, School support and Policies.

The four teachers who had used GeoGebra more after graduation from the SMGEM program than at present had allowed the students to explore on their own, and had suggested their students download the software at home. Teacher NU-09B pointed out:

> I use it in very small bits and pieces, but I can't integrate the creativity anymore because we don't have the computers for the students to explore. I just use the program in those little bits right now and I tell the kids to go home and try on their own, but I don't know if they are actually doing it. I promote it also in the math club, because those kids are more likely to go on a program on their own and explore. Since the parents sometimes are more receptive than the kids, I tell the parents to download GeoGebra during math night, and have the kids play on it. I also promote the program for use as help with homework. I don't know how far it takes off like that though.

When used in the classroom, the students had opportunities to discuss their discoveries in NU-09B classroom: "It was a puzzle for the kids to solve and the discussions that came up as a class as they solved it were great." The use of technology helped teacher NU-11A with her teaching: "It was a nice bridge between the concrete and the abstract. The students got the concepts faster and it just simplified my teaching."

Since observations were not done on these teachers, conclusions cannot be drawn on how their beliefs about how technology could influence the types of activities they created for their students. From their testimonies it would seem that four of them, NU-09A, NU-09B, NU-11A, and NU-11B, created student-centered activities when they were implementing GeoGebra as an instructional tool. In line with the definitions outlined by Noss et al. (1990), all these teachers seemed to be proactive users of technology since they used it with the goal to engage and motivate their students, and not because their school mandated them to use it. Nothing can be concluded in terms of how their belief about technology impacted their role in the classroom.

The responses from the Beliefs survey about the Nature of Mathematics indicated that five of them believe that math is a collection of rules and procedures that prescribe how to solve problems. NU-09A was the only teacher who was not sure about that statement. They all agreed that students perform better in math when they remember formulas, facts, and procedures, and that some problems can be solved in several ways.

Four of the teachers believe that students can discover rules and concepts when engaged in math tasks. Teacher NU-11A claimed that with the use of technology, students can rediscover concepts and get a better understanding: "When I used GeoGebra to demonstrate a concept, the students experience that 'Wow!' moment. It was something that they never have seen before." NU-09B related a similar experience when using GeoGebra with her students. She even experienced joy when her students got the 'ahas.'

NU-10A and NU-11B were the two teachers who answered not being sure about the statement in the survey about whether when engaging students in math tasks they can discover new things. These were the teachers with the recovery courses. They argued that the fast pacing guide they have to follow prevents them from having the time to introduce extra activities, which includes GeoGebra, in their classroom. They did not see how GeoGebra could help their students when they still were learning the basic rules and procedures of math. These two teachers expressed that math has been their favorite subject since elementary school. Teacher NU-11B was very enthusiastic when expressing her love for math, “Math is so wondrous, if only I can get more of the world to see that. It opens doors, it expands the mind, it has a regimen, it...is MATH.”

In summary, the participating teachers had similar beliefs about mathematics as a subject. Five of them feel math is a collection of rules and procedures. All six agreed that mathematics involves remembering those rules and procedures, as well as that math problems can be solved in different ways. The two teachers with the recovery courses were the only ones who did not agree that students can discover rules and procedures when engaged in math tasks.

Their responses in the Beliefs survey about Learning Mathematics were all very similar to each other. They believe that knowing why an answer is correct or wrong is more important than just getting the correct answer and that it is more important to focus on the conceptual aspect of mathematics than the computational aspect. Teacher NU-09A pointed out how the use of technology, like calculators, can benefit the children who have computational deficiencies overcome those blocks and concentrate more on the procedural aspect of math:

> If the students have problems with computations, I kind of want them to get past the barrier of computation and understand the overall concepts holistically. Sometimes they miss the concepts because they are bugged out on the computation, for which they did not get a solid foundation in the elementary school. So in 6th grade they are not allowed to use a calculator but there is a lot

> of computations that are even complex for these elementary kids who are coming in. So they are multiplying two- and three-digit decimal numbers and sometimes if they don't line them up, they get bugged out. So a lot of what I am doing is helping them with the computation because even if they understand the concept, they make a mistake in the computation and it is wrong. Whereas when they get to 7th and 8th grade, they have a little more wiggle room because if they make a mistake with the computation is not a big deal because they are allowed to use the calculator. So we can look through it, and look at concepts, try out some ideas and decide what to punch in the calculator. Now estimate this, is this reasonable? Is it close to what you should be getting? We can do that but with 6th grade is a little tougher.

They believe that teachers should allow students to figure out their own ways to solve math problems, and speed is not necessary. NU-10A and NU-11B were not sure whether students learned best by attending to the teacher's explanations, while NU-09A, NU-10B, and NU-11A disagreed with that statement. NU-09B feels that lectures are the best way to learn math, even though she expressed that was not the way that helped her learn best when she was in high school:

> As far as math classes, I can remember being bored sitting through algebra, and I even skipped some classes and decided it was faster and easier to teach the lessons to myself from the book rather than sitting through the class.

In the interviews, the teachers remembered being good math students and enjoying the experience of learning mathematics. Their responses are shown in Table 19.

Table 19

Non-Users as Math Students in Elementary and Secondary Schools

Teacher	Math Student Style
NU-09A	I was one of those students who would always follow precisely what the teacher prescribed. I would do it exactly the way they told me to do it. I never really struggled with math; I was pretty good at it.
NU-09B	I just did as I was told basically. I remember it being easy to the point that it was almost boring. I did not have to put too much effort into math. We had a lot of rote memorization in elementary school, as far as math goes.
NU-10A	I remember that I always liked math. I always wanted to succeed in everything so I did all my practice. I loved math, I would do my math homework first every night when I got home. In middle and high school teachers would always ask students when they were absent to come and get my notes.
NU-10B	I was a very confident math student. My experience as a whole was not very good because I attended so many different schools because I was constantly moving. So my education was disrupted with lots of changes of addresses but on the whole I was always placed in the highest levels of math classes.
NU-11A	Math was always my favorite subject. I was very lucky to have really good teachers.
NU-11B	I was a normal student. As a matter of fact, I kind of struggled a little bit in elementary school with some things, but I was good at memorizing others. I knew my math tables. That was not a challenge for me. When I went into high school, I started loving math.

Their Beliefs about Teaching Math also were very similar. They all agreed that math learning is dependent on the social environment. Only teacher NU-11B was not sure whether hands-on activities are worth the time and expense. NU-10A and NU-11B believe that in order to be good at math, you need to have a 'mathematical mind;' the others do not think it is necessary. They all agreed that effort is more important than natural ability, and that mathematical ability can change throughout a person's life.

The teachers' responses when asked to describe some of the past experiences teaching math with the help of GeoGebra are shown in Table 20.

One relationship that seems to emerge from the analysis of the findings in the area of beliefs about learning and teaching mathematics is between the two teachers who have the recovery or remedial courses and their belief that in order to be good at math, students must have a kind of 'mathematical mind.' It appears that the estimation of their students' potential to perform in math might be based on more than the actual performance of their students. A teacher's view of student potential can influence their decision on what math activities to incorporate in their teaching. From their testimonies of past teaching experiences using GeoGebra, it can be seen that NU-10A did not give a specific example and teacher NU-11B recalled an example of a GeoGebra file that she used to refresh students memory, but not with the intention to increase their level of understanding or thinking. The use of that file seemed to be more for her benefit of not having to repeat herself than for the students' benefit.

Table 20

Non-User Usage of GeoGebra in their Teaching

Teacher	Use of GeoGebra in Teaching
NU-09A	I loved it, I really did. There was a point in time where I housed the laptop cart in my classroom and could use them whenever I wanted. It was exciting because I could see the excitement in the kids faces when I would take them out, and they would be engaged and I did not have to worry about them being bored or not paying attention because they wanted to do it.
NU-09B	I made an HTML file involving adding and subtracting integers. The file I made had a few circles and inside the circles were different numbers and the kids were able to drag other integers into each circle to make the circles sum up to a certain amount. It was a puzzle for the kids to solve, and the discussions that came up as they solved it were great.
NU-10A	When I started the program, I was in another school in the magnet program teaching the GEM courses. I left that school, and I honestly, have not been able to use GeoGebra with my students since then.
NU-10B	I have not use GeoGebra in the classroom.
NU-11A	Last year I used GeoGebra to teach the students angle relationships. I used a file from the wiki and changed a few things so it would match my lesson better. The students enjoyed it and with the visual, they understood the relationships better. Before I used to fold a paper to show corresponding angles, but now I can just move the slider and the angles overlap.
NU-11B	I used certain examples over and over again so the children could always go back to the same example. I did that with GeoGebra, when I showed them that circular thing, I would go back and if I was talking about circumference or anything with a circle, I would go back and say remember the parallelogram that changes into a circle, what did you see there? I could go back to the same example and I knew they were visualizing it. I appreciated that because I hate repeating myself over and over and over on major concepts and I can use the same concept through out the year and build on their memory.

When the participating teachers were asked in their interviews if GeoGebra is better suited for GEM or advanced students than regular or remedial students, the responses were mixed. These responses can be seen in Table 21.

Table 21

Non-User Response to Whether GeoGebra Better Suited for Advanced Students

Teacher	Response
NU-09A	I think the advanced students have more resources at home, and a lot of them are more apt to explore at home, so when they come to the classroom, they are really on and they are ready, and they understand how to use the tools.
NU-09B	I do not think the level is an issue. Advance or remedial, I think the exposure is important. Any kid, regardless of their level, can learn from the program. I don't know how to get the kids the tools necessary though because of the money situation and the economy.
NU-10A	My students were having such a difficulty grabbing the basic concepts that introducing that extra…to me it would confuse them more, because their attention span and their capabilities are so low.
NU-10B	I think both. If I could get the GEM kids to use it, I would need to have it downloaded in the media center because I am sure they would pick it up very well and I could have them do lots of projects with it. For the regular kids I would use it to demonstrate some key concepts in math and maybe offer for extra credit for those who want to try it at home.
NU-11A	The students, advanced and regular, were able to comprehend the concept better because it was very visual for them, and it was easy to follow along.
NU-11B	I could only use it with the 6^{th} graders and their level of maturity was not there. It was good for the 7^{th} and 8^{th} graders but they had other work they needed to do.

Teacher NU-09B was the only teacher who agreed that students learn best when attending to the teacher's explanations, even though she skipped lectures as a math student herself. In her interview she kept referring to the use of discussions in her classroom:

> I strive to keep that type of mathematical discussion throughout my teaching because listening to the kids talk about the math is entirely different than teaching them. I enjoy that a lot, and I think they do also.

The six teachers described themselves as good math students who enjoyed the subject from early ages. Most of them recalled doing their math homework first, doing all the practice required, doing as they were told, and wanting to succeed. Although only two of the teachers have bachelor degrees in mathematics, they all had decided to become math teachers eventually. Their past experiences could be related to their beliefs that effort is more important than natural ability when doing math problems, that speed is not important, and that students can figure out their own ways of solving math problems.

A summary of their beliefs is shown in Table 22.

TPACK Development

The analysis on the TPACK development stages was based mainly on the teachers' interviews and also on their responses to the Cognitive Tool Survey. Since no observations were done, it was not possible to compare their practice to their verbal comments. The stages of the model, in ascending order of development, are: recognizing, accepting, adapting, exploring, and advancing. From their descriptions and using the Mathematics Teacher TPACK Development Model, some of the stages were more easily identified than others, as seen in the Table 23.

Table 22

Non-User Beliefs about Technology, Mathematics, and Math Learning and Teaching

Teacher	Belief			
	Technology	Mathematics	Learning	Teaching
NU-09A	• Use technology every day • Proactive users	• Remembering necessary • Students can discover rules	• Conceptual focus • Correct answer not important	• Hands-on activities effective • Math ability can change • Effort more important
NU-09B	• Use technology every day • Proactive users	• Collection of rules • Remembering necessary • Students can discover rules	• Conceptual focus • Correct answer not important • Teacher explanation necessary	• Hands-on activities effective • Math ability can change • Effort more important
NU-10A	• Use technology every day • Proactive users	• Collection of rules • Remembering necessary	• Conceptual focus • Correct answer not important • Not sure if teacher explanation necessary	• Hands-on activities effective • 'Math mind' necessary • Math ability can change • Effort more important
NU-10B	• Use technology every day • Proactive users	• Collection of rules • Remembering necessary • Students can discover rules	• Conceptual focus • Correct answer not important • Effort more important	• Hands-on activities effective • Math ability can change • Effort more important
NU-11A	• Use technology every day • Proactive users	• Collection of rules • Remembering necessary • Students can discover rules	• Conceptual focus • Correct answer not important • Effort more important	• Hands-on activities effective • Math ability can change • Effort more important
NU-11B	• Use technology every day • Proactive users	• Collection of rules • Remembering necessary	• Conceptual focus • Correct answer not important • Not sure if teacher explanation necessary	• Hands-on activities effective • 'Math mind' necessary • Not sure if math ability can change • Effort more important

Table 23

TPACK Development Stages of Non-Users

Teacher	TPAC Development Stage			
	Curriculum	Learning	Teaching	Student access
NU-09A	Advancing	Advancing	Advancing	Advancing
NU-09B	Accepting	Advancing	Advancing	Advancing
NU-10A	Accepting	Recognizing	Recognizing	Not determined
NU-10B	Recognizing	Not determined	Not determined	Not determined
NU-11A	Advancing	Advancing	Advancing	Advancing
NU-11B	Accepting	Recognizing	Recognizing	Accepting

The teachers from the 2010 cohort were the teachers who reported never to have used GeoGebra as an instructional tool. Since they never had used it, it was understandable that they did not seem to progress through the stages of development. Teacher NU-10B talked more about what she would do if the technology were available, but did not give any specific examples of past experiences. Therefore, it was difficult to determine a possible stage for her TPACK development. In the first survey, Teacher NU-11B responded that she never used GeoGebra as an instructional tool after graduation; however, during her interview she related instances of using the software with her students.

Teacher NU-10B recognized that mathematical ideas displayed with technology can be useful for making sense of topics addressed in the curriculum, but she had never used technology in her teaching. For this reason, she would appear to be in the first stage of development in the curriculum area. Three teachers appeared to be in the accepting stage for curriculum. They expressed desire but demonstrated difficulty in identifying topics in their own curriculum for including technology as an instructional tool. NU-09B said: “I wanted to bring it into my classroom and I couldn’t find a way to link it to the curriculum I had to teach.” For NU-10A, learning the basics of math does not mix with the use of technology: “They don’t know the basic facts, and I am still doing times drills with them.” When referring to her students using GeoGebra, teacher NU-11B related: “So to get them to construct in GeoGebra at the level that I enjoyed it, I am not there yet, I need to go to high school or something.”

In the same curriculum area, teachers NU-09A and NU-11A appeared to be in the advancing stage. They both have developed innovative ways to use technology to develop mathematical thinking in students, sometimes modifying the curriculum. NU-09A described how she used GeoGebra as a teaching and learning tool:

> I would give them a set of instructions to construct things, and a lot of them were doing them at home and they were using the constructions to help them with assignments. When we were looking at different types of equations with my algebra kids, they were really getting into it. They were really engaged.

Teacher NU-11A did not limit her use of technology as an instructional tool because of the lack of working computers; instead, she created activities where her students could use their own devices, like smart phones and tablets:

> I use the QR code app. It is an application that the students can download in their smart phone, if they have a tablet, they can download it in their tablet. As teachers we create math problems and put it in the QR code and the students use their own devices, they scan it to reveal the math problem and then they answer it.

Teachers NU-10A and NU-11B appeared to be in the recognizing stage of the learning area of the development model. They view technology as getting in the way of learning mathematics for their students. These are the teachers who are teaching the low level or remedial courses. They both expressed that the inclusion of GeoGebra would confuse their students. As NU-10A pointed out:

> I just found that it would be more confusing to them and I run those ideas past a few other teachers at the school who also went through the SMGEM program and we all came up with sort of the same thing, until they get the basics, we couldn't do GeoGebra with them.

NU-11B concurred: "It was enlightening experience for 10% of my students. The rest of them saw it as a game and saw no connections whatsoever."

Three teachers appeared to be in the advancing stage of the learning area. They plan, implement, and reflect on teaching and learning with concern and personal conviction for student thinking and understanding of math, allowing students to explore math topics. Teacher NU-09A described her students' behavior while working with GeoGebra: "They were engaged, they were excited, and they were picking up things that had not picked up before." Teacher NU-09B had a similar experience: "I know when I am able to bring up a GeoGebra file showing area and/or volume the students become much more involved, they ask more questions and they seem more interested." Teacher NU-11A felt that GeoGebra made her teaching easier as well:

> It made my life easier just to have this program and connect what the students were doing with hands-on activities to be able to connect it in a visual way. The

students, advanced and regular, were able to comprehend the concept better because it was very visual for them, and it was easy to follow along.

In the area of teaching, two teachers appeared to be in the recognizing stage and three in the advancing stage. Teachers NU-10A and NU-11B are concerned that the use of GeoGebra will take away from teaching mathematics to their current students. Teacher NU-11B expressed it best when she said:

> I find that in order to answer one question, it takes 10 times longer because they want to play with it. This is such a great tool for the kids, but I do not have the time. The instructional focus calendar for Broward County is so intense, I have pretty much a topic to do every day and I need to be able to move on to the next topic and I don't want to just cover things, I want to teach it, I want them to understand it.

The descriptor of the advancing stage of development in the area of teaching delineates teachers who consistently accept technology as instructional tools and manage technology-enhanced activities in ways that maintain student engagement and self-direction. The narratives of teachers NU-09B, and NU-11A exemplified those descriptors.

According to NU-09B:

> The kids would come up to the board, because I had a smart board at the time, and they would drag the integers into the circles, they would have to tell why they were putting that number into a particular circle, and the kids would have to talk about the integers and how they worked together with adding and subtracting. It created a mathematical discussion within the classroom where

> kids were actually able to talk with each other and learn something through it. It was nice as a teacher to not talk but listen to the reasoning of the kids.

NU-11A added: "I never thought of that visual way and how it could benefit me and my students to really comprehend the concept of integers, which is a concept that the students struggle with a lot. I used GeoGebra to teach that skill in a different way."

In the area of student access, one teacher appeared to be in the accepting stage and three in the advancing stage, while two did not describe any experiences where students had access to technology. Teacher NU-11B described past experiences of students using technology in limited ways during regular instructional times. She had a laptop cart available for sixth grade students' use; however, she seldom used it:

> The tech person uploaded the program in all the computers for me and it was accessible and easy but it was not what I needed to bring my students where they needed to be. It was good for the 7^{th} and 8^{th} graders but they had other work they needed to do.

The three teachers in the advancing stage had limited availability to computers. They encouraged their students, and still do, to upload the program at home and use it. The teachers see technology as an opportunity to challenge the mathematics that their students can master. According to NU-09A: "I would give them a set of instructions to construct things, and a lot of them were doing them at home and they were using the constructions to help them with assignments."

NU-09B stated:

> The lack of computers has been the main factor. Every single year I tell the kids they have to go home and download the program and see what it can do. I

> usually do this during the first few weeks of school, and if I have a pre-made file on surface area or something that I know everybody can sit as a group and watch on the screen I use the program during class. I use it in very small bits and pieces like that but I can't integrate the creativity anymore because we don't have the computers for the students to explore. I just use the program in those little bits right now and I tell the kids to go home and try on their own, but I don't know if they are actually doing it.

NU-11A also said: "The students found GeoGebra very interesting and they asked me if they can use it at home."

Experiences in the SMGEM Program

All six teachers described the graduate program as an enjoyable yet challenging experience. Many of them especially remembered the helpful collaboration they had with their classmates. NU-09B liked it more than her bachelor program experience: "I did it when I was older and I found it much more interesting and enjoyable because we were able to collaborate and there was no collaboration as far as I remember in any of my prior college experiences." NU-11B had a similar experience:

> I loved every minute of it, because I found out in graduate school, that in elementary and secondary school, I was not that challenged. I liked the idea of using math in so many intricate ways to figure out something. I think that is what I loved about graduate school because it was on the edge for me every minute I had to find everything.

The main goal of the program is to improve teacher content knowledge in mathematics. NU-09A remembered: "There was a lot of work involved, but it also helped me to conceptualize things that I had not conceptualized before and helped me really see the proof for the math." The use of GeoGebra in the program was a key component, since it helped them understand math concepts more deeply. NU-09A added:

> I can remember when I first saw the link of the algebra on the screen with the geometry picture we were drawing and the value it brought was indescribable. I was thinking how important it would be to show the kids this link between the geometry and the algebra.

Teachers were asked to recall a specific occasion in the program when they were using GeoGebra to explore a math concept. Table 24 shows their responses.

The level of the topics covered in the program reached beyond the middle school level many times, but the professors, assistants, and classmates always were ready to help. According to NU-10A:

> It was definitely more of the high school math in our assignments and in our programs, so that got me to go back to things that I liked. It was challenging, but we could search for help towards an answer or work with somebody that could clarify it, so I liked it. I thought it was right on track.

Table 24

Examples of Non-User Usage of GeoGebra in the SMGEM Program

Teacher	Use of GeoGebra in SMGEM Program
NU-09A	I did a project on the Pythagorean Theorem and I remember this whole presentation about how you can use GeoGebra to prove the Pythagorean Theorem. I think it was a pedagogy conference in the third year that we had to present. That really sticks out to me, really putting the project together. It sticks out because there was a lot of work involved, but also helped me to conceptualize things that I had not conceptualized before and helped me really see the proof for the math.
NU-09B	I can remember a time at the beginning of the program when we were learning how to use GeoGebra, and we were just playing around with the points and seeing what would happen when we were told to construct an equilateral triangle. I think it was one of the first constructions we did. I can remember when I first saw the link of the algebra on the screen with the geometry picture we were drawing and the value it brought was indescribable. I was thinking how important it would be to show the kids this link between the geometry and the algebra.
NU-10A	It had to be the lesson on tessellations. It was making the whole unit; we put it all together, and then having to rotate it and translated. I thought it was unique because instead of drawing and cutting, you had it all in front of you, you could pull pieces out. GGb did a lot of the work and made it easier for you to understand the concept. We could readjust everything… I really enjoyed that one.
NU-10B	There were so many. One example I can give you was when we were using GeoGebra to demonstrate why the formula of the volume of a cylinder uses the circumference of a circle using a parallelogram and you switched it you could see the parallelogram becoming smaller and smaller to become the circumference of a circle. I thought that was really cool how that was demonstrated.
NU-11A	I remember being amazed by the visual representations of math concepts that are so abstract and how easy can be understood with a picture. For example, the square of a binomial. Once you see the picture in the geometry window, you never forget that it has also those two rectangles.
NU-11B	I really enjoyed, and used in my classroom, the circle that opens up and shows all the triangular pieces and the triangular pieces turned around and fill a parallelogram.

The higher level of the topics and the disconnection with what they were teaching in their classrooms created some frustration for teacher NU-09B: “It was frustrating because at the time I was learning all of this about math and GeoGebra, I wanted to bring it into my classroom and I couldn’t find a way to link it to the curriculum I had to teach.”

Teacher NU-11A uses some of the topics and technologies she learned in her teaching: “I had the opportunity to learn additional programs that I can incorporate to make math more fun and attractive for the students, including GeoGebra as one of the programs.”

All of them miss the graduate program and the people involved in it. One teacher became a math coach in her school after graduating from the SMGEM program; another is in the process of being trained to become a department chair. One teacher used ideas from the program when applying for her National Board Certification, which she successfully received. During the interview, teachers were asked if they felt the program helped them increase their knowledge of mathematics and their knowledge in the use of technology that included GeoGebra. The responses were an absolute yes. They all asked to be informed if a similar professional development program opens up in the future.

Table 25 shows the findings in the theme of knowledge acquired in the SMGEM program.

Table 25

Knowledge Acquired by Non-Users in the SMGEM Program

Teacher	Knowledge Acquired	
	Mathematics	Technology
NU-09A	Yes	Yes
NU-09B	Yes	Yes
NU-10A	Yes	Yes
NU-10B	Yes	Yes
NU-11A	Yes	Yes
NU-11B	Yes	Yes

Discussion on the Findings on Seldom or Non-Users of GeoGebra

Table 26 shows the findings for each theme and how they relate to the research questions for this group of seldom or non-user of GeoGebra. The numbers in parentheses indicate how many teachers in this group agreed with the statement.

In this group of six experienced math teachers who graduated from the SMGEM program in three different cohorts, two admitted to never using GeoGebra as an instructional tool, and four said they used it more after graduation than at the present time. None of the teachers worked in the same schools, although three of them worked with other graduates of the program. Their bachelor degrees were varied, as indicated previously, with only two of them having a degree in math or math education.

The first research question asked for the perceived internal and external factors that prevented this group of teachers from using GeoGebra as an instructional tool.

Table 26

Themes and Research Questions for Non-Users

Themes	Categories	RQ 1: What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?	RQ 3: What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?
GeoGebra Knowledge	Basic	• Comfortable • Would like more training (5) • Would like to work with colleagues (5)	
	Implementation	• Difficult for 2 teachers teaching remedial classes • Could not design activities (3)	
Concerns	Personal	None	
	Management	• Classroom management (2) • Time demands (3) • Efficiency of GeoGebra (2)	
	Consequence	• Usefulness of GeoGebra with low-performing students	
External Factors	Resources	• More computers (6) • Technical support (5)	N/A
	School Support	• Professional development (5) • Leadership support (1)	N/A
	Policy	• FCAT • Pacing guide	N/A

Table 26 *continued*

Themes	Categories	RQ 1: What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?	RQ 3: What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?
Beliefs	Technology	• Use technology every day (6) • Proactive Users (6)	
	Mathematics	• Collection of rules (5) • Remembering necessary (6) • Students can discover rules (4)	
	Learning	• Conceptual focus (6) • Correct answer not important (6) • Teacher explanation necessary (1)	
	Teaching	• Hands-on activities effective (5) • 'Math mind' necessary (2) • Effort more important (6) • Math ability can change (6)	
TPACK	Curriculum	• Accepting (3) • Advancing (2) • Not determined (1)	
	Learning	• Recognizing (2) • Advancing (3) • Not determined (1)	
	Teaching	• Recognizing (2) • Advancing (3) • Not determined (1)	
	Student Access	• Accepting (1) • Advancing (3) • Not determined (2)	
Experiences in the SMGEM program	Math Content		Higher level than needed for middle school
	Technology Content		Extensive

They all felt that the SMGEM program gave them the basic knowledge of GeoGebra and other technologies. For this reason, they did not display any personal concerns.

The findings in their technology beliefs indicated that they believe technology can have a positive impact on students. The TPACK development level, in the area of curriculum, corroborated the fact that they believe that the use of technology can be useful for making sense of some mathematics topics.

All of them are proactive users of technology in their classrooms, using different technologies like Promethean boards and document cameras every day. However, four of them do not use GeoGebra as an instructional tool in their classrooms. Two of those four are using GeoGebra sporadically, to the point that they do not consider it as being used. There are two teachers who never had used it, not even after graduation from the program.

The two main reasons given for not using the software are lack of working computers and skill level of the students. According to four of the teachers, the laptops available for students' use are very old and slow, have missing parts, run on old operating systems, and do not support the newer releases of GeoGebra. Some testified that the few working computers available are being saved for computerized standardized testing. Lack of working computers is an external factor, and the teachers did not have any control over that fact.

The other reason given by two of the teachers was the performing level of their current students. Those teachers explained that the use of GeoGebra in their classroom would take valuable instructional time away from the students since they still were learning the basics, like the multiplication tables. One of the teachers was concerned about her students misbehaving when using GeoGebra. As she explained, her students are so immature that they only want to play when she tries to use GeoGebra as an instructional tool. Both of them agreed that they had difficulties designing GeoGebra-rich activities for the students. Their beliefs about math teaching indicated they agreed that students need to have some type of ‘mathematical mind’ to perform in math. It would seem that this belief might be related to their experience. They were the only two teachers who indicated having consequence concerns, as described by Hall and Hord (2011).

Other concerns expressed by the teachers were a consequence to the lack of working computers. Teachers indicated that the laptops in the cart that students use created disruptions in the classroom since some of them did not work properly and the teachers had to spend time fixing technical problems instead. Time was another management concern that teachers talked about. Some of them felt they need not only time to plan the activities but also more instructional time to implement the activities. They feel pressured for time since they have to get the students ready for FCAT and the pacing guide given by the district moves fast. The pacing guide and the FCAT are external factors that create management concerns in the teachers.

They all believe that, when learning mathematics, a conceptual focus is more important than a procedural one, and that the use of GeoGebra helps with the conceptual understanding of some topics. However, their current circumstances of lack of working computers and performing level of their students seems to prevent them from using GeoGebra as an instructional tool.

The second research question referred to what the teachers thought they learned from the SMGEM program with respect to GeoGebra, and how they were using what they have learned. Responses to the Cognitive Tool Survey and their own testimonies indicated that they believe they know the basics of GeoGebra. However, five teachers are interested in more professional development, and three are interested in help with designing instructional activities that include GeoGebra, and that are appropriate for their own classrooms. Some expressed that the level of mathematics taught in the program is more suited for high school teachers, making it difficult for them to transfer the newly learned skills into their practice.

In general, they enjoyed the program and they learned many math concepts. Some of the concepts they already had learned in high school; however they admitted they understood those concepts better after reviewing them in the program. The teachers said that guidance of the professors and teacher assistants and their expertise in the use of GeoGebra made it possible for the teachers to learn different math concepts. The two teachers with the remedial courses feel their acquired skills and knowledge would be better utilized in high school and they were open about their desire to have a new teaching position. A third teacher also talked about looking into moving to teach in a high school.

Classroom observations were not done with this group of teachers; therefore, the answers on their beliefs about technology, mathematics, and math learning and teaching could not be compared to their actual practices. It was not possible to determine how they are using what they have learned in the program.

Users of GeoGebra

Six math middle school teachers who described themselves as GeoGebra users were selected from the 25 teachers who answered the Cognitive Tool Survey. These teachers answered the Beliefs survey, were interviewed once, observed twice in their classrooms as they taught a lesson using GeoGebra, and participated in a focus group. The teachers were interviewed before and after each observation. The interviews and focus group were audio taped, and transcribed by the researcher. A copy of the transcribed interviews was sent to the teachers for member checking. All interviews followed a semi-structured protocol. The observation was done using an observation tool. One teacher, U-10A, was not able to participate in the focus group due to a death in the family.

The pseudonyms assigned followed a similar pattern from the non-users: U (user); year of graduation from the SMGEM program; and either A, B, or C since different teachers were selected from each cohort of graduates from the program. Table 27 summarizes their basic educational, professional background information, and their gender.

Table 27

Basic Background Information of Users

Pseudo-nym	Bachelor Degree	Gender	NSF Workshops Attended after Graduation	Years of Teaching Experience	Number of Years Using Technology in Classroom
U-08A	Sociology, minor in math	F	5	20	15
U-08B	Elementary Ed	M	5	9	6
U-09A	Business Administration	F	3	9	5
U-09B	Sociology	M	3	9	5
U-09C	Business Administration	F	4	10	10
U-10A	Mathematics	F	2	21	5

The teachers had a variety of technology tools in their classrooms. Table 28 lists the available tools as well as the average number of students.

The teachers' current teaching assignments and the observed classes by the researcher are listed in Table 29.

Teachers U-08A and NU-09A worked in the same school, and teachers U-08B and U-09B were colleagues in the same school.

Table 28

Technology Available to Users

Teacher	Number of Computers Available for Instruction	Location of Computers Available for Instruction	Instructional Technology Available	Average Number of Students Per Class
NU-09A	18	Classroom Computer cart	Promethean Board Document Camera Reader LCD Projector	22
NU-09B	1	Classroom	Promethean Board Document Camera Reader LCD Projector	20
NU-10A	1	Classroom Computer cart	Promethean Board Document Camera Reader LCD Projector	27
NU-10B	18	Computer cart	Promethean Board Document Camera Reader LCD Projector	20
NU-11A	20	Computer cart	Promethean Board Document Camera Reader LCD Projector	22
NU-11B	1	Classroom	LCD Projector	22

Table 29

User Current Teaching Assignments and Classes Observed

Teacher	Current Teaching Assignment	Observation 1	Observation 2
U-08A	7th grade remedial and regular	7th grade remedial	7th grade regular
U-08B	8th grader regular and Geometry GEM	8th grade GEM Geometry	8th grade GEM Geometry
U-09A	8th grade Geometry GEM and Algebra honors	8th grade GEM Geometry	8th grade GEM Geometry
U-09B	6th grade honors and 1 class of 8th grade Geometry GEM	6th grade advanced	6th grade advanced
U-09C	8th Grade Advanced and Geometry Honors (GEM)	8th grade advanced	8th grade GEM Geometry
U-10A	Math coach. Algebra and 8th grade regular math	8th grade Algebra	8th grade Algebra

As with the seldom or non-users, the rationale for investigating these six teachers was not to generalize the findings, but to have a deeper perspective of the teachers' perceptions of what is difficult about technology implementation. The gathered data were used to answer the following research question:

Why and how do middle school mathematics teachers integrate dynamic mathematics learning environments in their practices?

The following sub-questions were addressed:

1. What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?

2. What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?
3. What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?

The same themes and categories as with the non-users were analyzed, with the addition of a new theme, the instrumental orchestration. The instrumental orchestration was divided into three categories: pedagogical context, preparation, and pedagogical action. Figure 9 shows the breakdown of the themes into categories used for the users.

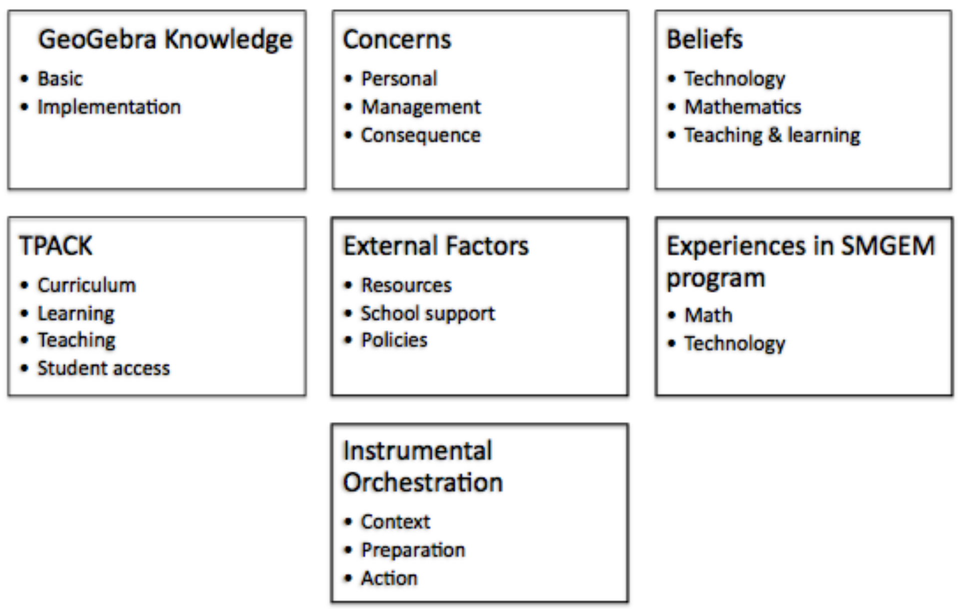

Figure 9. Themes and Categories for Users.

The themes used to find a possible answer to the questions are depicted in Figure 10.

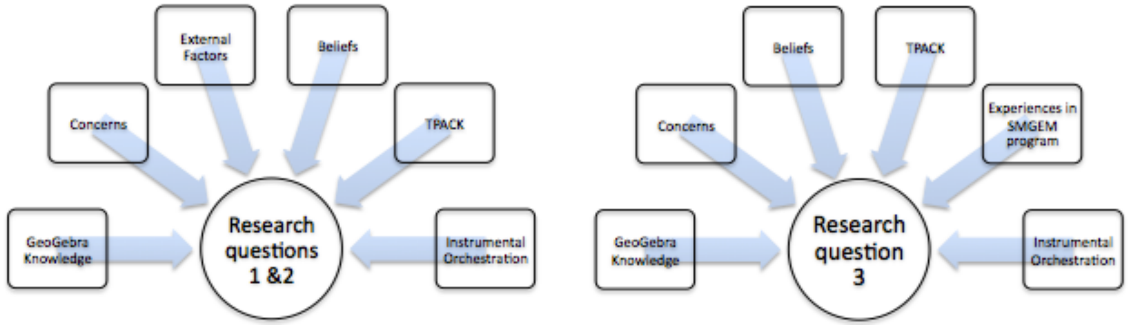

Figure 10. Themes for Each User Research Question.

Teacher Knowledge about GeoGebra

The researcher used the answers to the Cognitive Tool Survey, the transcribed interviews, and focus group responses to investigate the knowledge of these teachers about GeoGebra basic functions and its use in the classroom.

In their responses to the survey, they all indicated knowing the basics of GeoGebra; being able to demonstrate how to use GeoGebra for problem solving, investigation, and exploration; and being able to develop an expanded repertoire of strategies to promote student learning with GeoGebra. They agreed that they can design student-learning activities that integrate GeoGebra, and know how and when GeoGebra can enhance their classroom activities. They know how to locate the learning opportunities needed to advance their GeoGebra skills as new versions arrive.

In their interviews, teachers indicated not only using GeoGebra in their classrooms but also in extra curricular activities at their respective schools, as well as in the district and in a Florida collaborative platform called Collaborate, Plan, Align, Learn, Motivate, and Share (CPALMS). Teacher U-08A was a member of her school technology committee. She described the committee:

It is a group of teachers who want to volunteer their time and have some knowledge of technology to come and serve the school. Different people have different specialties. Some people are the go-to person for Excel, or for Word, or for iMovie, you name it. I am one of the members whose job is to troubleshoot the different software that we use. I am the go-to person for GeoGebra and Excel. If anyone in the school has questions about these two programs, they contact me.

Teachers U-08B, U-09A, and U-09C were working with the district on a special team to align Common Core practice standards to the school calendar, making sure that the alignments are strong. They were developing highlighted problems in Power Point and Flipcharts, and creating scripted podcasts of the lessons using GeoGebra as well as other web-based resources. Teacher U-09B was asked to participate but for personal reasons had to decline.

Teachers U-08B, U-09A, and U-09C also were developing professional development opportunities and digital resources for CPALMS, using the Common Core State Standards (CCSS) for mathematics and GeoGebra. Teacher U-09B also was asked by CPALMS to collaborate, but also had to decline.

During the focus group, the question was asked if they thought being involved in these projects after school helped them to continue using GeoGebra in the classroom. U-08B responded: "Yes, it helps me stay motivated. I see those cool files they make and then I go home and play with it. Now I am trying to do things in 3D." Teacher U-09C agreed:

> Have you seen the file that Duke (district math curriculum specialist) did for mixture problems? It is so cool. It is in the GeoGebra tube. You should see it. I am going to write my next lesson for CPALMS using it and I can't wait to use it with my students.

None of them expressed the desire for more professional development in GeoGebra. Teacher U-09C explained one of the reasons for not needing any professional development:

> We are lucky that we have people that we know we can ask if we don't know how to do something. Every Wednesday, when we get together for the planning of the Common Core materials, I can see what Duke and Guy are doing in GeoGebra and I can ask them. They are always ready to help.

They did not need to see examples of the use of GeoGebra in the classroom, and neither did they need reasons as to why they should incorporate GeoGebra in their classrooms. Teachers U-09A and U-10A would like to have more time to learn about the new releases of GeoGebra on their own.

The survey asked how they used GeoGebra in the classroom as well as how often. The teachers have never used GeoGebra in their classrooms for drill and practice or for testing. The responses on how often they use it in other instructional activities are shown in Table 30.

Table 30

Types of Instructional Activities and Frequency for Users

Teacher	Time Per Week Spent in the Preparation of GGb Lessons	Presentation of New Material	Demonstration	Enrichment	Experiments	Problem Solving
U-08A	More than 60 min	Weekly	Weekly	Weekly	Weekly	Weekly
U-08B	30 min	Weekly	Weekly	Never	Never	Weekly
U-09A	More than 60 min	Weekly	Every other day	Weekly	Every other day	Monthly
U-09B	15 min	Monthly	Weekly	Never	Never	Monthly
U-09C	15 min	Weekly	Weekly	Monthly	Monthly	Monthly
U-10A	15 min	Monthly	Monthly	Never	Never	Monthly

The instructional activity most frequently done by these teachers when using GeoGebra is demonstration, followed by presentation of new material, and problem solving. The interview asked them to describe how they use GeoGebra as an instructional tool. Table 31 shows their responses.

Their use of GeoGebra in the classroom was limited by the lack of laptops available for student use. They had the software downloaded on their teacher laptops, which they project to the students. Only teachers U-08A and U-09A, who teach in the same school, had access to a laptop cart. Teacher U-09A explained how she planned ahead of time when she should request the laptop cart during the school year:

Table 31

User Usage of GeoGebra in the Classroom

Teacher	Use of GeoGebra in Classroom
U-08A	I embed my GeoGebra files into my flip charts when I use for demonstration. Sometimes my students investigate by themselves using the laptops.
U-08B	I use GeoGebra for exploration of concepts, but not as much as I used to, especially since they have the new version because I am not as familiar with that as I was with the old version. I project my computer; I do not have laptops for them to use.
U-09A	I show GeoGebra using my computer. Depending on the concept, if I feel it is something they need to explore then I ask for the laptop cart once or twice a week if I am able to.
U-09B	Mainly it is just me showing GeoGebra on the Promethean board. Getting a laptop cart is not easy, the batteries last about 20 minutes, there are no power strips, you cannot use them for every class, it is not easy, so it is me showing on the projector on the Promethean board. Even if you show it to them it is such a great thing.
U-09C	At the current time, GeoGebra is used as a visualization tool. Individual students use the Promethean Board and GeoGebra to demonstrate for the entire class. My laptop cart is not working efficiently and using GeoGebra on individual student laptops has been hindered.
U-10A	I do not use GeoGebra very often anymore; I need to familiarize myself with the new version. When I use it, it is me showing the students a file, and asking them questions.

In the summer I look at the school schedule and the laptop cart calendar. I really have to be strategic and work out when I can use it, when it is best to use it for what lessons, and what lessons I can try if I can get the cart. It is not as readily available as it used to be, so I find that a lot of the concepts I am showing them from my own computer, whereas before I used the laptop carts more and the students did the investigations individually.

From the responses to the survey and in the interviews it can be seen that this group of teachers seem to be knowledgeable in GeoGebra, especially on version 3.0 of the software. All of them use it as an instructional tool in diverse activities, especially demonstration, and with different students, regardless of their level of performance in math. Three of them use it outside the school while collaborating on other educational projects, and that helps them to stay updated and motivated them even more to continue using it in the classroom. Table 32 shows the findings for the theme of teacher perception of GeoGebra knowledge.

Table 32

Summary of User GeoGebra Knowledge

Basic	Implementation
• Knowledgeable on the old version (6) • Need to learn new version (6)	• Presentation of new material (6) • Demonstration (6) • Enrichment (4) • Experiments (3) • Problem Solving (6) • Drill and Practice (0) • Testing (0) • Outside educational activities (3)

Teacher Concerns

The findings from the previous section illustrate that these teachers are well informed on the software and its demands. Nevertheless, U-08B, U09A, and U-10A displayed personal concerns due to the newer releases of the software. In U-08B's words: "I use GeoGebra but not as much as I used to, especially since they have the new version because I am not as familiar with that as I was with the old version." During the first observation, U-08B could not bring up the software from the web because a new release of GeoGebra was in place, and his laptop did not have the correct Java version to support the release. He was forced to do a software update on his laptop. Once the software was updated, which took a few minutes, he could not open the files he had prepared ahead of time for that lesson since he had used the older version of GeoGebra. He was able to make a new, simpler file from scratch while teaching the lesson. The new release includes new commands and tools. He mentioned during the focus group: "The developers have made it a little more complicated now to use. Before it was more for school and now it seems more for college math."

The new release not only contains new commands and tools but also the user interface has some changes. During the focus group, U-09A stated concerns about the new release as well: "They have been changing the software so much that sometimes I have problems finding the things that I knew where they were before." A few days before observing U-09A for the second time, a new version of GeoGebra was released. She had trouble showing her students how to turn on the grid on the graphics window. This forced the teacher to change her plans during the lesson. After the observation, U-09A reflected on what she would do different next time she taught the same lesson:

“Probably give them an HTML applet they are going to use as opposed to my own file. And just to make sure the grid is visible, so they wouldn’t have to move the screen.”

Teachers displayed some personal concerns on GeoGebra with relation to the new version. They did not have any personal concerns in relation to their role in the use of GeoGebra, their commitment, or the demands of its use.

Table 33 shows their responses on the Cognitive Tool Survey on items related to management concerns.

Table 33

Management Concerns of Users

Teacher	Management Concerns		
	Need more time to plan activities	Need to try pre-made GGb activities before trying in the classroom	Need more instructional time to incorporate GGB activities
U-08A	Agree	Disagree	Agree
U-08B	Agree	Disagree	Strongly Agree
U-09A	Agree	Disagree	Strongly Agree
U-09B	Disagree	Disagree	Disagree
U-09C	Strongly Agree	Disagree	Strongly Agree
U-10A	Strongly Agree	Strongly Agree	Strongly Agree

Teacher U-09B was the only teacher (17%) who felt they did not need any more time to plan or to incorporate GeoGebra activities:

> I can use files from the wiki or I can just make the files that I need from scratch in front of the students and it only takes a few seconds. I prefer to do it that way because I can also show them how the GeoGebra tools work in case they want to

try at home. When an activity is suggested in the book, I know how to change it to GeoGebra.

The other teachers felt pressured for instructional time, especially because of the FCAT testing. Teacher U-08A mentioned:

> I really wish that we did not have to focus so much on FCAT so we could do what we need to do for the students so they could understand the possibilities they have with technology that is available at their fingertips. It takes a while to teach them and we are always against the clock. When I started teaching and when I started the program in 2005, there was no such pressure on the test and the results being published. Now the test scores are being connected to teachers pay. Before we did not have that, so I could finish a unit on data analysis and then I could take a week for the students to complete a project were students could see how what they are studying in school is really connected to what we are doing in real life.

U-08B indicated that he did not use GeoGebra with some of his 8th grade classes, where he faced discipline problems. He wants the students to be occupied at all times and if he has technical problems while teaching them, he could lose control of the classroom discipline. Teacher U-10A felt the need to try pre-designed files before feeling comfortable designing her own. She felt that her files are too simple. During the post-interview of the first observation, she mentioned:

> Although I feel happy with the outcomes of the lesson and the discussions we had thanks to GeoGebra, I feel I am not using GeoGebra to its full potential. I

need to get more acclimated with the new version. I am sure there are new commands that will help my students understand even more.

The personal concern about not being as familiar with the new version as they were with the old version creates some management concerns. In summary, those management concerns referred to unfamiliarity with the newer versions that could potentially create more disruptions in the classroom in terms of discipline, difficulties with its use that could take more instructional time, and the feeling of inefficiency of not being able to use it at its full potential.

None of the teachers expressed any consequence concerns about the impact of the use of GeoGebra in their students. One of the questions in the interview asked the teachers to describe the impact of GeoGebra activities in their students. The responses are summarized in Table 34.

The responses show teachers experience positive reactions from their students and believe the use of GeoGebra as an instructional tool has a good impact on students of all achievement levels. Teacher U-09A said that her more advanced students showed some opposition to use GeoGebra at the beginning of the school year. She pointed out that the students were not familiar with the software unless they had teacher U-08A in previous years. She indicated that her students, in general, were used to doing work that emphasized procedural methods for solution, and were not too keen on taking the time to investigate the concepts behind the procedures. In her words:

Table 34

User Perceptions of Impact of the Use of GeoGebra on Students

Teacher	Perceptions
U-08A	My students are not very confident in their math ability. What I ask them to do first is to try to draw a picture of what they think it would look like. I give my students what I call conference time so they can discuss what they have and try to convince each other. Then when I show it in GeoGebra they say: "I told you, I told you." It is a sense of being affirmed that they could figure it out. It builds their confidence up.
U-08B	These students nowadays are so used to computers and technology that they do not get impressed very easily. Also my geometry students are very good and they really don't need much in order to understand the concepts. In general the students are now immune to the power of technology and they expect not just to see a file but a movie. For some of the regular students, GeoGebra helps them understand some of the abstract concepts because they can visualize it with the computer.
U-09A	They love it, even my ESE students.
U-09B	It is great. You see then getting that "aha" moment. The kids understand something that the other math teachers have been mostly covering in all the grades from 4^{th} grade up, but they don't really understand like π for example, until you show it to them. You think it is so obvious, but it is a big concept to get through to the kids. I have never done the coffee can and the string, but GeoGebra makes it so easy and they get it.
U-09C	When I use GeoGebra, my students get excited and want me to replay the demonstration many times. Any time a file uses transformations (translations, rotations), students carefully and excitedly participate in the lesson at hand.
U-10A	When we use GeoGebra, they are able to ascertain, and even formulate, a conjecture of what they think it is going to happen. It really creates more critical thinking questions and excitement. We have excellent classroom discussions.

My students know they get good results and they rather not do the explorations, they just want to know what they need to know factually. They prefer to do book work. It takes a good half of the year to really get them to see the benefit of it and enjoy it.

During the focus group the teachers were asked to discuss how the use of GeoGebra helped their students learn mathematics. Part of the conversation between teachers U-08A (the one teaching the remedial courses), U-09A, and U-09C follows:

> U-08A: It helps them visualize things that are not so clear for them in the textbook or in their heads. Sometimes I ask them to turn the book in different ways so they can see some pictures from different perspectives and with GeoGebra it can be done more easily.
>
> U-09A: I let my students use it so they can discover some concepts for themselves so they do not have to memorize things and can remember them much easier.
>
> U-08A: I will praise the Lord if my current students are able to discover anything in math by themselves. It just helps them see things that otherwise they could not imagine. I also ask them to use it to check things, like for example points on a line. If the points they found for homework are not on a straight line, then they know they made a mistake someplace.
>
> U-09C: My students like to work with computers and many times I use it for motivation because it is something different and they get hooked. I do not have computers for all of them so I ask them to come to the board and manipulate it using the smart board. They like it and they get involved. Anything to get their attention.

The internal factors or concerns shown by these teachers are very specific. They expressed personal concerns when talking about the new releases of the software. This personal concern created in some of them management concerns of not being able to handle instructional time properly and of having discipline problems. For this reason, five of them want more time to plan activities as well as more instructional time to implement GeoGebra-rich activities. They all agreed that its use as an instructional tool has a positive impact on their students, since the students assimilate math concepts faster, without memorization, and made them feel more confident about their math abilities. Teacher U-09C mentioned that she feels her students respect her more because they see her as an expert on something that not other teachers in her school are using. Table 35 summarizes these findings.

Table 35

Concerns of Users

Teacher	Concerns		
	Personal	Management	Consequence
U-08A	• Knowledge on new release	• More time to plan • More instructional time	• None
U-08B	• Knowledge on new release	• Classroom discipline • More time to plan • More instructional time	• None
U-09A	• Knowledge on new release	• Classroom discipline • More time to plan • More instructional time	• None
U-09B	• Knowledge on new release	• Classroom discipline	• None
U-09C	• Knowledge on new release	• Classroom discipline • More time to plan • More instructional time	• None
U-10A	• Knowledge on new release	• Classroom discipline • More time to plan • More instructional time	• None

External Factors

Table 36 shows the responses on the Cognitive Tool Survey that dealt with external factors.

Table 36

External Factors Expressed by Users in Cognitive Tool Survey

Teacher	External Factors				
	Access to more computers	More technical support	Opportunities to work with colleagues	Administration to provide PD	Administration to provide resources
NU-09A	Strongly Agree	Strongly Agree	Disagree	Disagree	Agree
NU-09B	Strongly Agree	Agree	Agree	Disagree	Agree
NU-10A	Agree	Agree	Agree	Disagree	Agree
NU-10B	Strongly Agree	Strongly Agree	Strongly Agree	Disagree	Agree
NU-11A	Strongly Agree	Strongly Agree	Agree	Disagree	Agree
NU-11B	Strongly Agree	Strongly Agree	Agree	Disagree	Agree

Table 36 illustrates that the teachers would like to have access to better computers for their students' use and that they feel they could use more technical support to make sure the computers are in good working condition. They all agreed that the administration should provide resources, even teachers U-08A and U-09A who teach in a school they described as not typical in terms of the technology available to them. Teachers U-08B and U-09B, who teach in the same school, described how difficult and inconvenient it is to bring the laptop cart to their classrooms. In U-08B's words:

> The carts are housed at the other end of campus. The laptops in the carts are very old. If I get them, the batteries do not last very long, maybe 20 minutes at the most, and then when I get the next group, the batteries run out. We do not have extension cords to recharge them and it is just a hassle.

Teacher U-09B corroborated:

> The operating system in those laptops is older than my students. The computers are so slow, they have better stuff at home; they just don't enjoy it. Students get frustrated; it actually turns some kids off technology. And then if you have three classes in a row, the batteries run out during the first few minutes of the first class. I guess I could by extension cords, but it just would not work.

Teacher U-10A described the computers in the cart in the same way:

> Oh, those are definitely outdated. In my class there are no desktops, the school is getting rid of all of them because they are either broken, or very old. They cannot be used at all. For Internet use, you have to go to the library and we hardly go to the library because it takes too much instructional time.

Teacher U-08B has several desktop computers in his classroom, which he was not using either. He mentioned:

> I have these computers in my room but they are so old that students get frustrated when they use them. Their smart phones are more powerful than those computers. They might use it to check their grades but not to search the web or do anything like that. These computers are not worth anything.

Teacher U-09C had the same problem. She had asked her students to download the program at home and they used it for homework assignments. She understood the students would benefit more from using individual laptops at school; nonetheless, she feels that just by projecting her computer in class, they are having meaningful mathematical discussions.

Teachers U-08B and U-09B both mentioned the possibility of using the computers in the library. Teacher U-09B does not take their students to the library since "those are used for online testing and stuff like that."

U-08A was the only teacher who did not see the need to work with other colleagues. She volunteers her time in the technical team at her school and, as she mentioned during the focus group, she always is eager to share any of her work with her colleagues. However, she sounded disappointed when she mentioned that none of them would come to ask her any questions. Three of the teachers who worked on educational projects after school would welcome the opportunity to work with other colleagues. During the focus group, they inquired about the possibility of the math department at the institute of higher education offering a specialist degree. They also mentioned the possibility of the district offering training once the new books covering the CCSS were adopted.

All six teachers agreed that the administrators should provide more resources; however, they all know how difficult that is, and how improbable it is that more teachers would use GeoGebra if the publishers do not include them in their upcoming textbooks. Part of the conversation from the focus group on resources follows:

U-08A: In a couple of years when we get the new textbooks all that money is going to be marked for training the teachers on how to use the new software. If there is no GeoGebra in those textbooks, then nobody is going to be using it or getting exposed to GeoGebra; that is the way it is going to be. You just have to look at the pattern, the ones with the money dictate what and how they want the money to be used and that is how everybody is going to get their resources.

U-09B: Yes, but we need more resources now. Administrators need to realize that if you invest in teachers it will filter down to the students.

In summary, the teachers expressed similar external factors that hampered the use of GeoGebra as an instructional tool. All six agreed that the outdated existing laptops that students use make it impossible for them to use. Consequently, they would like to have more technical support to make those laptops more useable or have access to better, newer laptops. Five of them would welcome the opportunity to work with colleagues, and they all agreed that administration should provide more resources. All of them had mentioned preparation for FCAT as an impediment to do the enrichment activities they felt would benefit the students. The pacing guide also was a factor that limited their instructional time and freedom to do different instructional activities that could take more time to complete. Table 37 shows these findings.

Table 37

Summary of External Factors for Users

Teacher	External Factors		
	Resources	School support	Policies
NU-09A	• More computers	• Work with colleagues • Leadership support	• FCAT
NU-09B	• More computers • Technical support	• Work with colleagues • Leadership support	• FCAT • Pacing guide
NU-10A	• More computers • Technical support	• Leadership support	• FCAT • Pacing guide
NU-10B	• More computers • Technical support	• Work with colleagues • Leadership support	• FCAT • Pacing guide
NU-11A	• More computers • Technical support • Time	• Work with colleagues • Leadership support	• FCAT • Pacing guide
NU-11B	• More computers • Technical support	• Work with colleagues • Leadership support	• FCAT • Pacing guide

Beliefs about Technology, Mathematics, and Math Teaching and Learning

Responses from both surveys and interviews were considered to analyze the tachers' beliefs in those areas. Observations and the pre- and post-observation interviews were used to compare their stated beliefs with their practices.

In terms of technology, they all feel skillful using productivity tools for professional use and use them every day in their classroom. Most of them started using technology in their practice a year or two after they started teaching, or at the time they started the SMGEM program. Teacher U-08A, one of the two teachers with the most years of experience, started using technology five years into her teaching career. In her interview, U-08A described how she started using technology in her teaching:

> I started typing my lesson plans, my grades, the activities and so forth and so on, so that the next year I did not have to work so hard. I wanted to work smarter, nor harder. But every summer I would take some classes and instead of using what I did last year, I was thinking of new activities. I am always trying new ideas, and giving myself more work. That was my introduction to technology. Then I got into the MSGEM program and the sky was the limit.

One question asked in the post-observation interviews was if they thought the use of GeoGebra as an instructional tool made them change their teaching style. Their responses are summarized in Table 38.

The responses show that some of them think that the use of GeoGebra as an instructional tool gives them more control over the lessons to their students. During the first observation, teacher U-09C made a student take control of GeoGebra and move points in a line while the others were trying to figure out the slope of the line. The students discussed among each other how to do it and what would be the correct answer, while the teacher observed the students interacting for most of the lesson.

Table 38

User Usage of GeoGebra and Changes in their Teaching Style

Teacher	Use of GeoGebra and Changes in Teaching Style
U-08A	Yes, because as you know GeoGebra helps you show in a vivid way the relationships because it is interactive, because it is dynamic. Actually, I already had that teaching style of wanting the students to understand and visualize concepts, but I did not have the tools to express it.
U-08B	With these students you have to be alert all the time. They many times surprise me because they already know some of the concepts or they can guess them correctly. They are the ones that make me change my style, not the software. When they ask questions that I am not expecting, we can investigate the answer together with the use of GeoGebra, it is a great tool to have with these bright students.
U-09A	It definitely changed my teaching style. It has changed and even now is still changing. The students are more in control so that is hard. Sometimes I have to sit down and count in my head to give them time to investigate instead of just telling them the answers. With the use of GeoGebra, students are able to investigate and explain things on their own, whereas before I would just tell the answer, so I think that is the difference.
U-09B	Yes, GeoGebra enables me to easily demonstrate concepts that are otherwise difficult to for me to explain and for the students to understand.
U-09C	It changes me from being a lecturer and a teller to getting the kids more involved, which is important. It makes them responsible for the material. This role is energizing to me, it is fun when you see the kids having fun, it makes it more interesting for me too. At first it is a little strange giving the control of the lesson to the students, but you have to trust your students, and you need to have a good discipline plan in place so things do not get carried away. I can see how a new user of GeoGebra can get a little nervous, if they don't know the technology and they are trying to do the classroom management at the same time, that can be challenging.
U-10A	Yes, with GeoGebra the students are more inquisitive and we end up having more classrooms discussions. With it we can always investigate the question" "what would happen if we change this?"

Teacher U-09A also gave control of the lesson to her students when they were trying to decide if the ratio of dilation was 2 or ½. During the second observation, teacher U-08B changed his lesson plan minutes after he started his lesson when he realized the students knew the relationship of special right triangles. U-10A's students were directing her lesson as they were questioning what would be the solution of different systems of inequalities. In general, the teachers used less lectures and more discussions with their students.

U-08A had more control of the lessons observed. She had given her students a sheet of clear directions, which they were supposed to follow precisely. Her lesson consisted of graphing linear function in GeoGebra. In the post-observation interview, she mentioned:

> I was satisfied with the lesson, because most of the students were able to do the constructions from start to finish by themselves. They 'conferenced' with each other of course, but that is OK. This was a chance for them to be empowered.

Teachers were asked the reasons for using GeoGebra in their classroom. Table 39 contains excerpts of their responses.

Table 39

User Reasons for Using GeoGebra in the Classroom

Teacher	Reasons for Using GeoGebra in Classroom
U-08A	GeoGebra makes the concept that I am teaching alive. For example, last year we were looking at rotations. The questions asked what picture would look like at a rotation of a given angle. Who knows that? I don't, because I am not a visual person at all. It did not make sense to me to come and tell the students this is wrong when they cannot see what it is wrong. With GeoGebra I could show them the same figure rotate at different angles.
U-08B	I think we were all amazed at how the professors used in the program, we were surprised by the capabilities and we wanted our students to experience the same things we did.
U-09A	I think it is due to time efficiency and also the effectiveness in some concepts that I think without it, it would take a lot longer for them to master and wouldn't be as interesting.
U-09B	Ease of use, and cost too. I am a visual learner. For example when we talk about the incenter, the kids question how could a center be on the outside of the triangle. It would take hours to draw different cases, an obtuse triangle with the circle outside. You have to be good drawing on the board, and approximate circles. I do not have those motor skills. GeoGebra makes it easy.
U-09C	As a student, I found GeoGebra to be very helpful and I know that my teaching has been positively affected by the use of technology.
U-10A	In the program, when GeoGebra was being introduced, it helped me with the understanding of the material. I know it also helps my students.

An interesting observation from these responses is that some of the reasons were related to their experiences as students in the SMGEM program. They felt that if it was beneficial to them, it would be beneficial to their students. They talked about their own characteristics as learners, and they seemed to expect the students to resemble them in their learning style. U-09B mentioned he was a visual learner, and he strived to make visualization of concepts possible for his students. In his first observed lesson, he used different files of areas of quadrilaterals and triangles to show his students visually how they are related to the formula of the area of a parallelogram with the same base and height. In the post-observation interview he commented:

> The students gained an understanding of why the formulas for various polygons work, how they are related to the formulas for the areas of other polygons, and hopefully saw visuals to help them memorize the formulas, as opposed to just relying on the reference sheet. The memorization is easy when you can visualize the concepts. The textbook does not show good examples of a parallelogram dynamically "slanting" while keeping the same base and height. This is extremely useful in both relating the area of a rectangle to that of a parallelogram and to explaining why slant height has no bearing on these areas.

The teachers believe that technology has a positive impact on their students' learning. As with the non-users, the teacher users seem to be proactive users of technology. The implementation of GeoGebra in their classrooms made them change their teaching style and provide more student-centered activities than teacher-centered activities. The teachers recognized that the use of technology helped them in the

SMGEM program and were hoping that technology would have the same effect on their students.

The responses in the survey about the beliefs of mathematics were the same for all six of these teachers. They believe math is a collection of rules and procedures, but it is not essential to know the right procedures in order to solve a math problem. One can discover many things in math on their own and try them out because math involves creativity and new ideas. Math requires practice, and it has practical relevance since it can help to solve everyday problems and tasks. They believe that knowing the concepts is more important than knowing the procedures.

The observations done showed that these teachers promoted inquire-based lessons. Even teacher U-08A, who preferred rules and step-by-step procedures, allowed her students to investigate on their own if points were collinear. Teacher U-08B used GeoGebra to help her students discover the geometric mean in right similar triangles. Teacher U-09A designed an activity that allowed her students to come up with an equation to find the sum of the interior angles in polygons. Teacher U-09B showed his students the relationship between equations of areas of different parallelograms. One of his students inquired about the area of a trapezoid, and with the guidance of the teacher, the students came up with the formula for the area on their own. Teachers U-09C started a lesson with a question to her students: "Is it possible to form a triangle with sides of length 1, 2, and 7 units? Why or why not?" In U-10A's first observation, the topic was lines of best fit. The students were using different points on the scatter plot to get the equation of a line that would connect those points. The teacher graphed the different lines and then students discussed which one would be a better fit for all the points. After this activity, they read the instructions given in the book for finding the line of best fit. Students gave their opinions on the appropriateness of those instructions. Her students seemed to understand the concept of line of best fit and how to approximate a line of best fit in a scatter plot.

When asked in the focus group if the sequence of the curriculum changed due to the use of GeoGebra, the teachers responded:

> U-08B: We have a tight calendar from the district that we have to follow. There might be some questions that come up and I try to deal with those questions but I pretty much follow the book.
>
> U-08A: With the FCAT coming up, the only changes that we make are to review concepts to get them ready for the test. Remember, I have the lowest of the lowest, we need to review and review. Sometimes it is hard to cover what we have to cover.
>
> U-09C: No changes really, just like he said, we might talk about other stuff that comes up but we have to keep going. Like for example, I might mention to them that they are going to see something again in Algebra 2, and they will talk about more specifics, but I do not go into showing them that. The objective of the day is written on the board each day and if administrators come they like to see that. Not that they will notice if I am doing something else because when they come to observe me for my evaluation, they send an administrator who used to be a biology teacher and he runs away when I mention some math concepts.
>
> U-09B: If we don't follow the calendar, we are in trouble with the administration and with other teachers in the team. The teachers in the 6th grade

> regular math team use the same tests, and we give tests on the same days. If I digress from the plan, there is no way I can catch up.
>
> U-09A: We have more freedom in the GEM courses; however, the students have to take the end of course exam along with the regular scheduled assessments from the district. I need to have those students ready.

Even though the teachers in this group teach within the parameters of what is in the curriculum, they prepare lessons that are focused on discovery and classroom discussions. They hold a conceptual view of mathematics, and in the observed lessons they stressed the understanding of concepts. At no time were they observed doing drill and practice activities with their students.

Interestingly, the beliefs on teaching and learning math are identical for these six teachers. They believe that when learning mathematics, correct answers are not important if students do not understand the problem, and that more emphasis should be placed on the process followed. They agreed that students could learn math even if they don't attend to teacher explanations, and that students should be allowed to figure out their own way of solving problems.

Their actions in the classroom agreed with those beliefs. The teachers were using GeoGebra to allow their students to find some type of relationship: collinear points, area formulas, interior angles in a triangle, slopes of lines, geometric mean, or systems of inequalities. They were emphasizing concepts, and not showing them procedures for solving problems. Even the teachers who were projecting GeoGebra through their own laptops were allowing students to take control of the lesson and manipulate GeoGebra in the Promethean Board or the teacher's laptop. Student-centered lessons forced some teachers to change their plan on the spot. After the second observation, U-09C was asked if she had made any changes to her original plan. She responded: "GeoGebra allowed me to have instant feedback from my students. When they were explaining and discovering the theorem, I could tailor the lesson. No major changes but I was lead but what the students needed me to go."

Teacher U-10A had a similar response:

> When I am letting the student discuss a concept, I don't know where they will go. I just try to keep the discussion on topic. I was not planning on taking about all those examples of systems of equations, but the students wanted to know and it was an important discussion.

In terms of their beliefs about teaching mathematics, they disagreed with the statement that the use of hands-on models and other visual aids is not necessary. They also disagreed with the statement that in order to be good at math, students need to have a kind of 'mathematical mind;' for them effort is more important than natural ability and ability could change throughout a person's life. They strongly agreed that all students can solve multi-step problems, not only the most able students.

As previously outlined, their teaching assignments were varied, ranging from remedial to GEM courses. During the focus group, they were asked if GeoGebra was better suited for high-performing students. Part of the conversation follows:

> U-09A: I think that more than the level of the students it depends on the curriculum. For example, it is great for geometry. However, the lower level students would do anything in order to get their hands on the computer. As long as they are not seating with a textbook and paper, anything different, it draws

their attention. I think at first they do not see the academic benefit, it is just fun for them, but I think that once it clicks and they get a concept they were struggling with before, they feel proud of themselves and start seeing math differently. For FCAT camp, I used to teach the slow 8th graders. Once they understand a concept with GeoGebra, they say: “Oh, this is so easy. I never knew it was this easy before.” These slow students enjoy it at first and then they see the benefits, while the more advanced students are just the opposite.

U-08A: My kids are in the low level and I always find a way to use it that is going to help them understand something better. I do not use it all the time, but there are some things that even with the lower kids the basic math can be used. I think that if you do not show things in GeoGebra because you have lower level kids, you are doing a disservice to your students. They all can benefit from it.

U-08B: In some of my regular 8th grade classes, I have discipline problems and I do not use it with them because I need to keep them constantly occupied. If I were to have technical problems like I did the first day that you came to observe, I would have serious problems with the behavior of the class, and it is not worth it.

U-09B: I think that the level of the kids is not the issue, but the willingness of the teachers to use it. We saw GeoGebra development. We got to meet Markus and see his enthusiasm, that motivated me, and we saw he used our feedback to change and modify the software. I met him, Dr. Peitgen, and Dr. Voss and remember what they taught us. I am not letting that go! Not matter what kids I have in my class.

These teachers believe that all students can do math and they did not express any beliefs of academic limitation in their students. Even if their FCAT scores are low, all students can show progress. Teacher U-08A is the only one teaching remedial courses. She described her students:

> Right now I am teaching 7th grade intensive math; that means I teach math to the students who score a 1 or 2 on the FCAT. The ones have a deficiency of two grade levels and the twos a deficiency of one grade below from where they are supposed to be. It so happens that those students not only are deficient in math but also deficient in reading because 70% of the test is given as word problems so when you compound the lack of math skills with the lack of reading comprehension skills, it's a recipe for disaster and heart breaks. So every year that is the group I work with and I choose to work with them because I strongly feel that those kids need somebody with experience, just like if you go to a hospital and the most pathological patients would not be given to a first year doctor, you would give them to somebody with experience and versed in those situations. And just like a doctor with the most pathological patients, you don't give up on them. I am glad to report that every year, looking at those subgroups, my students show student achievement. Are they where they are supposed to be? No, but as long as there is progress, that is what I am looking for.

Five of the teachers teach advanced or GEM courses, which typically have students who score high on the FCAT. Teacher U-09A explained how that always is not the case:

> Administrators put all the 3's in my algebra class whether they were ready or not. They do that because they do not have another classroom due to class size. Some of them ended up with D's in their previous math class and now they are in algebra. They have a few more math gaps and some don't have the study habits necessary. It is more work for me, but I don't want to give up on them. It is not the students' fault.

Except for the observation of U-08A, the rest of the observations were done either in advanced or GEM classes. The teachers did not seem to modify their use of technology to match their perception of some students' abilities. The students were all treated equally within the same classroom.

In the interviews, teachers were asked to describe one of their experiences teaching while using GeoGebra as an instructional tool and how that experience was for them and for their students. Table 40 shows what they described.

From their responses it can be inferred that the teachers used GeoGebra as a time saver since they can reuse the files, they do not have to draw on the board, and, when showing examples by hand, they can only show a few. With GeoGebra they were able to show accurate drawings, many more examples, and they did not have to use cumbersome manipulatives. The students' reactions were positive and they were able to comprehend more accurately concepts that have been taught in the past. Two of the teachers commented on the adverse reaction from more advanced students at first.

Table 40

User Past Teaching Experience Using GeoGebra

Teacher	Experience for Teacher	Experience for Student
U-08A	When teaching rotations, I take a screen shot of each picture and I connect each picture to an angle on GeoGebra so it would show exactly what the view is like at a specific angle. In the past I would tell them, take your book, put it upside down, and turn it around until it made sense. This was difficult for them and for me too.	My students feel empowered and they have a sense of accomplishment because I ask them to imagine what the rotated figure would look like. They conference with each other and they feel proud if it is what they imagined at first.
U-08B	Using GeoGebra is great. At first it took a long time to prepare the files but once I had them I could reuse them over and over. I want to show them things that were not shown in the book so they could visually see the math concepts.	Some of my students are so good they could teach math themselves. In general the students are now immune to the power of technology and they expect not just to see a file but a movie. For some of the regular students, GeoGebra helps them understand some of the abstract concepts because they can visualize it with the computer.
U-09A	For me it has been great, you know besides the technical glitches, like worksheets not opening, having to do updates and things like that. I feel it is so much easier to teach using it because it is more time efficient and I can show them neat drawing.	I feel that the high level students that I teach are at first really reluctant to use technology, while the regular students are more open to it. They would do anything that is not working with paper and pencil.

Table 40 *continued*

Teacher	Experience for Teacher	Experience for Student
U-09B	It is a huge time saver. When you teach kids about circumference and diameter and how they relate, in the book they recommend the teacher to bring a bicycle tire, a frisbie, a coffee can…I mean I do not have all that stuff. Before I did not have the same room every period and I could not bring all that stuff with me. With GeoGebra you show them the rolling circle. We do investigative learning, without needing all that expensive manipulatives recommended in the book. So the kids guess, and I go down to from 0 to 2 and they see what the diameter is, what the circumference will be, and I see them understanding that every year, they had never understood it until you show them that in GeoGebra.	It is great. You see then getting that “aha” moment. The kids understand something that the other math teachers have been mostly covering in all the grades from 4^{th} grade up, but they don’t really understand π, why π, until you show it to them. You think it is so obvious, but it is a big concept to get through to the kids.
U-09C	I remember the first time I used the files to show the proof of the Pythagorean Theorem. I used animations and the students loved it. I felt so proud of myself.	My students were amazed and love anything with animation. For them math always was a bunch of formulas in a book and now they see it coming alive.
U-10A	I remember using GeoGebra to show them the display of equations. I could have done it on the board but it saved me so much time and I was able to show them many more examples.	They were able to ascertain, and even basically formulate a conjecture of what they thought it was going to happen. It really created more questions.

According to U-08B and U-09A, advanced students are not impressed by the technology and expect to see more graphics and animations. They also prefer to go immediately to the use of the formulas and solution of the problems without going through investigations.

In the post-observation interviews the teachers were asked if they thought the use of GeoGebra helped the lesson and how. They also were asked their opinion on the students' reactions to the use of GeoGebra. The responses from one of the observations are shown in Table 41.

The responses show that the teachers see GeoGebra as a convenience for them; it helps them show better and more examples. The use of more examples helps students with the understanding of the concepts, and students are engaged and on task at all times.

Beliefs about technology, mathematics, and math teaching and learning were very similar for these six teachers. Their testimonies and observed lesson corroborated their expressed beliefs. They believe that technology in general is useful in their practices and the use of GeoGebra helps them to provide better student-centered activities. They believe that all students can solve math problems using different methods and discover math concepts on their own. In class, teachers allowed opportunities for their students to discover and experiment, either by themselves or together as a class through group discussions. Table 42 shows the finding in the area of beliefs. Since the stated beliefs were the same for all six teachers, only the summary of the beliefs is shown.

Table 41

Observed User Teaching Experience Using GeoGebra

Teacher	Why It Helped the Lesson	Student Reactions
U-08A	Absolutely, because of the visual representation.	They were very open. You could see they were very excited to work on it.
U-08B	I think GeoGebra helped me make very distinct scalene right triangles so that the students could really differentiate the short leg from the long leg of the triangles. We were able to really differentiate the similar triangles	Since they did not use the software and they have seen it before, I do not think they were impressed
U-09A	Yes, it just solidified the understanding of the formula that they figured out by themselves.	I think it kept them interested.
U-09B	Yes, the textbook does not show good examples.	They enjoyed it very much. As you saw they asked questions about the trapezoid that I was not planning on covering that day.
U-09C	It was a great visualization tool on the spot. There wasn't any delay or lag in time, and more models could be presented.	I think they enjoyed that lesson more that a lecture type lesson. It was more hands on and more interactive for them
U-10A	Well, I really could not have drawn all those inequalities on the board as nicely as GeoGebra does it with different colors and it took less time.	You could see the students were engaged and they were telling me what to do.

Table 42

Summary of User Beliefs about Technology, Mathematics, and Math Learning and Teaching

Technology	Mathematics	Learning	Teaching
• Uses technology every day • Proactive user • Changes in teaching style	• Math collection of rules & procedures • Students can discover rules • Involves creativity and new ideas • Requires practice	• Conceptual focus • Correct answer not important • Lectures not important • Students should be allowed to figure things out on their own or in groups	• Hands-on activities effective • No need to have a 'math mind' to do well • Effort more important • Math ability can change • All students can solve multi-step problems

TPACK Development

The interviews and the two observations were used to determine the stage of development for the teachers who use GeoGebra as an instructional tool. Table 43 shows the determined development stage. Explanations and examples follow according to each area.

Table 43

TPACK Development Stages of Users

Teacher	TPAC Development Stage			
	Curriculum	Learning	Teaching	Student access
U-08A	Exploring	Advancing	Advancing	Advancing
U-08B	Advancing	Advancing	Advancing	Advancing
U-09A	Advancing	Advancing	Advancing	Advancing
U-09B	Advancing	Advancing	Advancing	Advancing
U-09C	Advancing	Advancing	Advancing	Advancing
U-10A	Exploring	Exploring	Exploring	Exploring

Curriculum. In the area of curriculum, U-08A adapted specific math lessons to include technology, a descriptor of the exploring stage. The other five teachers developed their own files to promote math thinking in their students, sometimes going beyond the specified curriculum as much as they could under the restrictions of the district focus calendar. Examples of how they dealt with the curriculum while using GeoGebra are in Table 44.

Learning. In the area of learning, all teachers seemed to be at the advancing stage. They planned, implemented, and reflected on teaching and learning with concern for student thinking and understanding. They enhanced their instruction with the use of GeoGebra. The descriptor of this stage relates to viewing technology as an integral part of the development of student understanding, and not just as an addition. Table 45 presents some examples of this behavior.

Table 44

User Behavior in the TPACK Curriculum Area

Teacher	Testimony	Observation
U-08A	I take a screen shot of each picture in the book and connect each picture to an angle on GeoGebra so it would show exactly what the view is like at a specific angle.	Teacher had a file showing homework problems from the book and they were investigating if they were similar figures. No more examples were examined.
U-08B	I use the book every day and if I see they recommend an activity, then I know how to change it for GeoGebra.	His prepared file did not open. As teacher created a new file in front of the students, he explained Thales' theorem.
U-09A	Before, when I was teaching quadratics, it was awful, it was a mess, I had to do everything by hand. Now with GGb I feel the kids get the concept so much more from that single investigation.	Teacher gave a handout to the students with a table to find the sum of the interior angles of a polygon. Students used GGb to investigate and come up with a general formula.
U-09B	If my students had access to working computers, I would have greatly preferred to let the students do their own investigation using GGb worksheets. The html versions of these files contain higher-level, thought-provoking, investigative-style questions.	Student asked about area of trapezoids. Teacher guided a discussion that concluded in students finding an answer. Topic was not going to be covered that day.
U-09C	I could see how important it was to create my own files for exploration.	Students came to the board and manipulated GeoGebra creating examples of their own and analyzing those examples as a class.
U-10A	The use GeoGebra in my algebra class is awesome, definitely awesome. The students are able to see the concepts more clearly and go beyond.	Students discussed which one was a better approximation of a line of best fit in a scatter plot. They later critiqued the effectiveness of the steps given in the textbook.

Table 45

User Behavior in the TPACK Learning Area

Teacher	Testimony	Observation
U-08A	The book only shows you the rules. With GeoGebra you can test the rules.	Students were able to verify that in order for figures to be similar, not only the corresponding sides have to be proportional but the corresponding angles must be equal.
U-08B	My advance students could teach themselves but my regular students need the visuals.	Teacher changed the right triangle several times to help the students visualize the similar triangles.
U-09A	I want students to explore dilations without having to spend time using the distance formula. The students are also asking how to find the center of dilation, which they will discover by themselves.	Students used GeoGebra to explore dilated figures and figured out what was the dilation ratio. With the use of lines connecting corresponding vertices, they conjectured on where was the center of dilation.
U-09B	I use it to show kids things that can be better shown in GeoGebra than in the book, sometimes they are not even shown in the book. For example they are showing you the centers of the triangle and they say: this is the circumcenter, and the incenter, but they do not show you the circles inside or around the triangle, nowhere in the book, and nowhere in the resources that come from the publisher. If you don't use GeoGebra, you get them all confused.	Teacher showed how some formulas for area of polygons can be derived from the formula for area of a rectangle. He did this in an effort to help students with the understanding and memorization of those formulas.
U-09C	My students are struggling with the concept of slope, so I definitely need to go back. I can't just let it go. It is too important of a concept for this year and for algebra next year so I am going to go back.	Students were graphing at the board using GeoGebra by dragging the points around. Students discussed the relationship between the points they were dragging and the rise and run concepts of the slope.
U-10A	GGb really brought your imagination to life with some of the concepts. It helped to give you a basic understanding of certain concepts and topics that we were doing and not clearly defined in the book.	Students discussed the solution of systems of inequalities they were creating with the use of GeoGebra.

Teaching. In the TPACK development area of teaching, the six teachers were in the advancing stage. They accepted technology as tools for learning and teaching math in ways that translate mathematical concepts into forms understandable by students. They were seen as resources in their school for helping students learn with technology, especially with GeoGebra. They all developed GeoGebra-rich activities in ways that maintain student engagement and self-direction. The teacher users were all accomplished teachers who were seen as leaders in their respective schools and districts. Teachers U-08A and U-09C hold National Board Certifications. Teachers U-09A and U-09C had been nominated for the prestigious Presidential Award for Excellence in Math and Science Teaching, with U-09C becoming one of three state finalists. The same teachers were selected as teachers of the year by the local affiliation of the National Council of Teachers of Mathematics. Three of them contributed to a special district team that was designing lesson plans aligned to the CCSS using GeoGebra. Teacher U-10A is a math coach in her school. During the focus group, the topic about how the administration viewed them came up. The responses were:

> U-08B: For me that changes every year because we have had so many administrators in the last few years. I know they think well of me and not just because of what I do in the class in terms of academics but because I do not send any students with referrals to the office. I take care of my own discipline and do not involve them. I know they like that!
>
> U-09A: I think we are viewed as teacher leaders in the county. The evaluation that we get with the FCAT scores does not reflect that unfortunately.

U-09C: When the administrator asks what do you think of this or that, that is when I know they value me because they value my opinion.

Student Access. In the area of TPACK development in student access, the teachers were in the exploring stage. Teachers permitted students to use technology for exploring specific mathematical topics. The stress of staying within the limits of the curriculum seemed to prevent the teachers from progressing to the advancing stage. The descriptor of the advancing stage specifies that technology is seen as an opportunity to challenge notions of what mathematics students can master and it is used to expand the curriculum. The responses of the Cognitive Tool Survey showed that these teachers use GeoGebra for a variety of instructional activities. They do not use it for testing or drill and practice. Only two of the six teachers, U-08A and U-09A, have access to a computer cart, allowing the students to do their own explorations. The other four teachers are confined to projecting GeoGebra using their laptops. U-09C allows one student at a time to come up to the Promethean board and manipulate GeoGebra. The other three teachers are in control of the computer, with U-10A doing what students suggest.

The teacher users seemed to be in the latest stages of TPACK development. The limitations of the curriculum they have to cover and the lack of computers available make it difficult for some to develop further in the area of student access. Teacher U-08A seems to be in a different stage in the area of curriculum from the others. One of the reasons for this difference could be the level of the students she is teaching and her style of teaching with precise rules and instructions.

Experiences in the SMGEM Program

The six teachers expressed they have had positive experiences in the program. With the exception of U-10A, the other teachers also were invited to participate in an additional program sponsored by the National Science Foundation. After graduation, the teachers were chosen to participate by the principal investigator of the SMGEM program, Dr. Peitgen. They participated in six additional courses: four in mathematics and two in pedagogy. Their participation in the supplemental program lasted six academic semesters. As with the SMGEM program, the supplemental program relied heavily on the use of technology.

The six teachers had met and received classes from the GeoGebra developer Dr. Markus Hohenwarter during their participation in the SMGEM program. Many of them remembered Dr. Hohenwarter, whom they mentioned during their interviews. U-08A was delighted she was able to receive one-on-one help from Markus. U-08B described being in a class with him as one of his best experiences in the program. U-09A recalled her first workshop with Markus in a summer institute before she was part of the program. He used GeoGebra and U-09A recognized at that moment that as good as she is in math, there are many things she still has to learn. U-09B recognized that Markus' expertise inspired him to create his own website and use technology in his teaching. U-09C also was motivated by his enthusiasm. U-10A started the program after Markus left but was able to experience workshops with him during one of the summer institutes.

In their interviews, teachers described having good experiences as math students, especially in elementary and secondary schools. Some of them struggled in some courses in college while doing their bachelor degree. All of them seemed to enjoy their experiences in graduate school. In the interview they were asked to recall a time in the SMGEM program when they were using GeoGebra to explore a math concept and describe that experience. Parts of their responses are shown in Table 46.

Teachers' past experiences as learners of math using technology were transferred to their own classrooms. Teacher U-08A needs step-by-step instructions and she provides her students with the same kind of instructions. Teacher U-08B encourages his students to use GeoGebra at home and create their own files. Teacher U-09A teaches geometry to her GEM students with investigation activities. U-09C is considered a GeoGebra expert in her school. U-10A uses GeoGebra not only as a visual display but also as a way to promote student discussion.

The teachers acknowledged that the SMGEM program helped them with their content knowledge in mathematics and also with their teaching. Four of them had not been teaching for very long before starting the graduate program. The experience of being in the program helped them to develop their teaching style. U-08A admitted: "I was looking for a way of teaching math and being in the group with the other teachers helped me a lot because I could ask questions and get feedback from the more experienced teachers as well as from the professors."

Some of their comments about the graduate program are in Table 47.

Table 46

User Experience in the SMGEM Program Using GeoGebra to Explore a Math Concept

Teacher	Experience in the SMGEM Program Using GeoGebra
U-08A	I remember my first semester meeting Markus, and getting individual help from him. I needed more step-by-step instructions than other people, but GeoGebra helped me see things in a way I had never seen before. I thought I knew math until the professors made us really explore in depth those concepts.
U-08B	My best learning experiences with GeoGebra were when we were in that first class with Markus and we had to develop our own materials. I remember I wanted to do thing in 3D and for that I had to trick the system because the 3D was not available yet. I had to use a lot of ellipses and other figures I really did not know much about. I learned a lot of math while trying to do those files. We had a great time in that class and we were doing things that we wanted to do.
U-09A	I don't remember much from my geometry class that I took in high school, so that was my weakness. I think it was our first workshop with Markus in a summer institute, and I remember they asked us to construct a square. I had no idea how to do it. I remember learning with GeoGebra the difference between a drawing, a sketch, and a construction, you don't use measurement tools, and you use arcs, and circles. I remember feeling at that point that as good as I was in math, there were a lot of things I needed to learn.
U-09B	Oh, I remember being impressed by Heron's formula. Just to know that there was such a brilliant formula out there that worked and we saw it in GeoGebra, and not taught in our curriculum, it was amazing. Why do we only teach kids right triangles when most of the world isn't right triangles? And there is a formula for all triangles. That was one of the most monumental things.
U-09C	I felt as if I was surrounded by "experts" and had to work very hard (especially with any project involving if-then statements). With GeoGebra, the abstract of math was gone. I wanted to do that magic in my classroom.
U-10A	All math concepts were illuminated with GeoGebra because of the visual display and presentation. It really brought your imagination to life with some of the concepts. It helped to give you a basic understanding of certain concepts and topics that we were doing.

Table 47

User Comments about the SMGEM Program

Teacher	Comments about SMGEM Program
U-08A	I had been teaching for more than 10 years when I started the program but I was always looking into different ways to improve my teaching and innovative ways to present the information. I just incorporated many new ideas into my teaching. I had always used technology in my teaching even before the program but after being in the program, my use of technology increased considerably.
U-08B	In those classes I did great. I remember having to work to learn it but I also remember a lot of the students before and after class would ask me questions because I knew what I was talking in class. The math was not easy, but the writing was what was always hard for me. To write explanations in math and what I was thinking about was not easy, but it always helped to bounce off some ideas with other students. That is probably how I got through most of those courses, working on other people, sharing ideas, and cooperating with them. We did a lot of learning that way by using their ideas and my ideas. The hardest was the writing but I learned a lot on those courses.
U-09A	The math to me was very new, using the technology was very interesting but also the very abstract things we did like paper folding. It was difficult but I felt that the collaboration that we had, and getting to see people's different perspectives helped a lot. It was cool to see things that I teach in middle school, the basic ideas, being taught in different ways, and the reasoning behind everything and the different methodologies, that was interesting.
U-09B	Graduate school reignited the flame that I had for math because in college I kind of was all over the place and the graduate program gave me the chance to get into something that I loved and also raised me to another level. In my other schools I was always the best and in this program I was the worst, so it was quite a change. It was nice to see people that excel in math, I felt like the tables had turned on me. It was a lot of tough work but I enjoyed it.
U-09C	I enjoyed my classes, but feel that I had to work harder than my classmates to understand material possibly due to my own diligence but also due to my lack of previous experience.
U-10A	I really enjoyed it. When I got my bachelor's was in 1989, so from 1989 to 2010, when I graduated, there were some things that were lost, I had to go back and review, but I enjoyed the instructors. I didn't feel lost, I felt very comfortable with the information, even though some of the information I wasn't taught. Some of the information did transcend my level of teaching at the middle school level. I think that because of the program, I am a better teacher.

The teachers expressed their gratitude to the program in their personal interviews as well as in the focus group. They expressed their desire to participate in a Ph.D. or specialist program.

When asked what could be done to promote the use of GeoGebra as an instructional tool, some teachers mentioned the amount of available materials found in the district wiki as well as the GeoGebra wiki. U-08B pointed out that teachers need to have the GeoGebra basics before they could look for materials. The suggestions continued in the focus group:

> U-09C: One comment I have is that we have to figure out a way to keep going with the training at the district level, and ask Duke and Guy to help us. Because at first there was this big push, and we were all doing GeoGebra trainings, and now there is nothing.
>
> U-09A: That is because they had that publisher's money for trainings, but that is now gone.
>
> U-09B: They have trainings all the time, it is just that they are not all focused on GeoGebra. In a couple of years when we get the new textbooks all that money it is going to be marked for training the teachers to use the new software. If there is no GeoGebra in those textbooks, then nobody is going to be using it or getting exposed to GeoGebra, that is the way it is going to be.
>
> U-08B: Remember the semester when we did the research? Many of the conclusions of the studies were that you needed to invest in the teachers' knowledge in their subject area. So in order to get teachers to use it, if you give training, and the person that is giving the training uses GeoGebra to train the

teachers to learn this new concept in their subject, the teachers would be more likely to turn around and use it the same way they were taught. I remember in college in my education classes they said that teachers mostly teach the way they were taught. So if you want to make a change, that is what you have to do. We have to renew our certificate every five years and we have to get some kind of in-service training to do that. If we could put in the principals' ears that we have to do this training in order to get the certificate, and as the instructors give their lessons using GeoGebra, the teachers can come back to their schools with the certification renewed, as well as with new materials and the supporting files.

The teachers understood how difficult this proposition was to implement because the administrators had their own initiatives that they felt were what the school needed most. These teachers felt that the students were top priority and would do anything necessary to help them get the best education they could receive.

Table 48 shows the findings in the area of knowledge acquired by this group of teachers from the SMGEM program.

Table 48

Knowledge Acquired by Users in the SMGEM Program

Teacher	Mathematics	Technology
U-08A	Yes	Yes
U-08B	Yes	Yes
U-09A	Yes	Yes
U-09B	Yes	Yes
U-09C	Yes	Yes
U-10A	Yes	Yes

Instrumental Orchestration

Each teacher was observed twice and answered to pre- and post-observation interviews. The data allowed some insight into their instrumental orchestration in terms of the pedagogical context, their preparation, and their pedagogical actions.

Pedagogical context refers to the arrangement of teaching settings and artifacts available in the environment. The teaching settings for five of the six teachers were the same; U-10A was the only one who had a different setting. Five classrooms were arranged in neat rows of desks facing the Promethean Board. They had an LCD projector that allowed them to display their laptops as well as the document cameras. Teacher U-10A had the desks arranged in a U shape and did not have a Promethean Board. She was projecting her computer directly to a white board. The instructional approaches used on the observed lessons were either interactive instruction, with heavy emphasis on whole classroom discussions, or discovery where students were mostly working on their own on individual laptops. Table 49 shows what instruction approaches and topics were used in each observation.

Table 49

User Instructional Approaches Used and Topics Covered During Observed Lessons

Teacher	Obs. 1	Topic Covered	Grade	Obs. 2	Topic Covered	Grade
U-08A	Discussion	Similar Figures	7th grade remedial	Discovery	Graphing linear equations	7th grade regular
U-08B	Discussion	Geometric mean	8th grade GEM Geometry	Discussion	Special Right triangles	8th grade GEM Geometry
U-09A	Discovery	Interior and Exterior angle sum in Polygons	8th grade GEM Geometry	Discovery	Dilations	8th grade GEM Geometry
U-09B	Discussion	Formulas for areas of parallelograms	6th grade advanced	Discussion	Formulas for areas of parallelograms	6th grade advanced
U-09C	Discussion	Slope	8th grade advanced	Discussion	Triangle Inequality Theorem	8th grade GEM Geometry
U-10A	Discussion	Linear Regression	8th grade Algebra	Discussion	Systems of inequalities	8th grade Algebra

The preparation mode represents the ways in which teachers decide to make full use of pedagogical preparation for the benefit of instruction. During the pre-observation interviews, the teachers were asked to describe their plans for the lesson, which included how the activity was going to be introduced and worked out, how GeoGebra was going to be used, and how they thought GeoGebra would help achieve the goal of the lesson. Their answers are in Table 50, which shows the preparation for lesson 1, and in Table 51, which shows it for lesson 2.

Table 50

User Preparation of Observed Lesson 1

Teacher	Introduction	GGb Use	Math Concepts Enhanced by the Use of GGb	How GGb Will Facilitate Learning
U-08A	Review of concept	Teacher demonstration Students discussions	Investigation of the two properties of similar figures	Visualization of properties. Help with calculations
U-08B	New Concept	Teacher demonstration Students discussions	Verification of proportional sides of similar triangles	Discovery of geometric mean. Help with calculations
U-09A	New concept	Students working on laptops. Teacher provided chart	Discovery of the formula	Ownership of the investigation of the formula to avoid memorization
U-09B	New concept	Teacher demonstration Students discussions	GeoGebra will help the students see why the area of a triangle includes the '1/2' and see why a rectangle has the same formula for area as a parallelogram.	It shows the students concepts that are difficult to understand from the algebraic formula alone. A picture is worth a thousand words.
U-09C	Review of concept	I am going to make them get up from their seats and make them actually graph at the board using GeoGebra by dragging the point around.	They will see a relationship between the points they are dragging and the rise and run concepts of the slope.	The use of any technology gets their attention, so if you get their attention, you have a chance of helping them.
U-10A	New concept	Teacher demonstration Students discussions	Verification of solutions to systems of inequalities	Faster verification with the use of different colors. It can show the difference between less than and greater than in inequalities

Table 51

User Preparation of Observed Lesson 2

Teacher	Introduction	GGb Use	Math Concepts Enhanced by the Use of GGb	How GGb Will Facilitate Learning
U-08A	Continuation of previous lesson	Students working on laptops. Teacher provided written instructions.	Graphing of functions using technology	Verification of collinear points found on homework assignment.
U-08B	New concept	Teacher demonstration. Student discussions.	Verification of properties of special right triangles	Generalization of properties of right triangles.
U-09A	Continuation of previous lesson	Students working on laptops. Teacher provided written instructions.	Easy of graphing and the visual, instead of doing it by hand	By being able to see the grids, to see the distance, the lengths of the sides without having to use the distance formula. The students are also asking how to find the center of dilation.
U-09B	New concept	Teacher demonstration. Student discussions.	GeoGebra will help the students see why the area of a triangle includes the '1/2' and see why a rectangle has the same formula for area as a parallelogram.	It shows the students concepts that are difficult to understand from the algebraic formula alone. A picture is worth a thousand words.
U-09C	New concept	Teacher post questions to the students of what possible sideline lengths are needed to create a triangle using sliders so they can see many models at one time.	It is an exploration and discovery lesson, instead of just giving them the theorem we are going to let them make guesses, hypothesized and then use GeoGebra to check if they were right.	They will be able to see many models in a short period of time rather than to draw things or use physical models they can use the technology they love so much.
U-10A	New concept	Teacher demonstration. Student discussions.	Ease of graphing of linear functions.	The different lines will create a discussion on the best fit line.

Pedagogical action involves ad hoc decisions made during teaching about how to actually perform the chosen pedagogical configuration and preparation mode. In the post-observation interview, teachers were asked for their opinion on how the lesson went and if they had to make any changes. Table 52 shows observations made and teacher responses during either one of the observations.

Disruptions in the classroom happened either due to technical difficulties with the software or due to the students not knowing how to manipulate the different tools in GeoGebra. These are experienced teachers who know how to handle these types of situations. U-08B had a major technical setback with his laptop during the first observation, which did not allow him to use the files he had prepared. He was able to manage the situation in a calm manner and modify his plans without any interruption of the learning.

Drijvers et al. (2010) identified six orchestration types that they termed technical-demo, explain-the-screen, link-screen-board, discuss-the-screen, spot-and-show, and Sherpa-at-work. Most of the teachers employed the link-screen-board orchestration, since the relationship between what happened in GeoGebra and how it translated to an abstract representation that could be written with a formula was stressed. Teacher U-10A was able to have whole class discussions about what happened on the computer screen. This was an example of what Drijvers et al. called the discuss-the-screen orchestration.

Table 52

Pedagogical Actions of Users

Teacher	Action	How Did the Lesson Go?	Did You Change Anything?	Student Reactions
U-08A	Students asked for a counter-example. Teacher showed two figures with same angles but were not similar.	This was a review, but I think it clarified the properties to the students.	Not really, I had to change some examples but I was expecting that.	You hear the "ahh" from the students.
U-08B	Students already know the relationship of special right triangles.	Well, I had to change my plans because the students already knew the relationships.	I used the lesson I had prepared for the next day.	They wanted me to change the triangle to check their guesses. They like it when they are right.
U-09A	The grid could not be shown.	Besides the technical issues, I think it was useful.	No. I had to write an instruction I forgot on the handout.	I thought it was good besides all the trouble-shooting we had.
U-09B	Students asked for the formula for area of trapezoid.	I feel it went well. A majority of the students now understand why the formulas work as opposed to just how to plug in values for unknowns using the reference sheet they are given for standardized testing.	We explored the area of a trapezoid in one of the classes in response to a question by one of the students.	They enjoyed it very much and, as every time we use it, learned a great deal.

Table 52 *continued*

Teacher	Action	How Did the Lesson Go?	Did You Change Anything?	Student Reactions
U-09C	Students only focused on the upper range of the triangle inequality	I liked the lesson. I took a very dry subject, a "who cares" kind of concept, and I made it more interesting so I think that was a good one.	I knew the students would not find the lower range. We had to work on more examples than expected	They loved it. These are my GEM kids, my computer experts, and they really enjoyed it. They had a good time, which is good because learning is supposed to be fun.
U-10A	Teacher showed the line of best fit that GeoGebra would draw. Students confused that it did not go through any of the given points.	I think it went well. This students are always interested and engaged.	Not really but I wish I could have explained better the technique that the software used for finding the line.	They loved it, they always do. They enjoyed seeing their lines graphed in the same scatter plot and the discussion of which one was the best.

Discussion of the Findings on Users of GeoGebra

A summary of the findings for each theme and categories for this group of individuals is shown in Table 53. The numbers in parentheses indicate how many teachers in this group agreed with the statement.

Table 53

Themes and Research Questions for Users

Themes	Categories	RQ 1: What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?	RQ2: What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?	RQ 3: What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?
GeoGebra knowledge	Basic	• Knowledgeable on the old version (6) Need to learn new version (6)		
	Implementation	• Presentation of new material (6) • Demonstration (6) • Enrichment (4) • Experiments (3) • Problem Solving (6) • Drill and Practice (0) • Testing (0) Outside educational activities (3)		
Concerns	Personal	Knowledge on new release (6)		
	Management	• Classroom discipline (5) • More time to plan (6) More instructional time (6)		
	Consequence	• None		
External Factors	Resources	• More computers (6) Technical support (5)		N/A
	School Support	• Work with colleagues (5) Leadership support (6)		N/A
	Policy	• FCAT Pacing Guide		N/A

Table 53 *continued*

Themes	Categories	RQ 1: What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?	RQ2: What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?	RQ 3: What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?
Beliefs	Technology	• Use technology every day (6) • Proactive Users (6) Changes in teaching style (6)		
	Mathematics	• Math collection of rules & procedures (6) • Students can discover rules (6) • Involves creativity and new ideas (6) Requires practice (6)		
	Learning	• Conceptual focus (6) • Correct answer not important (6) • Lectures not important (6) Students should be allowed to figure things out on their own or in groups (6)		
	Teaching	• Hands-on activities effective (6) • No need to have a 'math mind' to do well (6) • Effort more important (6) • Math ability can change (6) All students can solve multi-step problems (6)		

Table 53 *continued*

Themes	Categories	RQ 1: What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?	RQ2: What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?	RQ 3: What do the teachers think they learned from the SMGEM program with respect to GeoGebra, and how are they using what they have learned?
TPACK	Curriculum	• Exploring (2) Advancing (4)		
	Learning	• Exploring (1) Advancing (5)		
	Teaching	• Exploring (1) • Advancing (5)		
	Student Access	• Exploring (1) • Advancing (5)		
Experiences in the SMGEM program	Math Content			Extensive
	Technology Content			Extensive
Instrumental Orchestration	Context	• Desks in rows facing front (5)		
	Preparation	• Discussion (4) • Discovery (2)		
	Action	• Effortless resolution of technical or content situations (6) • Link-Screen-Board (5) • Discuss-the-screen (1)		

This group of teachers used GeoGebra as an instructional tool in their classrooms since graduating from the SMGEM program. Five of them had been invited to participate in an additional program with some of the same instructors as the SMGEM program. In this additional program, called NOYCE program, teachers received four extra classes on mathematics and two on pedagogy. The math classes were heavily infused by technology, especially GeoGebra.

Three of these teachers used GeoGebra in other educational projects in which they were involved outside their schools. A fourth teacher volunteered in a technology team in her school, and a fifth teacher was the math coach at her school. The sixth teacher had been asked to participate in the same educational projects as the first three, but due to the arrival of a baby in his family, he accepted tutoring jobs instead.

Only one teacher had a bachelor degree in mathematics, while another had one in elementary education. Four of them had decided to become teachers after working in other professions. Only one of them is teaching remedial courses, while the others are teaching a combination of regular, advanced, or GEM classes

The first research question asked for the perceived internal and external factors that foster their use of GeoGebra as an instructional tool. They all felt that the SMGEM program gave them the basic knowledge of GeoGebra and other technologies, like Excel and Power Point. Since they are frequent users, they are aware of the rapid changes that the GeoGebra developers are making on the software. All of them are concerned that they have not kept up with the improvements done on GeoGebra; therefore, expressing personal concerns about their knowledge on the newer releases.

They know where to find the information needed, but all agreed that they need time to research the information.

These teachers do not think that GeoGebra is better suited for advanced students. They argued that the regular and remedial students enjoy the use of GeoGebra more than the advanced students, since the advanced students want to do the work quickly and are more interested in the procedural aspect of mathematics. According to some of the teachers, the regular students enjoy the opportunity to do work that is different from the traditional paper and pencil work. As teacher U-09A said, “regular or ESE students may have more fun at first and then they see the benefits of using GeoGebra later.” Teacher U-09C sees the use of GeoGebra as an opportunity to get the students interested and engaged, while teacher U-08A sees it as an opportunity to empower her remedial students. Their responses on the Beliefs survey about mathematics teaching showed they do not think students need a ‘mathematics mind’ in order to perform well in mathematics.

Their positive experiences as students of mathematics in the SMGEM program inspired them to continue using GeoGebra with their students. Five of the teachers recalled good memories from having the opportunity to work with Dr. Markus Hohenwarter, the main developer of GeoGebra, who worked in the SMGEM program for two years and was an inspiration to the teachers.

Their lack of knowledge on the newer releases of GeoGebra, the lack of available working laptops for students, the pressure of the FCAT, and the stressful pacing guide they have to follow did not stop this group of teachers from using GeoGebra as an instructional tool. They themselves had been engaged students who enjoyed the use of technology as an instructional tool. The teachers are using GeoGebra outside their classrooms, as they are members of groups with other colleagues who are users as well. Working on those activities motivates the teachers, who expressed being inspired by the SMGEM program. As U-08B said: “I think we were all amazed at how the professors used in the program, we were surprised by the capabilities, and we wanted our students to experience the same things we did.”

After graduation from the SMGEM program, five of the teachers had participated in the NOYCE program, which lasted an additional three academic years. During the NOYCE program, they received four additional classes in mathematics where GeoGebra was used extensively. U-10A was the only teacher who did not participate in the program. Teacher U-10A was the least frequent user of GeoGebra. She indicated that she uses it on a monthly basis on some instructional activities and never in some others while the other teachers are using it in some activities on a weekly basis or every other day. The five teachers have kept in contact with each other after finishing the NOYCE program, either by working together on academic activities after school, or by working in the same schools.

Teacher U-10A is the only teacher in this group who had not experienced semester long classes with the GeoGebra developer, Markus Hohenwarter, and who had not been part of the NOYCE program. She also is the only user who graduated from the SMGEM program in the second to last cohort. She is the math coach of her middle school and was teaching only one section of algebra at the time of the study. She uses GeoGebra in her algebra class, but not as often as the other users. She is teaching in the same school as NU-10A, who is not a user.

The second research question examined the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments. Even though the teacher users felt behind in their knowledge of the newer releases, they described themselves to be knowledgeable using GeoGebra in different instructional activities, with presentation of new materials and demonstrations being the most popular activities. The responses about the technology beliefs confirmed that they believe the use of technology in the teaching and learning of mathematics has a positive impact on students. However, some expressed concerns about having behavioral problems in the classroom when they are using GeoGebra and encounter technical difficulties with the software or with the laptops. They felt they needed to keep the students busy at all times, and any possible changes in the routine could create problems managing student behavior. For this reason, they do not use GeoGebra in some classes.

The factor that made the biggest difference in their approach to the use of GeoGebra was the way the students had access to GeoGebra. Only two teachers who work in the same school have the availability of laptops for their students' use. They give handouts to their students with some GeoGebra instructions and students are engaged in activities where they are discovering math concepts, formulas, and theorems on their own. The other four teachers are projecting GeoGebra from their own laptops. They recognize that the laptops available in their schools are old and have antiquated operating systems older than some of the students, as U-09B stated. However, this fact does not stop them from using GeoGebra in their classrooms. These four teachers have managed to create GeoGebra activities that foster classroom discussions, and they keep their students engaged in discovery activities as well. Their responses on the Beliefs survey on mathematics, math learning, and teaching showed that they feel students can discover mathematics facts, and that a conceptual focus is more important than a procedural one. The observations confirmed that they emphasize the understanding of concepts and let the students discover concepts on their own.

According to Drijvers et al. (2010), the teachers' views on mathematics teaching and learning and the opportunities technology offers can be seen in their justification of orchestration types. These teachers showed the connection between what GeoGebra is showing and the mathematical representation, and they employed discussion techniques with their classes. These techniques are in accordance to their stated beliefs.

The teachers indicated that the use of GeoGebra helps them in their practices. They feel it is easy to use, it is free so they could recommend their students download it at home, it saves them time since they can show students a multitude of examples in a short amount of time, and they are able to show their students perfect pictures without having to draw freehand on the white boards.

The third research question referred to what the teachers thought they learned from the SMGEM program with respect to GeoGebra, and how they were using what they have learned. The teachers had expressed their beliefs that the program had made them the teachers they are. Some of them had only been teaching for one or two years before entering in the program and, as they said, it helped them solidify their teaching style. Two of the teachers had been working a few years before entering the program, and they also shared the belief that the program changed them. As U-08A stated: "My use of technology increased considerably." It was a great refresher for U-10A since she had graduated with a math degree in 1989. Even she admitted that some topics covered in the program she had never seen before. She said: "I think that because of the program I am a better teacher."

The teachers acknowledged that the SMGEM program helped them with their math knowledge. In the classes observed the teachers mastered the topic at hand and were able to anticipate student confusion and questions, which they handled with ease. They displayed knowledge of the math the students were going to take in future years. U-09C stated that not only did she want her students to know how to use GeoGebra for her class, but for when they were taking Algebra 2 as well.

Comparison on the Findings of the Seldom or Non-Users and the Users of GeoGebra

Table 54 shows the findings of the non-user and users in all themes. The information was used to compare the findings in an attempt to see the similarities and differences.

In the area of GeoGebra knowledge, the users and non-users felt they knew the basics of the software. The non-users did not talk about the newer releases, while the users were aware of it and felt they needed to learn more about it. Even though they all expressed to knowing where to find the help needed in the use of GeoGebra, five of the non-users wanted to have more training, and all of the users wanted to learn more about the new releases on their own. The users also knew of other people in the district whom they could ask for information, as well as how to use the international GeoGebra Forum.

Table 54

Comparison of Findings of Non-Users and Users

Themes	Categories	Non-Users	Users
GeoGebra Knowledge	Basic	• Comfortable (6) • Would like more training (5) • Would like to work with colleagues (5)	• Knowledgeable on the old version (6) • Need to learn new version (6)
	Implementation	• Difficult for 2 teachers teaching remedial classes • Could not design activities (3)	• Presentation of new material (6) • Demonstration (6) • Enrichment (4) • Experiments (3) • Problem Solving (6) • Drill and Practice (0) • Testing (0) • Educational activities (3)
Concerns	Personal	• None	• Knowledge on new release (6)
	Management	• Classroom management (2) • Time demands (3) • Efficiency of GeoGebra (2)	• Classroom discipline (5) • More time to plan (6) • More instructional time (6)
	Consequence	• Usefulness of GeoGebra with low-performing students	• None
External Factors	Resources	• More computers (6) • Technical support (5)	• More computers (6) • Technical support (5)
	School Support	• Professional development (5) • Leadership support (1)	• Work with colleagues (5) • Leadership support (6)
	Policy	• FCAT • Pacing guide	• FCAT • Pacing Guide

Table 54 *continued*

Themes	Categories	Non-Users	Users
Beliefs	Technology	• Use technology every day (6) • Proactive Users (6)	• Use technology every day (6) • Proactive Users (6) • Changes in teaching style (6)
	Mathematics	• Collection of rules (5) • Remembering necessary (6) • Students can discover rules (4)	• Math collection of rules & procedures (6) • Students can discover rules (6) • Involves creativity and new ideas (6) • Requires practice (6)
	Learning	• Conceptual focus (6) • Correct answer not important (6) • Teacher explanation necessary (1)	• Conceptual focus (6) • Correct answer not important (6) • Lectures not important (6) • Students should be allowed to figure things out on their own or in groups (6)
	Teaching	• Hands-on activities effective (5) • 'Math mind' necessary (2) • Effort more important (6) • Math ability can change (6)	• Hands-on activities effective (6) • No need to have a 'math mind' to do well (6) • Effort more important (6) • Math ability can change (6) • All students can solve multi-step problems (6)

Three of the non-users felt they had difficulty designing activities for student use. Two of them expressed not knowing how to design activities suitable to the remedial students they are teaching. The users used GeoGebra in a variety of instructional activities like introduction of new topics, demonstrations, and explorations. The users felt that the use of GeoGebra can help all students, including the remedial students. One of the teacher users is teaching these students and she noted that GeoGebra not only helps them with the mathematical understanding but also empowers them and helps them feel more confident of their math ability.

In the areas of concerns there are some differences. The non-users did not express any personal concerns while the users were concerned about not being so familiar with the newer releases and therefore not being able to use GeoGebra to its full potential. Both groups expressed some management concerns. Two non-users felt that the use of GeoGebra would create disruptions with their students. One of them argued that her students were immature and when GeoGebra was used, they misbehaved and treated the experience as a game. The other non-user felt the disruption in the class came from using computers that did not work properly. Teacher U-09A explained that the computer screens would not show the same things, perhaps because some of the computers might have viruses, and as she was trying to help her students with the technical difficulties, too much instructional time was wasted. This was the same concern expressed by five of the users. They also pointed out that since they were not so familiar with the new release, they did not want to spend time in front of the students trying to figure how the new changes worked. Also, the new releases sometimes would come unexpectedly from one day to the next and surprise the teachers, as happened during one of the lessons observed.

Two of the non-users had concerns about the efficiency of GeoGebra with low-performing students. None of the users had this type of concern. Even though most of them are teaching advanced or GEM classes, they used GeoGebra with their regular students as well as with students in the FCAT preparation classes. This concern also was converted into a consequence concern for the two non-users. They had questions about the impact of GeoGebra on their students. The users believed GeoGebra had a positive impact on all their students as it had on them when they were students in the SMGEM program.

The two groups mentioned the same external factors. They all talked about the need for newer laptops for the students, and the need of more technical and leadership support. They all argued about the stress they felt with the FCAT testing and with the fast pace of the calendar guide given by the district. Teacher U-08A wanted the FCAT to be over so she could do more activities with her remedial students where they could see, in a fun way, some of the real life applications of mathematics.

The beliefs of the teachers were very similar. In terms of technology they all felt they are skillful and use some sort of technology every day, either the Promethean board, or document camera, or their LCD projector. Some of the non-users used clickers and QR reader applications with the students. They used the technology because they believed it could help their teaching as well as help students with their learning. In the area of Mathematics, one difference was that some non-users questioned if students could discover mathematical rules and procedures on their own while all users believed all students could do so.

The beliefs in math learning and teaching were very similar. The biggest difference was with the statement on whether students needed to have some kind of ‘mathematical mind’ in order to do well in math. Two of the non-users felt this was true, while the other four non-users and all the users didn’t. The two non-users who felt differently were the teachers with the remedial students.

It is not possible to compare the TPACK development of the teachers since the non-users were not observed in their classroom while teaching with the help of technology and the stages were determined using only their testimonies. Understandably, the users seemed to be in higher stages of development.

The experiences both groups had in the SMGEM program were very similar. In terms of technology knowledge, both groups felt the program prepared them very well. In terms of mathematics, both groups expressed having learned a great amount, even the ones who had a bachelor degree in mathematics. Both groups felt that the program helped them to become stronger teachers. Three of the non-users felt they were prepared to teach higher levels and openly were seeking opportunities to teach in a high school. The users did not express any unhappiness with their teaching assignments. Teacher U-08A felt the program helped her be the strong teacher that she was. She compared herself to a doctor, and felt that experienced teachers should be working with the students who need the most help.

Except those three non-users who were unhappy with their assignments, the other teachers seemed to be happy with their jobs. They all felt the program and the professors changed them into strong, knowledgeable teachers.

Chapter 5 summarizes the findings organized by the research questions of the study, their connection to the theoretical framework described in Chapter 1, and the relationship to the literature reviewed in Chapter 2. It also includes a discussion of the implications that can be drawn from the study. The chapter concludes with recommendations for further study and final remarks.

CHAPTER FIVE: DISCUSSIONS AND IMPLICATIONS

The purpose of the study was to describe, analyze, and compare information on the different ways that 12 middle school mathematics teachers, who graduated at different times from the same 2-year long, technology-rich master's degree program in mathematics, integrated a dynamic mathematics learning environments called GeoGebra in their practice. This program was funded entirely by the National Science Foundation (NSF) and the enrolled teachers did not have to pay tuition to the higher education institution. After graduation, some of the teachers were invited to participate in a supplementary program, also funded by NSF, where they received four more semesters of mathematics instruction. The decision as to which teachers would participate in the additional program rested on the principal investigators and the district mathematics curriculum specialists.

Applying a qualitative approach of multiple case studies, the research focused on the different reasons for the decision whether to integrate GeoGebra as an instructional tool. The researcher of the study looked at teachers' concerns, beliefs, external factors, and behaviors and actions in the classroom while integrating technology in their practice, in an attempt to understand what they perceived as the role of technology when teachers facilitated activities that fostered student's construction of knowledge.

The study was framed by a review of current research in the field and the literature on the subject of technology integration in the mathematics classroom, the internal and external barriers that could foster or prevent this integration, and possible changes that the implementation of technology could introduce. Throughout this review of the literature, the researcher intended to understand why and how middle school mathematics teachers integrated technology tools in their practices.

In the course of the review of the literature, it increasingly became clear that much of the research, the writing, and the thinking and actions of individuals focused more strongly on the effects of technology in the learning of mathematics by students than on the possible factors impacting the implementation of dynamic mathematics learning technology by teachers. For example, there have been many studies that have shown technology is under-utilized in mathematics classrooms (Becker & Ravitz, 2001; Conlon & Simpson, 2003; Cuban, 2001; Reed et al., 2010). Other studies have shown the effect of technology on student' achievement in mathematics (Becker & Ravitz, 2001; Carter & Smith, 2001; Li & Ma, 2010; Means, 2010; Shapley et al., 2011; Wenglinksy, 1998). Only a few studies have investigated Technological Pedagogical Content Knowledge (TPACK) of teachers (Koehler & Mishra, 2009; Niess et al., 2007), a term that has been used only since 2006. Some of the studies concentrated on the use of calculators, and even less studies were focused on the use of dynamic mathematics learning environments as cognitive tools in the teaching and learning of mathematics (Laborde, 2001). Examples of dynamic mathematics learning environments are the Cabri Geometer (developed in 1988-92), the Geometer's Sketchpad (developed in 1992), and GeoGebra (developed in 2001-02). These cognitive tools can turn classrooms into laboratories for the discovery of algebraic and geometric relationships. The few studies found, however, did not describe the reasons why middle school math teachers who have had intensive training in one of those cognitive tools decided whether to implement those tools into their practices. This pointed to a gap in the understanding of the internal and external factors that teachers had undergone in their professional lives when deciding how to use and implement dynamic mathematics learning environments like GeoGebra.

Vygostky (1978), an advocate of social constructivism, proposed that the use of tools and signs changes all psychological operations, and the use of tools expands the range of activities within which the new psychological functions may operate. The belief in the importance of understanding the dynamic between the use of tools, the factors and concerns about their implementation, and the range of activities that they can foster framed the study.

The research question was: Why and how do middle school mathematics teachers integrate dynamic mathematics learning environments in their practices? The following sub-questions were addressed:

1. What are the teachers' perceptions of the internal and external factors that foster or prevent the use of dynamic mathematics learning environments in the classroom?
2. What are the differences and similarities with respect to how and why teachers use dynamic mathematics learning environments?
3. What do the teachers think they learned from the SMGEM program in terms of GeoGebra, and how are they using what they have learned?

What follows are the conclusions drawn from the findings described in the previous chapter and organized by the research questions. Also, connections to the theoretical framework are discussed. Finally, the chapter concludes with a discussion of implications that can be drawn from the study, and recommendations for further research in this topic.

The Cognitive Tool survey was sent via email to 52 math teachers who graduated from the Standard Mapped Graduate Education and Mentoring (SMGEM) program. The researcher excluded the first 14 graduates from the program since those teachers were chosen to participate and they did not represent the rest of the teacher population. From the 25 (48%) teachers who responded, 12 middle school math teachers were selected according to the criteria defined in Chapter 3. Those 12 selected were divided in two groups: 6 teachers who responded they seldom or never used GeoGebra in their classrooms, and 6 who were using GeoGebra as a cognitive tool in their classrooms. Those 12 teachers answered a Beliefs survey and were interviewed once by the researcher using a semi-structured protocol. Interviews were transcribed by the researcher and sent back to the teachers for member checking. The six teachers who were using GeoGebra were observed in their classrooms twice while teaching a lesson that implemented GeoGebra. The researcher used another semi-structured protocol to conduct pre- and post-observation interviews, and also used an observation tool to collect extensive notes. When the observations were done, the researcher met five of the six teacher users for a final focus group. One teacher could not attend the focus group due to a death in her family.

Using the analytic progression ladder recommended by Carney (as cited in Miles & Huberman, 1994), the researcher summarized and packaged the data, repackaged and aggregated the data, and developed and constructed an explanatory framework. During the first step of the ladder, the transcribed interviews were sent back to the teachers for their final approval. Survey responses, approved transcribed interviews, and observation notes, when available, were read and coded using *a priori* themes related to their perceived internal and external factors that foster or prevent the use of cognitive tools in the classroom: teachers perceived knowledge of GeoGebra, teachers' concerns, their beliefs, their Technological Pedagogical Content Knowledge (TPACK) development, external factors, experiences in the SMGEM program, and their instrumental orchestration.

For the second level of the analytical ladder, after the data for each teacher were triangulated, they then were used to write in-depth summaries of each participating teacher. Completed summaries were read and coded again according to each theme. Themes were subdivided into categories. For GeoGebra knowledge, the categories were basic and implementation. For concerns, the categories were personal, management, and consequence. The categories for beliefs were technology, mathematics, mathematics teaching, and mathematics learning. The TPACK development areas were curriculum, learning, teaching, and access. The external factors were resources, school, and policy. Experiences in the SMGEM were broken down into math content and technology content. Finally, the instrumental orchestration consisted of context, preparation, and action.

For the third and final level of the ladder of analytical abstraction, the researcher combined the summaries of the teachers in two different groups: the seldom or non-users of GeoGebra, and the teachers who were using GeoGebra as an instructional tool. The data for each group were clustered according to themes, and categories were developed with the intention of answering the research questions of the study. The findings for each group of teachers were summarized according to each research question and findings for each group were compared for similarities and differences.

Internal and External Factors

Responses to the two surveys and interviews were used to gather information about the internal and external factors that foster or prevent the use of GeoGebra in the middle school math classrooms. Data were analyzed in terms of knowledge on GeoGebra, concerns, external factors, beliefs, TPACK development, and experiences in the SMGEM program.

Hall and Horde (2011) described teacher' concerns as the feelings and perceptions they have about an innovation and/or a change process. They defined seven stages of concerns: one for unconcerned, two related to the person implementing the innovation, two related to the task, and two related to the impact of the innovation. Most of the literature reviewed on concerns of teachers when implementing technology used one concern from each of the last three areas: personal, management, and consequence. The study looked into those three stages as well.

Teachers at the personal stage might feel uncertain about the demands of the technology, feel inadequate to meet those demands, and be unsure of his or her role with the technology. In the study, three of the non-users worried that they could not design GeoGebra activities for their low-performing students and they did not think that GeoGebra could help their students. This outcome was consistent with the study of Wiske and Houde (1993), which found that teachers who were interested in providing more active inquiry activities to their students were concerned with the pre-planned activities and, as a result, missed opportunities to see how the technology also could help with their lectures. Mitchell et al. (2008) found that teachers have many concerns about integrating technology when they are in the later stages of a successful teaching career. The findings in the study contradict that conclusion, since the teachers with the most years of experience did not express any personal concerns. When information about technology continually changes, personal concerns might remain high for an extended period of time (Wedman et al., 1986). The users in the study had personal concerns of not being familiar with the changes in the new releases of GeoGebra and, as a consequence, felt they were not utilizing the software to its potential.

Hall and Hord (2011) explained that teachers are in the management concern stage when their attention is focused on the processes and tasks of using the technology. One of the non-users expressed concerns of using GeoGebra and having discipline problems due to the immaturity of the students. Wiske and Houde (1993) found that if teachers did not provide enough structure for students to understand how to work with the technology, or provided students with the prior knowledge necessary for the exploration of the activities, they would miss the teaching opportunities. Since observations were not done on non-users, it was not possible to determine the cause of the discipline problems experienced by this non-user teacher. Another non-user and five of the users expressed concerns about discipline problems due to the lack of computers that worked properly. None of the studies reviewed were related to the use of malfunctioning computers.

Hall and Hord (2011) described teachers in the consequence concern stage as being focused on the technology and its impact on their students' learning outcomes.

Wiske and Houde (1993) found that as teachers cope with their personal and management concerns, they slowly move to the consequence concern stage. However, this was not the case in the present study. Teacher users did not have any concerns about the impact of GeoGebra in their students. Even though they felt pressured due to the fast paced calendar they had to follow, they did not think they had to alter the pre-determined number of weeks that were supposed to be devoted to certain topics, or the sequence in which topics were presented. Wiske and Houde observed teachers, like NU-10A and NU-11B, who expressed concerns about how adequately students could reason inductively in order to understand and apply the new process of learning mathematics.

Teachers' beliefs about technology influence the types of activities they create for their mathematics students. Noss et al. (1990) observed that teachers who liked structured activities and step-by-step directions argued that the students would benefit from a similar approach. This was verified in the present study. Teacher U-08A was a student who liked structured activities and step-by-step directions in the SMGEM program, and she was observed teaching her students in that way. The other teacher users enjoyed more open-ended projects in the program and that is what they did in their classes. Although the non-users also expressed to having enjoyed open-ended activities with GeoGebra, they were not using it. Therefore, the results in this area were inconclusive.

Heid et al. (1999) concluded that teachers also could be influenced by their estimation of student potential, without realizing that unintended learning can occur when using technology. The two non-users who were teaching low-achieving students commented that they could not use GeoGebra until their students learned the basics of mathematics first. However, the teacher users felt that GeoGebra especially could benefit those types of students. The users and the other non-users believed that all students could discover rules and procedures on their own and that was not necessary to have a ‘math mind’ in order to do well in mathematics. Teacher U-08A modified the GeoGebra displays she showed her students, hiding the algebra window so they would not get confused. This action supports Kendal and Stacey’s (2001) findings, which suggested that some teachers modify their use of technology to avoid things they perceive to be beyond students’ abilities.

Integration of technology in the classroom is not only dependent on a teacher’s internal factors like knowledge, concerns, and beliefs, but also is influenced by factors external to the teacher (Zbiek & Hollebrands, 2008). The main external factor found to be an impediment in the use of GeoGebra by the non-users was the lack of working computers. This finding is similar to Pelgrum’s (2001) finding that reported on the results of the Second Information Technology in Education Study (SITES), which surveyed representative samples of schools from 26 countries from 1997 to 1999 and found that the most mentioned obstacle was the insufficient number of computers available. Teacher NU-09A was frustrated when she tried to use the laptops with her students and had to deal with the technical problems, which made her use up too much instructional time (Sandholtz & Reilly, 2004). Four of the teacher users also reported frustrations when using the laptops available for students’ use; nevertheless, they still found a way to use GeoGebra in their classrooms.

Five of the non-users felt they wanted more professional development. Four of them indicated they had used GeoGebra in their classroom after graduation but then stopped using it. The other two non-users had never used GeoGebra in their classrooms. It would seem that the lack of use made them feel they needed to refresh their knowledge on the software. Five of the non-users were interested in professional development that helped them design activities suitable for their own students. Garet et al. (2001) found that effective professional development that focused on content and skills gave teachers opportunities for hands-on work, and were highly consistent with teachers' needs. Teacher NU-10A complained that GeoGebra could not be downloaded in any of the computers because the administration did not give permission for that. She argued that the administrators should be made aware of the benefits of using GeoGebra as an instructional tool. Sandholtz and Reilly (2004) reported that leaders who have a clear vision of the benefits of technology are able to take several steps toward securing successful implementation of technology in school classrooms. Some of the teacher users also mentioned the lack of administrative support, since the administrators felt that other areas like reading took priority on their agendas.

Van Braak (2001) reported that teachers' resistance to adopt technology could be influenced not only by the introductory professional development they received, but also by on-going professional development that provided them with support when they needed it. Five of the teacher users were part of the additional NOYCE program that provided four more semesters of math instruction and GeoGebra usage. Only one of the non-users was part of this program. Four of the teacher users were highly involved outside their schools in groups that used GeoGebra to develop lesson plans for the district as well as for the state. They felt they had a support system that not only provided help when needed but also inspiration to continue using GeoGebra in the classroom. This finding was consistent with Becker and Riel's (2000) study. The authors also saw a relationship between involvement in activities outside the school and the use of technology: the more involved teachers were in professional activities, the more likely they were to have teaching philosophies compatible with constructivist learning theory and to use technology in exemplary ways. The teacher users in the present study were observed using technology to engage their students in cognitively challenging tasks, and promoting communication and collaboration among the students, actions that resonate with the constructivist learning theory.

Teachers users and non-users mentioned the FCAT as a barrier that prevented them not only from using GeoGebra more in the classroom, but also from implementing enrichment activities that they felt could benefit the students but that would take too much instructional time. Butzin (2001) also found the same and noted that the pressure to score high on standardized tests, along with the need to cover vast amounts of materials in a short amount of time, created such a challenge to teachers. Consequently, teachers felt they could cover more material if they use traditional instructional approaches, like lectures. The non-users were not observed and they were not asked what their preferred instructional method was. The FCAT and the district pacing guide forced the teachers to adhere to the given curriculum. Three of the non-users found it difficult to design GeoGebra activities for their students that were aligned with the curriculum they had to cover. Monaghan (2000) concluded that the availability of written curriculum materials aligned to the technology seemed to be a limiting factor for teachers implementing technology in their classrooms. Teacher U-08B predicted that if the new textbooks aligned with the CCSS did not include the use of GeoGebra, teachers would not be interested in its use since they were going to follow more closely the textbook suggested activities.

The TPACK development of non-users seemed to be in earlier stages than the development of the users. The difference in the TPCAK development could be a consequence of the use or non-use of GeoGebra, but it also could be the cause of its implementation. Niess et al. (2007) suggested that TPACK development seemed to be directly related to Pedagogical Content Knowledge (PCK), since teachers with more content and pedagogical knowledge have a better understanding of how to integrate technology into their practice. The determination of the PCK of the teachers was beyond the scope of the present study, and therefore nothing could be concluded in that aspect.

Teachers in the present study were teaching in the same district, had to follow the same policies and pacing guide, and had to use the same textbooks. These external factors did not seem to be the most influential factors in their decision whether to use GeoGebra, since six of them were implementing the software in their practices and six of them were not. Their knowledge of GeoGebra when they finished the program was comparable since users and non-users had the same training. The teachers were exposed to the same mathematical content knowledge; however, their PCK was not determined. The use or non-use of GeoGebra as a cognitive tool created different concerns for the teachers. Some of the non-users had concerns about the effectiveness of GeoGebra with their students and for this reason they stopped using it or did not try to use it. The users had more personal concerns because they were aware of the changes in the new releases of GeoGebra. However, the users felt they had a support system on which they could count when they had difficulties with GeoGebra or with the design of technology-enriched activities. The support system also provided inspiration and motivation on the use of GeoGebra in the classroom.

Differences and Similarities with Respect To How and Why Teachers Used GeoGebra

In order to answer this question, the theoretical framework of instrumental orchestration was used during the observations conducted on the teacher users. Instrumental orchestration consists of three elements: pedagogical context, preparation mode, and pedagogical action. Pedagogical context refers to the classroom arrangements, the topics to be covered, and what artifacts will be available, as well as what instructional approach will be used. The preparation mode represents the ways in which teachers decide to make full use of pedagogical preparation for the benefit of instruction. It includes decisions about the way an activity is introduced and worked out as well as the schema and techniques to be developed. Pedagogical action involves ad hoc decisions made during teaching about how to perform the chosen pedagogical configuration and preparation mode. In the mathematics classroom, this could involve a teacher deciding how to deal with unexpected aspects of technology or with the activity itself, as well as determining how to deal with students' questions and actions.

The pedagogical context was very similar for all six teachers. The student desks were arranged in rows facing the Promethean or white board, except for teacher U-10A who used a U-shaped configuration for the desks. The teachers had the same type of technology available to them: Promethean Board, document cameras, and LCD projectors.

Several studies had suggested that the use of technology could bring about changes in the curriculum (Fey et al., 2010; Heid, 1997; Kilpatrick and Davis, 1993). The classroom observations done in the present study showed teachers planned lessons in accordance with their curriculum. Furthermore, the teachers did not want to deviate much from the given curriculum due to the pressure on preparing students for the standardized testing. In this study, the curriculum determined the possibilities of the use of technology, and the use of technology did not alter the curriculum. Heid (1997) indicated that with technology, teachers tended to present concepts and applications before skills. The teacher users stressed conceptual understanding more than procedures in their lessons and in none of the observed lessons were students doing drill-and-practice. In the Beliefs survey, teachers indicated they believed it was more important to focus on concepts rather than procedures. The non-users had the same beliefs. The researcher wondered if this belief was due to the prolonged exposure to the professors in the SMGEM program, or if they had that belief before entering the program, since the beliefs of users and non-users were about the same in all areas.

Pea (1985) and Kilpatrick and Davis (1993) suggested that the use of technology in mathematics classrooms can change the instructional activities in two ways: it can increase the number and range of examples with which students can come into contact, or it can alter the arrangement of the mathematics content. The first type of technology is called an amplifier; it mostly is used for drill-and-practice or remediation. The second type of technology, called a reorganizer, is used for investigation and exploration. In the study, the use of GeoGebra was a combination of an amplifier and a reorganizer. The teachers were observed providing investigation and exploration activities to their students, even when they had whole classroom discussions. Part of the discovery process included some inductive reasoning where students could formulate conjectures after exploring several examples. The teachers saw one of the advantages of GeoGebra was the fact that they could provide numerous examples and counterexamples in a short amount of time.

Some of the classes observed were in Algebra 1 and geometry, classes that most of the non-users felt were most appropriate for the use of GeoGebra. Two of the teachers were teaching algebra classes. Heid (1997) classified the types of technology used in algebra as one of three: Computer Algebra Systems (CAS), graphing tools, and multiple representation tools. Following this classification, the teachers used GeoGebra as a graphing tool and a multiple representation tool. GeoGebra connects graphing and algebra, making the investigation of functions a very rich experience with multiple representations. Teachers were observed helping their students with the graphing of linear equations, the graphing of the solutions of systems of inequalities, and explorations on the line of best fit in a scatter plot.

Three of the teacher users were teaching lessons related to geometry; two of them were teaching geometry courses, while the third was teaching a sixth grade advanced course covering a geometry topic. They had turned their classrooms into places of discovery of geometric relationships like the geometric mean, discovery of formulas for areas of polygons, and understanding of the formula for the sum of interior angles in polygons. Teachers or students used GeoGebra to dilate, move, rotate, or translate figures without changing their significant properties. The uses of GeoGebra helped students generate and test their own hypothesis. According to the teachers, this visualization helped the students with the understanding of the rigorous proofs.

The classroom observations of teacher users showed some commonalities among the teachers in the environment, nature of the tasks, nature of the teachers' actions, and nature of the students' actions. According to Duarte et al. (2000), technology enhances the learning environment by providing opportunities for students to investigate ideas, verify their thinking, construct graphs and diagrams, and discuss their ideas with peers and adults. Sheets and Heid (1990) believed that even if teachers do not plan for group work, technology fosters the development of collaboration. The six teachers were able to create an environment where students were a community of learners that collaborated together in order to make sense of the math concepts. Teacher U-08A gave her students what she called 'conference time,' which Hoyles and Noss (1992) identified as distancing, one of the four benefits of computer-enhanced discussions aimed at promoting learning. Distancing occurs when students discuss their computer work with each other, representing their thoughts and raising them to a conscious level. The other three benefits according to Hoyles and Noss are conflict, scaffolding, and monitoring. Conflict occurs when the computer or the group discussions provoke a reconsideration of initial perceptions. When observing U-10A, students were confused when GeoGebra showed the line of best fit that did not pass through any of the given points in the scatter plot; the teacher then led a discussion of different methods for finding the line. The group provided scaffolding, or a way of reasoning, that individuals would not have been able to construct on their own. Many of the teachers referred to this as students having more 'aha' moments when GeoGebra was used. Groups monitor their discussions, thus facilitating metacognition. Vygotsky (1978) asserted that knowledge is built within a community through the social interactions of its peers. Social interaction has been shown to have a positive impact on learning, social behavior, and motivation (Zurita & Nussbaum, 2007). The teachers in the study fostered social interactions among the students and provided constructivist-learning opportunities to their students.

"Tasks are the projects, questions, problems, constructions, applications, and exercises in which students engage. They provide the intellectual context for students' mathematical development" (NCTM, 1991, p. 20). The tasks that the teachers provided to their students covered topics provided in their textbooks; the tasks were not to model or solve real world problems. The tasks mostly were about making connections of different representations, and analyzing different examples with the intention of formulating a conjecture. Since the tasks were not to solve word problems, teachers did not have to provide a context for their tasks. Using the classification provided by Choi and Hannafin (1997), the six teachers used tasks that were simple decontextualized problems. They were simple problems because the tasks isolated the key data, and they were decontextualized problems because they provided minimal, non-meaningful information. However, the teacher users provided tasks to their students that engaged them. The observed teachers facilitated the learning process by providing rich learning environments, assigning engaging tasks, incorporating GeoGebra, and stimulating interaction and communication in the classroom.

The observed teachers were facilitators of learning, and not lecturers in their classrooms. The teachers who promoted classroom discussion asked questions to their students constantly; they even started the lesson with questions. Teacher U-10A liked to ask 'what if' questions to her students. The teachers who had the students working on individual laptops were helping their students on an individual basis as well as letting the students help each other. Teachers' questions can help students focus on the interpretation and understanding of the results (Doerr & Zangor, 2000). The students seemed to feel comfortable in their classrooms and were not afraid to ask questions or to comment on other student suggestions. According to Reed et al. (2010), what students learn when using technology appears to be related to the teacher's attitude towards mathematics as a subject as well as the teacher's attitude toward the technology used. These teachers believed that technology was beneficial to students and that all students could do math and discover concepts when engaged in mathematical tasks.

Vygotsky (1978) suggested, "Learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and when in cooperation with his peers" (p. 90). In the observed classes, there were two types of interactions: student-teacher and student-student. Using the list of roles developed by Zbiek and Hollebrands (2008), the teachers in the observed classes became facilitators, collaborators and fellow investigators, planners and conductors, a resource, and technical assistants. Students also had roles in these classrooms; they were cooperating with each other, they were investigators, resources for other students, and technical assistants. The atmosphere always was relaxed, with respect shown at all times to all the people in the classroom.

Experiences in the SMGEM Program

Reports from the SMGEM concluded that the program raised the content knowledge, creativity, enthusiasm, and pedagogical awareness of in-service middle grade mathematics teachers. Of all 66 graduates, 37 had assumed leadership positions in the district as department heads, math coaches, team leaders, and mathematics curriculum specialists. Program participants delivered hundreds of workshops, including technology workshops introducing GeoGebra to other national programs similar to the SMGEM. In addition, four participants have received Teacher-of-the-Year awards, nine have obtained National Board Certification, three were nominated for the Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST), and one was a state finalist of the PAEMST. SMGEM's contribution to a hierarchical group of teacher leaders in the school district had provided crucial and necessary support for the ongoing professional development in the county's schools.

From the group of SMGEM graduates, 12 were selected for the present study. There was a great difference in the accomplishments of the teachers in both groups. From the group of non-users, one of them had National Board Certification; one was a math coach; and another, who had been a trainer for the district, was being trained to be a math coach. From the group of users, two had National Board Certification, three had been nominated Teachers-of-the-Year in the district, two were nominated for the PAEMST award with the state finalist in this group, one was a math coach, and four of them had been district trainers. All the users had presented a variety of workshops in the district, and three of them had been speakers in the National Council of Teachers of Mathematics (NCTM) annual convention. Three of the users were contributing lessons to CPALMS, and one was a recipient of their first awards. Five of them were sponsors of the math competition teams in their schools. Three were part of a special district team that, along with the district math curriculum specialists, were designing lessons that included GeoGebra and were aligned to the upcoming Common Core State Standards. These lessons are available to all teachers in the district via a special designed wiki.

SMGEM reported to have created and delivered a unique standards-aware and technologically literate curriculum for graduate-level middle grade mathematics teacher education, as an extension of the Master in Science in Teaching (MST) degree program of the Department of Mathematical Sciences at the higher education institute. Results from evaluations of the program concluded that its curriculum enhanced the mathematical background of middle grade mathematics and provided multidisciplinary connections. In comparison to matched non-participating teachers, graduates from the SMGEM program: (a) saw their students make significant mathematics content gains in categories related to the Florida Sunshine State Standards, as verified by district benchmark achievement tests; (b) implemented a wider range of pedagogies in their classrooms, including more effective use of specific technologies, especially GeoGebra; and (c) increased their mathematical self-efficacy. With respect to technology, the addition in 2006 of Dr. Markus Hohenwarter, the developer of GeoGebra, to the project's instructional leadership team dramatically enhanced the already strong modeling of pedagogical effectiveness by program personnel. Dr. Hohenwarter helped to generate technology leaders in the district who were able to provide GeoGebra workshops to other projects sponsored by the National Science Foundation (NSF) around the nation.

The positive impact of Dr. Markus Hohenwarter was expressed very clearly by the study participants who graduated in cohorts 2008 and 2009, since the graduates in these cohorts had the opportunity to interact with him in the classes given in the program, and accompany him to several of the workshops given nationally. The non-users group in the study did not have any graduates from the 2008 cohort, and two from the 2009 cohort. The users group consisted of two graduates from the 2008 cohort and three from the 2009 cohort. The only respondents to the Cognitive Tool survey from the 2008 cohort were three teachers who were all users. Two of them participated in the study, the third one declined participation. It would seem that the SMGEM program and the personal interactions with Dr. Hohenwarter had a bigger impact on the teachers who had graduated in earlier cohorts and their use of GeoGebra.

The participating teachers expressed the belief that the program prepared them well in terms of mathematics and technology. Both groups felt that the program helped them become stronger teachers. Three of the non-users felt they were prepared to teach higher levels and openly were seeking opportunities to teach in a high school. The users did not express any unhappiness about their teaching assignments. Except those three non-users who were unhappy with their assignments, the other teachers seemed to be happy with their jobs. They all felt the program and the professors changed them into strong, knowledgeable teachers.

The study's findings corroborate some of the important recommendations given by the SMGEM principal investigators, the evaluation and research teams, and the participating teachers for both the professional development of teachers and their own classroom effectiveness: (a) appropriate use of technology to engage students' interest and to clarify difficult points of comprehension, (b) the need to master multiple representations for presentation by the instructor as well as multiple constructions for the students to create, (c) a familiarity and comfort with student-centered inquiry-based instructional methods, (d) use of hands-on instruction that favors digital and concrete manipulatives, (e) adoption of a style of questioning that identifies student misconceptions and encourages exploration, and (f) an openness to "risk taking" so that instructional strategies do not become stagnant.

Discussion

The study was based on social constructivist theory that implied that learning mathematics in a technology-rich classroom involved an active process of students constructing new knowledge. The teachers who used GeoGebra in their classroom as an instructional tool facilitated knowledge construction by interacting continuously with students and allowing them to take control of the lesson through whole class discussions and investigations while using GeoGebra as the starting point of discussion or by answering 'what if' questions throughout the lesson. Even though the tasks used by the teacher users were simple and decontextualized, due to the restrictions of the curriculum and the impending FCAT, the instrumental orchestration of these teachers indicated that teachers' actions and behaviors while guiding students were ones of assuming the roles of facilitators, collaborators, fellow investigators, and technical assistants. Students in these classrooms also had assumed similar active roles. All this was done while maintaining a safe and relaxed atmosphere.

The use of GeoGebra as an instructional tool had a positive impact on the six teachers who used it and the teachers felt it also had a positive impact on their students.

The teachers expressed the use of GeoGebra helped them with the teaching tasks since it was easy to use and they were able to provide multiple examples in multiple representations to the students; they felt they had become stronger teachers as a result of its use. On a more personal level, they experienced empowerment by being recognized by the students as experts in the field who used tools no other teachers were using. They expressed feelings of accomplishment as professionals and had been recognized as such by the several awards they received as well as by the added responsibilities they had undertaken. On the other hand, the non-users expressed more frustration with their jobs. The comments of the teacher users about students showed respect for students at all times; this fact was verified by the observed interactions in the classroom. For teacher U-08B, student recognition was more appreciated than the administrator evaluations, as he indicated he did not work for the administrators, but for the students. Teacher U-08A stated that her 'mission' was to figure out ways to make her students understand the math concepts. Teacher U-09C mentioned she planned activities to make her students interested and engaged in the learning.

The personal concerns reported by the teacher users of GeoGebra in the classroom included the desire to continue learning the new features of GeoGebra as well as the desire to connect themselves and others to a driving sense of purpose that led to engaging teachers' efforts in a common endeavor for the benefit of other teachers and ultimately the students. A consequence of this connection to an external purpose had the effect of increasing a sense of ownership of GeoGebra in those involved in the effort. The involved teachers also shared the feeling that individual efforts were ineffective unless they were an integral part of a combined endeavor of a greater community that involved the publishers of future math books.

The external factors did not impede but restricted their use of GeoGebra in the classroom and there was a consensus among the teacher users that they had to strike a balance between their professional goals and the available resources. They did not over-emphasize the challenges they encountered and tended to downplay them, with the result of engaging students and providing them with the best learning experiences they could.

With regard to the implementation of GeoGebra by the teachers, it was clear that it was not enough to know the basics of the software and to have had some personal experiences with it as learners of mathematics, as was the case of the non-users. Teachers must feel ownership of the software and feel that it can work not only for them but also it can work with them; they need to see the software as an instrument. Rabardel (1995), following Vygotsky's (1978) understanding of tools, expressed this relationship when explaining the difference between a tool and an instrument. An artifact is a bare tool that is available to the user to solve a problem, but one that can be useless if the user does not know how to apply it. Only when the user realizes how the artifact can be used for a specific purpose can that artifact become an instrument that mediates the activity. A bare tool, or artifact, becomes an instrument when the user establishes a relationship between the artifact and the activity to be mediated. This relationship or interaction requires a mental process from the user. Therefore, an instrument is both an artifact and the necessary mental schemata that the user develops for a particular activity (Drijvers & Trouche, 2008). Through the years after the program, the non-users stopped making and regenerating those mental processes to use the tool in question, GeoGebra, while the users had the internal drive and the external support to continue generating and strengthening their mental processes with respect to the use of the tool.

The teacher users usage of GeoGebra in and outside the classroom, as well as their continuous participation in educational projects, transformed the teacher users into leaders in their district. However, it also could be said that their active involvement and connections made during the SMGEM program in the district made them belong to a group of leaders who share educational goals in tune with the current demands of math education and who utilize educational instruments, including GeoGebra, that help them achieve their goals. An important implication for teachers is to become part of a group of professionals who share similar goals and allows its participants to grow even further in their knowledge.

Einstein asserted that current and future problems cannot be solved with the thinking that created them. New ways to solve problems must be thought of and new approaches must be found to satisfy needs that are not grounded in any known system. In an effort to improve mathematics education in the United States, professional organizations have promoted the integration of technology in the classroom with the goal of engaging students in the process of learning and understanding mathematics (AMTE, 2009; ISTE, 2008; NCTM, 2008). The effect of technology especially is evident when students work in small groups and engage in student-centered activities (Becker & Ravitz, 2001; Carter & Smith, 2001; Li & Ma, 2010; Shapley et al., 2011). The current economic situation of the country has made it more difficult for schools to allocate money to replace and improve their resources. It will require a combined effort of teachers and administrators to find a way to modernize mathematics education in a way that can prepare students for a future that cannot be easily foreseen, given the rapid changes of today. Administrators must find ways to modernize the equipment available to the students and provide teachers with the possibilities of working in professional learning communities that involve teachers from the district. Collaboration with peers allows teachers to increase their knowledge about content, strategies, students, and resources, as well as empower them in their profession.

An organization engaged in connecting teachers to purpose and meaningful work is a model that appears to offer great potential for evolving toward higher degrees of sustainability, even if the resources are not optimal. However, in order to sustain its momentum within the district, it is essential that new teachers join with the basic skills and the mindset necessary to carry on. Teacher educators should include the effective use of cognitive tools in their teacher education programs. Research shows that since some teachers do not have the experience of learning their course content with technology, they are not prepared to implement new technologies in their teaching (Niess, 2011), and therefore can have concerns about the implementation of any innovation (Hall & Hord, 2011). The joint crucial task of helping pre-service teachers and in-service teachers is two-fold: to know about the best instructional practices and possible tools available, like GeoGebra, and to help them with the initial creation of the necessary mental schemata that teachers would need for a particular activity. This educational process does not end after graduation, but should continue throughout their careers as they share work and ideas with colleagues. Only then can a free flow of creative ideas for any innovation be sustained in a safe and highly productive environment, as the small group of teacher users in the present study demonstrated.

From the findings of this study, there are clear indicators of the need to do things very differently in math education in order to help out students now and see them become accomplished adults 20 or 30 years from today. What will the schools of the future look like? What are the skills and knowledge needed to impart to students now so they can be successful and responsible citizens of the future?

Developing teacher leaders who can bring about new and revolutionary changes in math education that use the innovative technology available in the world is possible, but it requires not only professional development but also continuous investment in those teachers. In order to provide adequate support and training, many local groups of teachers, mathematicians, and math educators are working together in developing and adapting mathematics curriculum, materials, and resources to serve the needs of all math teachers and students.

GeoGebra has created a global community of institutes that collaborate with each other in developing and using open source technology in schools and in teacher education. Several GeoGebra institutes also are involved in pioneering projects featuring the use of netbooks and laptop computers. GeoGebra developers also are committed to reaching out specifically to users in developing countries who otherwise could not afford to pay for its use. The growth and commitment of this community brings hope and encouragement to math teachers from all over the world to enhance math education for students at all levels.

Implications for Future Professional Development

The study's findings suggest a series of concrete suggestions for organizations like the NSF and higher education institutions. When considering the needs of teachers, it is important to give special consideration to the beliefs teachers have with respect to technology, mathematics, and math teaching and learning, prior to the professional development. It is important to know what teachers' predispositions are in order to tailor the professional development to the needs of the teachers. The needs of the teachers do not only include the need to provide technology-rich activities that promote the construction of knowledge to their students, but also the internal needs of teachers to satisfy their concerns and provide activities that are aligned with their beliefs and teaching styles.

The reasons teachers have for taking professional development are important to consider as well as if a teacher is proactive, reactive, or a combination in terms of the use of technology. Proactive teachers personally are interested in the integration of technology in their practice and might not have a problem seeing the educational value it can bring. Reactive teachers more likely are to be taking professional development as a demand from the school or district administrators and not because of a personal conviction that technology can be of an educational benefit in their classrooms. If reactive teachers are taking professional development, the trainers should spend more time with them, provide more concrete examples, and possibly share studies that show the teachers the benefits that using technology can bring to them and to their students.

Teachers' predispositions and teaching styles are very important to consider. If teachers like structured activities and provide students with step-by-step directions, they would prefer to be introduced to technology in the same manner. Structured teachers will feel more comfortable learning how to use any innovation if they are taught in a style that reflects their preferred teaching style. Teachers might have concerns about the use of technology in the classroom and how it can change the routine as well as the behavior and role of the students. Professional development trainers should demonstrate an organized routine for using technology that can be reproduced easily in the teachers' classrooms given the resources available to them in their schools, and that can minimize classroom disruptions. Teachers with positive attitudes toward technology have organized routines and have less discipline problems (Means, 2010; Shapley et al., 2011).

Teachers' beliefs are important to consider as well. If teachers believe mathematics is a collection of rules and procedures that should be memorized, they will be inclined to use technology as a computational tool and not as a tool for exploration and discovery. Studies have shown that when technology is used for drill and practice in the classroom, it can have negative effects on the students (Wenglinsky, 1998). In those cases, trainers should provide technology activities where teachers can experience the discovery of concepts in math. The use of questioning techniques that help with the discovery process and the construction of knowledge would be important to stress in professional development. Trainers also should be aware whether teachers believe if mathematics performance is related to students having a 'mathematical mind' and if the level of performance could change over time. Studies could be shared where teachers could see how technology can help students with learning disabilities when it is used in small group settings and implementing student-centered activities (Li & Ma, 2010).

In order to develop teachers' TPACK, trainers should be aware of the teachers' TPACK development stage and their Pedagogical Content Knowledge (PCK). It is important that teachers experience activities that help move them along the different TPACK development stages in the areas of curriculum, learning, and teaching. These areas are closely related to their own beliefs. For example, if teachers acknowledge that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum, they already are in the recognizing stage of development. If teachers express concerns about how to adapt the technology to their own curriculum, they are in the accepting stage. Teachers in the accepting stage could be shown activities where students use technology to verify and reinforce mathematical concepts, and then are asked to develop their own lesson. This would bring them to the adapting stage. As teachers are trained and shown how to adapt their own lesson to include technology that enhances their current curriculum, they move to the exploring stage. Finally, when teachers are able to modify their own curriculum to incorporate technology efficiently and effectively as a teaching and learning tool, they would arrive at the advancing stage, the last stage in the TPACK development model (Niess et al., 2009).

The current study findings show that teacher users tend to give decontextualized tasks to their students. International mathematics testing, like the Program for International Student Assessment (PISA), uses problems that are contextualized. If our students are to succeed and compete internationally in those tests, they should be proficient in solving problems that provide context and useful information and do not isolate the important information. Professional development trainers should provide examples of contextualized problems that teachers could use in their own classrooms and that could be solved easily with the use of technology.

A well-designed professional development in technology that takes into account the concerns and beliefs of teachers might not be enough to ensure the continuous use of technology in the teachers' classrooms, as the study showed in the case of the non-users. Institutes of higher education as well as funding institutions like NSF should plan for continuous support of teachers in their own classroom and provide them with a support system that will help them implement technology successfully and effectively. This support system could be in the form of professional learning communities in their district, regular pedagogy conferences where teachers could present their own tested technology lessons and earn in-service points that could be applied towards their state certification, and regular follow-up workshops where teachers could be informed about the new releases of the software. The activities described would have the added benefit of creating teacher leaders who can be the new leaders of future professional development opportunities.

The external factors, like lack of properly working computers and FCAT pressure to cover vast amounts of curriculum, were the same in the district, and yet these obstacles did not stop the users from integrating GeoGebra in their practice. The reasons to continue using the software were internal reasons that were reinforced by their collaboration and participation in educational activities outside their classroom. The beliefs and concerns of teachers should be addressed before and after professional development. A carefully planned professional development that targets the internal and external needs of the teachers, combined with support that continues after the initial training, can influence the resistance to adopt technology in the classroom (Van Braak, 2001).

Recommendations for Further Research

This study showed some important factors that influenced the implementation of cognitive tools like GeoGebra in the mathematics classroom. The following are some suggestions for consideration to build upon the findings of this study.

Admittedly, the sample used in the study was too small for any conclusions to be generalized. Therefore, it would be beneficial to locate programs in other countries, like in Europe, where GeoGebra is very popular, and conduct the study with a larger sample. This would allow for generalizations of the findings as well as an interesting comparison of possible factors.

This study only looked at middle school teachers who graduated from the SMGEM program. A future study could include high school teachers who graduated from the program, in an effort to understand if the content and the math subject have influence on the use of GeoGebra as an instructional tool. Another interesting study would be finding middle school math teachers who use GeoGebra but who have not participated in the SMGEM program.

It can be noted that many of the teachers who participated in this study did not have a bachelor’s degree in mathematics or mathematics education. The scope of this study was beyond the determination of the teachers’ math content knowledge. A future study could look to control the content knowledge and examine how the teachers are implementing dynamic mathematics learning environments.

These types of cognitive tools are new in math education; for example, the Cabri Geometer was developed in 1988-92, the Geometer's Sketchpad in 1992, and GeoGebra in 2001-02. More research that involves teachers as well as students must be done about the use of these dynamic learning environments. Specifically, more research should be done on the possible factors impacting the implementation of dynamic mathematics learning environments by teachers in an effort to help them implement these innovations successfully in their classrooms.

Conclusion

The use of technology in the mathematics classroom has been scarce and disappointing (Reed et al., 2010). However, when technology is implemented, it seems to have a positive effect on student achievement (Becker & Ravitz, 2001; Carter & Smith, 2001). The success of the implementation of technology is highly dependent on teachers' concerns and feelings (van den Berg & Ros, 1999).

This study confirms that teachers are the key component in the implementation of technology as an instructional tool. Their concerns and feelings had a greater impact toward the use of GeoGebra in the classroom than the external factors. The external factors were common to all teachers in the study; however some were using GeoGebra regardless of the lack of resources and adverse instructional time limitations. The continuous involvement of teacher users in professional activities outside of the classroom was a source of inspiration and motivation for their constructivist actions in the classroom. Their students, schools, district, and communities recognized these teacher users as exemplary teachers. In turn, these recognitions fueled them to continue in their mission to provide engaging instructional tasks to their students, which included the use of GeoGebra.

Technology is ubiquitous, and students need to be locally and globally connected, anywhere and at any time. In an ideal scenario, school facilities encourage not only the students but their most valuable resource: the teachers.

APPENDICES

Appendix A

Proposed Mathematics Teacher TPACK Standards and Indicators[1]

I. Designing and developing digital-age learning environments and experiences
Teachers design and develop authentic learning environments and experiences incorporating appropriate digital-age tools and resources to maximize mathematical learning in context.

Teachers…

1. Identify, locate, and evaluate
    a. Mathematical environments, tasks, and experiences in the curriculum to integrate digital technology tools for supporting students' individual and collaborative mathematical learning and creativity;
    b. Appropriate technological resources and tools for these mathematical environments, tasks, and experiences.
2. Design appropriate mathematical learning opportunities that incorporate worthwhile mathematical tasks, based on current research and that apply appropriate technologies to support the diverse needs of all students in learning mathematics (considering diverse learning styles, working strategies, and abilities using digital tools and resources).
3. Plan strategies to facilitate equitable access to technology resources for all students in learning mathematics.

II. Teaching, learning and the mathematics curriculum
Teachers implement curriculum plans that include methods and strategies for applying appropriate technologies to maximize student learning and creativity in mathematics.

Teachers …

1. Incorporate knowledge of all students' understandings, thinking, and learning of mathematics with technology.
2. Facilitate technology-enhanced mathematical experiences that foster creativity and encourage all students to develop higher order thinking skills while promoting discourse among students as well as among teacher and students.
3. Use technology to support learner-centered strategies that address the diverse needs of all students in learning mathematics as these strategies help students become responsible for and reflect on their own learning.
4. Advocate, model and teach safe, legal, and ethical use of digital information and technology use by all students in learning mathematics.

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[1] Source: Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education, 9*(1), 4-24. *Reprinted with permission.*

III. Assessment and evaluation
Teachers apply technology to facilitate a variety of effective assessment and evaluation strategies.

Teachers …

1. Apply appropriate technologies to assess all students' learning of mathematics, reflect upon the assessment results, and communicate those results using a variety of tools and techniques.
2. Assess students' appropriate and ethical use of technology resources in learning and communicating mathematics.
3. Use formative assessment of technology-enhanced student learning to evaluate students' mathematics learning and to adjust instructional strategies.
4. Align the technology expectations for assessment tasks and practices with that of mathematics classroom activities and expectations.

IV. Productivity and professional practice
Teachers use technology to enhance their productivity and professional practice.

Teachers …

1. Evaluate and reflect on the effective use of existing and emerging technologies to enhance all students' mathematical learning.
2. Exhibit leadership by demonstrating a research-based vision of integrating technology in teaching mathematics.
3. Demonstrate and promote safe, legal and ethical use of technology for learning and exploring mathematics with students, parents, and colleagues.
4. Use technology to communicate and collaborate with parents, colleagues, and the larger community in order to nurture student mathematical learning.
5. Regularly participate and interact in ongoing professional activities, taking advantage of new and emerging digital age communication resources, to improve their technological, pedagogical, and content knowledge for promoting student creativity and learning in mathematics.

Appendix B

Mathematics Teacher TPACK Development Model[2]

CURRICULUM & ASSESSMENT

C: Curriculum descriptor A: Assessment descriptor Ex: Mathematics Example

Recognizing

C: Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.

Ex: Creates graphs of multiple linear functions using graphing calculators to provide a visual representation for varying slopes. Considers these visuals as making sense of the idea of slope but is unsure of how this might help students learn the basic concept.

A: Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.

Ex: Does not allow calculator use when assessing students' understanding of solving linear equations.

Accepting

C: Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.

Ex: Attends and participates in mathematics dynamic geometry system workshop to identify curricular ideas for incorporating the technologies as learning tools. Mimics the incorporation of a dynamic geometry system idea from the workshop to display measuring the sum of the angles of a triangle that upon multiple changes of the triangle suggests that the sum of the angles of any triangle is 180 degrees.

A: Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).

Ex: Attends and participates in a mathematics assessment professional development to consider ideas for assessing students' understanding of solving systems of linear functions using the calculator as a tool. Mimics the assessment idea to explain the use of the calculator for solving systems of linear functions by using the trace function to identify the intersection. Often retests technology questions with paper and pencil questions to be sure that the concept was learned the 'right' way.

Adapting

C: Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.

Ex: Targets key topics students investigate with technology. Develops lessons to demonstrate mathematics concepts with technology and activities for students to use technology to verify or reinforce those concepts. After students have learned to create graphs of specific linear functions, students are challenged to use the spreadsheet to verify the graphical representation of the ordered pairs.

A: Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).

Ex: Allows use of calculator in an assessment but designs the assessment to focus on gathering students' conceptual understanding of solving systems of linear functions in addition to their procedural understanding.

Exploring

C: Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.

Ex: Adapts own previous mathematics lesson to include technology.

Ex: Develops own ideas about using technology to enhance current curriculum; thus, begins altering

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[2] Source: Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education, 9*(1), 4-24. *Reprinted with permission.*

preexisting activities or creating new activities for current curriculum.

A: Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).

Ex: Designs assessments where students are expected to show their understanding of mathematical ideas using an appropriate technology that extends beyond paper and pencil type questions.

Advancing

C: Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.

Ex: Develops innovative ways to use technology to develop mathematical thinking in students such as using virtual algebra tiles to extend ideas of handheld manipulatives to focus on variables in algebraic expressions.

Ex: Modifies and advances curriculum to take advantage of technology as a tool for teaching and learning such as using CAS to explore more complex algebraic expressions.

A: Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.

Ex: Develops innovative assessments to capture students' understandings of the mathematics embedded in the particular technology.

LEARNING

M: Mathematics learning descriptor C: Conception of student thinking descriptor Ex: Mathematics example

Recognizing

M: Views mathematics as being learned in specific ways and that technology often gets in the way of learning.

Ex: Mathematical exploration with technology rarely seen.

C: More apt to accept the technology as a teaching tool rather than a learning tool.

Ex: Technology is used only outside of normal classroom activities, such as checking homework, calculating large numbers, etc.

Accepting

M: Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.

Ex: Limits student technology use, particularly during the introduction and development of key topics.

C: Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.

Ex: Activities that use technology are almost always redone without technology to be certain students really learned the particular concept.

Adapting

M: Begins to explore, experiment and practice integrating technologies as mathematics learning tools.

Ex: Students explore some mathematics topics using technology.

C: Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.

Ex: Although students use technology for most topics, assessing student thinking remains mostly technology free.

Exploring

M: Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.

Ex: Students explore numerous topics using technology, sometimes ranging outside the topic at hand.

C: Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.

Ex: Technology activities are implemented and evaluated with respect to student learning of mathematics and student attitudes toward mathematics.

Ex: Manages technology-enhanced activities towards directing student engagement and self-direction in learning mathematics.

Advancing

M: Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.

Ex: Students explore mathematics topics, integrating various technologies in attempts to better understand mathematical concepts.

C: Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.

Ex: Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.

Ex: Technology is used to develop advanced levels of understanding of mathematical concepts.

TEACHING

M: Mathematics learning descriptor I: Instructional descriptor E: Environment descriptor PD: Professional development descriptor Ex: Mathematics example

Recognizing

M: Concerned that the need to teach about the technology will take away time from teaching mathematics.

Ex: Students use technology on their own and little or no instruction with technology is present.

I: Does not use technology to develop mathematical concepts.

Ex: Technology, if used in class, is used for menial or rote activities.

E: Uses technology to reinforce concepts taught without technology.

Ex: Focus on linear functions where students practice creating graphs by hand to explore different functions. After students have demonstrated competence with linear functions, summarize the knowledge, with a spreadsheet example or a graphing calculator example.

PD: Considers attending local professional development to learn more about technologies.

Ex: Attends local workshops that focus on gaining skills with the technology; context of the learning activities is mathematics.

Accepting

M: Uses technology activities at the end of units, for “days off,” or for activities peripheral to classroom instruction

Ex: Technology-enhanced activities are not used for topics that require more advanced technology skills.

I: Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.

Ex: Introduces the Pythagorean Theorem algorithmically; teacher use of dynamic geometry to verify the Pythagorean Theorem; students find solutions to example problems using paper and pencil.

E: Tightly manages and orchestrates instruction using technology.

Ex: Technology is directed, in a tightly sequenced, step-by-step process. Skill-based, non-exploratory technology use.

PD: Recognizes the need to participate in technology related PD.

Ex: Seeks out technology-related professional development, workshops that are directed at developing the technology in the learning of mathematics.

Adapting

M: Uses technology to enhance or reinforce mathematics ideas that students have learned previously.

Ex: Students use technology to reinforce previously teacher-taught concepts.

I: Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.

Ex: Technology-based lessons are incorporated that are tailored to students’ needs.

E: Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.

Ex: Begins to adapt instructional approaches that allow students opportunities to explore with technology for part of lessons.

PD: Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).

Ex: Shares ideas from professional development with other mathematics teachers in the building.

Exploring

M: Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.

Ex: Teachers share classroom-tested, technology-based lessons, ideas, and successes with peers.

I: Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.

Ex: Students use technology to explore new concepts as the teacher serves mostly as a guide.

E: Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.

Ex: The teacher incorporates a variety of technologies for numerous topics.

PD: Seeks out and works with others who are engaged in incorporating technology in mathematics.

Ex: Organizes teachers of similar mathematics and grade level in investigating the mathematics curriculum to integrate appropriate technologies.

Advancing

M: Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.

Ex: Teacher is seen as a resource as novel ideas for helping students learn mathematics with technology.

I: Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.

Ex: The teacher helps students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics.

E: Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.

Ex: The teacher forms and reforms learning groups where individual and group learning is valued and encouraged.

PD: Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.

Ex: Engages teachers in the district in evaluating and revising the mathematics curriculum to more seamlessly integrate technology throughout the grades, adjusting the curriculum for a 21st century mathematics curriculum with appropriate technologies.

ACCESS

U: Usage descriptor B: Barrier descriptor A: Availability descriptor Ex: Mathematics example

Recognizing

U: Permits students to use technology ‘only’ after mastering certain concepts.

Ex: Mathematical exploration with technology tools is challenged by beliefs about how students need to learn mathematics.

B: Resists consideration of changes in content taught although it becomes accessible to more students through technology.

Ex: Student access to technology is limited to ‘after’ they have learned the given concepts using paper and pencil procedures and only for rote activities.

A: Notices that authentic problems are more likely to involve ‘unfriendly numbers’ and may be more easily solved if students had calculators.

Ex: Assigns some mathematics problems using school and community data but saves then for “extra credit” work if students have calculators.

Accepting

U: Students use technology in limited ways during regular instructional periods.

Ex: Student activities with technology are limited to brief tightly controlled situations.

B: Worries about access and management issues with respect to incorporating technology in the classroom.

Ex: Students can only use technology in isolated situations or non-important learning situations.

A: Calculators permit greater number of examples to be explored by students.

Ex: Student use calculators to investigate patterns and functions.

Adapting

U: Permits students to use technology in specifically designed units.

Ex: Access to and use of technology is available for exploration of new topics, usually with the teacher's demonstration.

B: Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.

Ex: Concepts learned with technology are not assessed with technology.

A: Concepts are taught differently since technology provides access to connections formerly out of reach.

Ex: Students use dynamic geometry software to investigate and make connections between trigonometry functions.

Exploring

U: Permits students to use technology for exploring specific mathematical topics.

Ex: Access to and use of technology is available and encouraged for mathematics exploration during most class times.

B: Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges.

Ex: Technology is used extensively in assessments. Seeks out ways to obtain technology for classroom use and begins creating methods for technology management issues.

A: Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.

Ex: Simultaneous equations are developed from an authentic situation, solved, and interpreted using graphs, tables, symbols and data.

Advancing

U: Permit students to use technology in every aspect of mathematics class.

Ex: Technology is seen as an opportunity to challenge notions of what mathematics students can master.

B: Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.

Ex: Technology is used to expand the mathematics concepts that can be accessed by students.

A: Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.

Ex: Using the Internet to find interesting mathematical problems, students investigate the role that technologies can play in finding solutions to the problems.

Appendix C

Approval from FAU Institutional Review Board

Institutional Review Board
Mailing Address:
Division of Research
777 Glades Rd., SU-80, Suite 106
Boca Raton, FL 33431

Tel: 561.297.0777 Fax: 561.297.2573

http://www.fau.edu/research/researchint
Nancy Aaron Jones, Ph.D., Chair

DATE:	August 10, 2012
TO:	Yash Bhagwanji, PhD
FROM:	Florida Atlantic University IRB
IRBNET ID #:	310786-3
PROTOCOL TITLE:	[310786-3] Middle school mathematics teachers' usage of dynamic mathematics learning environments as cognitive instructional tools
SUBMISSION TYPE:	Amendment/Modification
ACTION:	APPROVED
EFFECTIVE DATE:	August 10, 2012

Thank you for your submission of Amendment materials for this research protocol. The Florida Atlantic University IRB has approved your request to modify your protocol as outlined below:

- *Change of dissertation chair for Ana Escuder's protocol from Dr. James McLaughlin to Dr. Yash Bhagwanji.*

Please use the stamped, revised consents that accompany this approval letter.

If you have any questions or comments about this correspondence, please contact Angela Clear at:

Institutional Review Board
Research Integrity/Division of Research
Florida Atlantic University
SU-80, Suite 106
Boca Raton, FL 33431
Phone: 561-297-0777

* Please include your protocol number and title in all correspondence with this office.

This letter has been electronically signed in accordance with all applicable regulations, and a copy is retained within our records.

Appendix D

Approvals from School Board

THE SCHOOL BOARD OF [redacted], FLORIDA
INSTITUTIONAL REVIEW BOARD
[redacted]

[redacted]
Institutional Review Board (IRB) Chair
[redacted]

SCHOOL BOARD

Chair
Vice Chair

Board Members

[redacted]

October 10, 2012

Mrs. Ana Escuder
1135 Yellowheart Way
Hollywood, FL 33019

Dear Mrs. Escuder:

Thank you for submitting your research proposal #703 — ***Middle School Mathematics Teachers' Usage of Dynamic Mathematics Learning Environments as Cognitive Instructional Tools*** — for consideration by [redacted]. Staff has reviewed your research proposal and approval has been granted for you to *contact the Principals at the following Middle Schools **only***.

[redacted]

This approval means that we have found your proposed research methods to be compatible with a public school setting and your research questions of interest to the school District. However, approval is contingent upon consent forms specifying tat data will be kept for 5 years in accordance with [redacted] standards. Also, reviewers expressed concern with teacher interviews being conducted at the researcher's home. They would prefer an alternate location. The expiration date on your proposal is ***Thursday, October 10, 2013.*** If you are unable to complete your research by the expiration date, you must submit a Request for Renewal, ([redacted]), to the Student Assessment & Research Department ***four weeks*** prior to the expiration date. If a renewal is granted, all identification badges must be updated through the Security Clearance Department.

Implementing your research, however, is a decision to be reached by the affected school-based staff on a ***strictly voluntary basis***. To assist the school-based staff in their decision to participate, please outline the operational steps to be performed by staff at their school. Based upon this information, each school-based staff would then be ***asked to make a decision to participate or not*** and ***inform you or the requesting research parties of their decision at the time of your/their request.*** School-based staff have been instructed not to cooperate unless you **provide this District Security Approval Letter and the Principal Security Approval Memorandum**.

Note that you or any member of your research team who is *not* a current [redacted] employee and who will interact with students and/or District staff must comply with the District's Security Clearance procedures by completing the attached Security Forms as instructed below. **Please call the District's Security Clearance Department at [redacted], if you have any questions.**

Educating Today's Students for Tomorrow's World
[redacted] ***Public Schools Is An Equal Opportunity/Equal Access Employer***

Research Request #703
Mrs. Ana Escuder
October 10, 2012
Page 2

Each person involved with this project must complete and sign the **Security Clearance Form.** Return the form, via facsimile or U.S. Postal Service to **Ms.** [redacted] to obtain IRB Chair's signature. The signed form will be returned to you via facsimile or U.S. Postal Service.

To complete the registration process, ***each researcher and/or member of the research team must*** bring with them the ***full*** contents of this Security Approval Packet and a Photo ID, such as a valid State Driver's License, or U.S. Passport. If you did not receive a Security Background Check Form or a Security Clearance Form, **please contact Ms.** [redacted] **or via e-mail at** [redacted].

Please be aware that the Identification Badge(s) assigned to you and/or members of the research team for this project, are the property of The School Board of [redacted] Florida, and as such, ***must be returned*** to the Student Assessment & Research Department upon completion of the research project or soon after the Research Expiration Date whichever occurs first. Identification badges should be returned to: **The School Board of** [redacted] **Student Assessment & Research,** [redacted].

The anticipated date for submitting an electronic copy of your research findings is ***Monday, February 10, 2014.*** If additional assistance is needed from our staff, **please contact me at** [redacted]

Sincerely,

[redacted]

[redacted]

Attachments

THE SCHOOL BOARD [redacted]
STUDENT ASSESSMENT & RESEARCH

[redacted]

Telephone: [redacted] Facsimile: [redacted]

PRINCIPAL SECURITY APPROVAL EXPIRES THURSDAY, OCTOBER 10, 2013.

October 10, 2012

TO: Principals at the Middle Schools listed below

FROM: [redacted]
Institutional Review Board (IRB) Chair

VIA: [redacted]
Chief School Performance and Accountability Officer

SUBJECT: **PRINCIPAL SECURITY APPROVAL MEMORANDUM FOR RESEARCH PROPOSAL #703 — MIDDLE SCHOOL MATHEMATICS TEACHERS' USAGE OF DYNAMIC MATHEMATICS LEARNING ENVIRONMENTS AS COGNITIVE INSTRUCTIONAL TOOLS**

Staff has reviewed research proposal, #703 — ***Middle School Mathematics Teachers' Usage of Dynamic Mathematics Learning Environments as Cognitive Instructional Tools*** — submitted by Mrs. Ana Escuder, a Doctoral Student at Florida Atlantic University, and approval has been granted for the researcher and/or members of the Research Team to *contact the Principals at the following Middle Schools **only***:

[redacted]

The recently completed review of the proposed research involved school- and/or district-based staff, Institutional Review Board (IRB) approvals, and a review of the proposed research methods. These steps were taken to determine if the proposed methods demonstrated reasonable promise of generating data/analyses that will accurately answer the main research questions of interest.

Your ***participation*** in this research project is ***strictly voluntary***. To aid in your decision, Mrs. Escuder has been instructed to share, with each selected school-based staff, a complete description of research activities, as well as **provide the District Security Approval Letter and this Principal Security Approval Memorandum** for their review. Based upon this information, each school-based staff would then be ***asked to make a decision to participate or not*** and ***inform the requesting research parties of their decision at the time of their request.***

[redacted]

Appendix E

Cognitive Tools Use Survey

TITLE: Middle School Teachers Usage of Dynamic Learning Mathematics Environments as Cognitive Instructional Tools

Investigator(s): Ana Escuder and Dr. James McLaughlin

COGNITIVE TOOLS USE SURVEY

Thank you for participating in our research study. The purpose of this study is to describe, analyze, and compare information on (1) the practices of technology integration by a selected group of 12 middle school mathematics teachers, and (2) on their decision making processes regarding technology integration. You were selected to take this survey because you graduated from the NSF-funded Program *Standards Mapped Graduate Education and Mentoring*.

It should take you no more than 15 minutes to complete this survey. Your participation in this study is your choice. You may skip any questions that make you feel uncomfortable and you are free to withdraw from the study at any time without penalty.

There are minimal risks involved with participating in this study. **You will not be identified by name and this information will not be shared with your administrators or District personnel.** We do not know if you will receive any direct benefits by taking part in this study. However, this research will contribute to a greater understanding of why and how middle school mathematics teachers integrate cognitive technology in their practices.

By completing this online survey, you are consenting to participation in this study.

If you experience problems or have questions regarding your rights as a research subject, contact the Division of Research at (561) 297-0777. For other questions about the study, you should contact Ana Escuder, (954) 558-8784 or my advisor, Dr. James McLaughlin, (561) 297-3965. By completing and submitting the online survey, you give consent to participate in this study. We will give you a copy of this statement for your records.

Instructions:
Your response should be based on your experience as a teacher in your current teaching discipline. **Please answer all questions as best as you can.**

Section I: Personal information

1. My name is: ________________
2. Including this year, I have _________ years of full-time teaching experience.
3. I have used a computer for ________ years at home.
4. I have used a computer for ________ years in my teaching.
5. I graduated from the NSF program _____ years ago.
6. Since graduation, I have participated in _____ NSF-sponsored Pedagogy Conferences.
7. I am currently teaching ____ middle school, ____ high school.

Section II: Inventory

8. Number of computers available to you for your instructional activities._________
9. Average number of students in your class/classes? _____________
10. Approximately how much time each week do you spend in the preparation for and utilization of GeoGebra for instructional purposes?
    _____a. 15 min   _____c. 45 min   _____e. more than 60 min
    _____b. 30 min   _____d. 60 min   _____f. do not use

Please check the ones that are most appropriate in the questionnaire.

Section III: GeoGebra Usage

	Please answer the following questions by checking the appropriate box using the following coding: Y – Yes, N – No, NA – Not available	**Y**	**N**	**NA**				
11	Since graduation from the program, did you use GeoGebra for instructional purposes?							
12	Do you use GeoGebra for instructional purposes now?							
13	Did you use GeoGebra more frequently after graduating from the program than now?							
	(if your answer to question 12 is no, go to question 16)							
	Approximately how often do you use GeoGebra for the following types of instructional activities now? Please check the appropriate box using the following coding: D – Daily, ED – Every other day, W – weekly, EW – Every other week, M – monthly, EM – every other month or less, or N – never to the following questions	**D**	**ED**	**W**	**EW**	**M**	**EM**	**N**
14	I use GeoGebra for:							
	Presentation of new material							
	Demonstration							
	Enrichment activities							
	Drill and practice							
	Experimentation/simulations							
	Preparation of tests							
	Testing							
	Remediation							
	Problem solving							
	Collaborative learning							
	Other (please specify)							

	What type/types of computer access do you have? Please check the appropriate box using the following coding: C – Classroom, L – computer Lab, CC – Computer Carts, M – Media center, H – Home, or N – none. You may have more than one answer	**C**	**L**	**CC**	**M**	**H**	**N**
15	The computers I use for instruction are in the:						

Section IV: Technology Needs
Please circle the number which best reflects your needs.
What do you need to make GeoGebra a more integral part of your classroom's curricular activities? Please put an "X" in the space in each row which best reflects your belief.

1	2	3	4
Strongly Disagree	Disagree	Agree	Strongly Agree

16	I need more time to learn to use GeoGebra on my own	1	2	3	4
17	I need more time to plan activities that incorporate GeoGebra	1	2	3	4
18	I need more training with GeoGebra	1	2	3	4
19	I need more examples in the use of GeoGebra in my math classes	1	2	3	4
20	I need access to more computers for my students	1	2	3	4
21	I need prompt technical support to keep the computers working	1	2	3	4
22	I need to be able to try out already designed GeoGebra activities in my classroom before I am comfortable designing my own	1	2	3	4
23	I need more opportunities to work with colleagues to become more proficient using GeoGebra-enhanced curriculum units	1	2	3	4
24	I need good reasons why I should incorporate GeoGebra into the classroom.	1	2	3	4
25	I need the administration to provide staff development activities in GeoGebra to help me integrate it	1	2	3	4
26	I need the administration to provide resources for integrating technology into the classroom	1	2	3	4
27	I need more instructional time to incorporate GeoGebra activities in my classroom	1	2	3	4

Section V: Expertise in GeoGebra

Please take a moment to answer a few questions about your present use of GeoGebra. Please put an "X" in the space in each row which best reflects your belief.

1	2	3	4
Strongly Disagree	Disagree	Agree	Strongly Agree

28	I can find my way around GeoGebra (using menus, tools, and basic functions with confidence)	1	2	3	4
29	I know how and when GeoGebra may enhance my classroom activities (support student learning and help me meet curriculum standards)	1	2	3	4
30	I can locate the learning opportunities I need to advance my GeoGebra skills as new versions arrive	1	2	3	4
31	I can identify the connection between information literacy (making sense of information) and my curriculum goals	1	2	3	4
32	I can demonstrate how to use GeoGebra, for problem-solving, investigation, exploration, presentation and group work	1	2	3	4
33	I can design student learning activities that integrate GeoGebra into the daily life of my classroom	1	2	3	4
34	I can develop an expanded repertoire of strategies to promote student learning with GeoGebra, including problem-based learning	1	2	3	4
35	I am skillful in using productivity tools for professional use, including word processing, database, spreadsheet and graphic skills	1	2	3	4

Section VII: Levels of Use

Please read the descriptions of each of the six stages related to adoption of technology. Circle the number of the stage that best describes your level.

1	**Mechanical**
	I use it more to meet my needs (drawings for tests) than directly for my students' learning
2	**Routine**
	Use of GeoGebra is stabilized; little preparation is needed for its use
3	**Refinement**
	I vary the use of GeoGebra to increase its impact on students
4	**Integration**
	I combine my own efforts to use GeoGebra with related activities of colleagues
5	**Renewal**
	I keep up with the new improvements of GeoGebra and explore new goals for self and my students

This is the end of the survey. Thank you very much for your effort and input.

Appendix F

Beliefs Survey

TITLE: Middle School Teachers Usage of Dynamic Learning Mathematics Environments as Cognitive Instructional Tools

Investigator(s): Ana Escuder and Dr. James McLaughlin

BELIEFS SURVEY

Thank you for participating in our research study. The purpose of this study is to describe, analyze, and compare information on (1) the practices of technology integration by a selected group of 12 middle school mathematics teachers, and (2) on their decision making processes regarding technology integration. You were selected to take this survey because you graduated from the NSF-funded Program *Standards Mapped Graduate Education and Mentoring*.

It should take you no more than 20 minutes to complete this survey. Your participation in this study is your choice. You may skip any questions that make you feel uncomfortable and you are free to withdraw from the study at any time without penalty.

There are minimal risks involved with participating in this study. **You will not be identified by name and this information will not be shared with your administrators or District personnel.** We do not know if you will receive any direct benefits by taking part in this study. However, this research will contribute to a greater understanding of why and how middle school mathematics teachers integrate cognitive technology in their practices.

By completing this online survey, you are consenting to participation in this study.

If you experience problems or have questions regarding your rights as a research subject, contact the Division of Research at (561) 297-0777. For other questions about the study, you should contact Ana Escuder, (954) 558-8784 or my advisor, Dr. James McLaughlin, (561) 297-3965. By completing and submitting the online survey, you give consent to participate in this study. We will give you a copy of this statement for your records.

Technology Beliefs
To what extent do you agree or disagree with the following beliefs about technology in education? Please put an “X” in the space in each row which best reflects your belief.

1	2	3	4
Strongly Disagree	Disagree	Agree	Strongly Agree

		1	2	3	4
1	Textbooks will be replaced by electronic media in the near future				
2	The role of schools will be dramatically changed because of the Internet in the near future				
3	My role as the teacher will be dramatically changed because of the educational technology in the near future				
4	I am a better teacher with GeoGebra				
5	I attend technology-based professional development when my district offers it				
6	GeoGebra-assisted instruction aids learning				
7	GeoGebra can help accommodate different learning styles				
8	A mathematics teacher should know how to use GeoGebra				
9	The use of GeoGebra is just another way to teach, such as using a whiteboard, textbook, etc.				
10	Using GeoGebra is a strategy employed by effective mathematics teachers				
11	GeoGebra should be used when needed; otherwise, traditional methods should be used				
12	Teaching with GeoGebra is as good as, if not better, than traditional forms of teaching				
13	My students are more motivated to learn when using GeoGebra				
14	My least fear of using GeoGebra is embarrassment in front of my students				
15	The use of GeoGebra can be an effective substitute for a real-life learning experience in the classroom				
16	Using GeoGebra increases student learning				
17	A teacher who plans lessons that are learner-centered in mathematics uses GeoGebra.				

Beliefs about the Nature of Mathematics

To what extent do you agree or disagree with the following beliefs about the nature of mathematics? Please put an "X" in the space in each row which best reflects your belief.

1	2	3	4
Strongly Disagree	Disagree	Agree	Strongly Agree

		1	2	3	4
1	Mathematics is a collection of rules and procedures that prescribe how to solve a problem				
2	Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures				
3	Mathematics involves creativity and new ideas				
4	In mathematics many things can be discovered and tried out by oneself				
5	When solving mathematical tasks you need to know the correct procedure or else you would be lost				
6	If you engage in mathematics tasks, you can discover new things (e.g. connections, rules, concepts).				
7	Fundamental to mathematics is its logical rigor and preciseness				
8	Mathematical problems can be solved correctly in many ways				
9	Many aspects of mathematics have practical relevance				
10	Mathematics helps solve everyday problems and tasks				
11	To do mathematics requires much practice, correct application of routines, and problem-solving strategies				
12	Mathematics means learning, remembering and applying				

Beliefs about Learning Mathematics

To what extent do you agree or disagree with the following beliefs about learning mathematics? Please put an "X" in the space in each row which best reflects your belief.

1	2	3	4
Strongly Disagree	Disagree	Agree	Strongly Agree

		1	2	3	4
1	The best way to do well in mathematics is to memorize all the formulas				
2	Students need to be taught exact procedures for solving mathematical problems				
3	It doesn't really matter if you understand a mathematical problem as long as you can get the right answer				
4	To be good in mathematics you must be able to solve problems quickly.				
5.	Students learn mathematics best by attending to the teacher's explanations				
6	When students are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed				
7	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct				
8	Teachers should allow students to figure out their own ways to solve mathematical problems				
9	Non-standard procedures should be discourage because they can interfere with learning the correct procedure				
10	Hands-on mathematics experiences aren't worth the time and expense				
11	Time used to investigate why a solution to a mathematical problem works is time well spent				
12	Students can figure out a way to solve mathematical problems without a teacher's help				
13	Teachers should encourage students to find their own solutions to mathematical problems even if they are inefficient				
14	It is helpful for students to discuss different ways to solve particular problems				

Beliefs about Teaching Mathematics

To what extent do you agree or disagree with the following beliefs about teaching mathematics? Please put an "X" in the space in each row which best reflects your belief.

1	2	3	4
Strongly Disagree	Disagree	Agree	Strongly Agree

		1	2	3	4
1	Since older students can reason abstractly, the use of hands-on models and other visual aids become less necessary				
2	To be a good at mathematics you need to have a kind of "mathematical mind"				
3	Mathematics is a subject in which natural ability matters a lot more than effort				
4	Only the more able students can participate in multi-step problem solving activities				
5	Mathematical ability is something that remains relatively fixed throughout a person's life				
6	Some people are good at mathematics and some aren't				
7	There is a best way to solve a math problem				
8	Mathematics is easy to teach because the answer is either right or wrong				
9	Mathematics requires logic, not intuition				
10	Mathematics is a series of rules to be memorized and followed				
11	Mathematics learning is independent of the students' social environment				
12	Students should solve mathematics problems individually				

Appendix G

Interview Protocol for Seldom or Never GeoGebra Users

I appreciate you letting me interview you. I have some questions related to the use of technology in your classroom. This interview should take no more than 60 minutes. Would you mind if I tape the interview? It will help me stay focus on our conversation and it will ensure I have an accurate record of what we discussed.

During this interview, please do not refer to any student or teachers using their names. If you need to refer to a student, teacher, or other person, please use other identifiers. You can choose not to answer a particular question or to end this interview at any point.

You will be provided with a complete transcript of the interview for your approval. You can modify or add information to the transcript. Participation in this study is voluntary. Taking part in this interview is your agreement to participate.

1. Please describe your experience as a math student in elementary and secondary school.
2. What is your degree and what math classes did you take in college?
3. Please describe your experience as a math student in college and graduate school.
4. How and why did you decide to become a mathematics teacher?
5. Please describe your current teaching assignment.
6. Think of a time in the NSF program when you were using GeoGebra to explore a math concept, and describe that in as much detail as possible.
7. Please describe one of your experiences as a teacher using GeoGebra, and tell me what that was like for you.
8. Please describe in as much detail as possible one of your students' experiences when you used GeoGebra in the classroom.
9. What factors most influenced your decisions to not use GeoGebra as an instructional tool?
10. Are you currently using any other instructional technology in your classroom?
11. What would need to change in your school or classroom for you to consider using GeoGebra as an instructional tool?

Possible follow up questions for any of the previous questions:

a. You mentioned---------------, tell me what that was like for you.
b. You mentioned---------------, describe that in more detail for me.
c. You mentioned---------------, describe a specific example of that.

This is the end of the interview. You will receive a transcript of this interview in the next few days, and you will be able to inform me of any additional information or changes you feel are necessary. Thank you for your participation.

Appendix H

Interview Protocol 1 for GeoGebra Users

I appreciate you letting me interview you. I have some questions related to the use of technology in your classroom. This interview should take no more than 60 minutes. Would you mind if I tape the interview? It will help me stay focus on our conversation and it will ensure I have an accurate record of what we discussed.

During this interview, please do not refer to any student or teachers using their names. If you need to refer to a student, teacher, or other person, please use other identifiers. You can choose not to answer a particular question or to end this interview at any point.

You will be provided with a complete transcript of the interview for your approval. You can modify or add information to the transcript. Participation in this study is voluntary. Taking part in this interview is your agreement to participate.

1. Please describe your experience as a math student in elementary and secondary school.
2. What is your degree and what math classes did you take in college?
3. Please describe your experience as a math student in college and graduate school.
4. How and why did you decide to become a mathematics teacher?
5. Please describe your current teaching assignment.
6. Please describe how did you start using technology in your teaching?
7. Think of a time in the NSF program when you were using GeoGebra to explore a math concept, and describe that in as much detail as possible.
8. Please describe one of your experiences as a teacher using GeoGebra, and tell me what that was like for you.
9. Please describe in as much detail as possible one of your students' experiences when you use GeoGebra in the classroom.
10. Please describe the role that technology plays in your classroom. How do your students use GeoGebra to learn mathematics in your classroom?
11. What factors most influenced your decisions to use GeoGebra as an instructional tool?
12. Are you currently using any other instructional technology in your classroom?

Possible follow up questions for any of the previous questions:

d. You mentioned---------------, tell me what that was like for you.
e. You mentioned---------------, describe that in more detail for me.
f. You mentioned---------------, describe a specific example of that.

This is the end of the interview. You will receive a transcript of this interview in the next few days, and you will be able to inform me of any additional information or changes you feel are necessary. Thank you for your participation.

Appendix I

Interview Protocol 2 for GeoGebra Users

I appreciate you letting me interview you. I have some questions related to the use of technology in your classroom. This interview should take no more than 45 minutes. Would you mind if I tape the interview? It will help me stay focus on our conversation and it will ensure I have an accurate record of what we discussed.

During this interview, please do not refer to any student or teachers using their names. If you need to refer to a student, teacher, or other person, please use other identifiers. You can choose not to answer a particular question or to end this interview at any point.

You will be provided with a complete transcript of the interview for your approval. You can modify or add information to the transcript. Participation in this study is voluntary. Taking part in this interview is your agreement to participate.

<u>Pre-observation</u>

1. Tell me about the ability level of the students in this class.
2. In your opinion, how do they compare to students in the school as a whole?
3. Are there any students with special needs in this class? Can you describe the student's need?
4. Please describe what the students have been learning in the past few days.
5. Please describe the objectives for today's lesson.
6. What led you to teach the math concept/skills in this lesson?
    a. It is included in the district curriculum/course of study.
    b. It is included in the state/district math assessment.
    c. It is included in an assigned textbook or program designated for this class.
7. How are you planning to use GeoGebra in today's lesson?
8. What mathematical concepts will be enhanced with the use of GeoGebra?
9. How do you think GeoGebra will facilitate students' understanding?
10. Have you used GeoGebra to teach this concept before?

<u>Post-observation</u>

1. How do you feel about how the lesson played out?
2. What do you think the students gained from today's lesson?
3. Did you make any changes from your original plan? Why?
4. Do you think the use of GeoGebra helped the lesson? Why?
5. What do you think about the students' reaction to the use of GeoGebra?
6. What would you change on today's lesson?
7. Does GeoGebra make you change your teaching style? How?

Possible follow up questions for any of the previous questions:

a. You mentioned---------------, tell me what that was like for you.
b. You mentioned---------------, describe that in more detail for me.
c. You mentioned---------------, describe a specific example of that.

This is the end of the interview. You will receive a transcript of this interview in the next few days, and you will be able to inform me of any additional information or changes you feel are necessary. Thank you for your participation.

Appendix J

Focus Group Protocol

I appreciate you participating in this focus group. I have some questions related to the use of technology in your classroom. This focus group should take no more than 90 minutes. Would you mind if I tape the interview? It will help me stay focus on our conversation and it will ensure I have an accurate record of what we discussed.

During this interview, please do not refer to any student or teachers using their names. If you need to refer to a student, teacher, or other person, please use other identifiers. You can choose not to answer a particular question or to end this interview at any point.

You will be provided with a complete transcript of the interview for your approval. You can modify or add information to the transcript. Participation in this study is voluntary. Taking part in this interview is your agreement to participate.

In this focus group, I will pose questions and you may respond to them individually as well as comment about each other's responses.

1. How does GeoGebra help your students learn mathematics?
2. Did your teaching style changed after graduation from the NSF program? How?
3. Do the students behave differently when using technology as an instructional tool? How?
4. Do you change the sequence of the curriculum when implementing GeoGebra in the classroom? How?
5. What is your future plan with regard to the use of GeoGebra in the classroom?

Clarification questions and transition statements:

a. You have said that----------. Is that accurate?
b. You talked about------------. Does that mean that----------?
c. When you said-------------, do you mean-------------?
d. You mentioned--------. Tell me a bit more about what you mean by that term.
e. You talked about----------. Does anyone have an example of that?
f. Does anyone have any other stories about----------- that you would like to share?
g. We have heard ----------, what are other views about that?
h. Before we move on, would anyone else like to add to what has been said?
i. You have told me about-----------, now I'd like to learn about your views on-------.

Summarizing questions and closing questions

a. Now just to sum up what you've said. What I've heard is ---------------. Is there anything else you would like to add?
b. Are there any relevant topics that you would like to discuss that I have missed?
c. Are there any questions that I have not asked that we should talk about?

This is the end of the interview. Thank you for your cooperation.

Appendix K

Observation Tool

Date ____________________
Teacher ____________________________________ School ________________________

Classroom description (including demographics, seating arrangements, available technologies, etc.)

Time	Notes specific to GeoGebra usage	General notes

Theme	Indicator
Curriculum & Assessment	• Technology dependent or independent lesson • Formal or informal assessments • Alignment to framework
Learning	• Student use of technology • Awareness of student prior understandings and misunderstandings • Student engagement in Process Standards (NCTM, 2000)
Teaching	• Role of the teacher and instructional methods • Questions posed during lesson • Relating technology to mathematical goals
Access	• Technologies available and context of use • Student and teacher familiarity with technology • Access to representations

Appendix L

Example of A Teacher's Profile

Teacher NU-09A. Teacher NU-09A described herself as a good student in elementary and secondary school and never struggled with math: "I was one of those students who would always follow precisely what the teacher prescribed; I would do it exactly the way they told me to do it. I never really struggled with math, I was pretty good at it." She has a degree in elementary education and took the required math courses for that major, which were "not really rigorous math courses in college, I did the basic algebra." Her first teaching job was as a math teacher in middle school. She enjoyed teaching math and pushed herself to obtain the middle school certification. "I started teaching math by accident, and then I wanted to get better so I started going to workshops and I accidentally ended up in the summer institute and then that is how I ended up getting a masters degree." She has been teaching math for 16 years.

Teacher NU-09A's responses to the first survey indicated that she believes she has a working knowledge of GeoGebra and does not need any more examples, professional development, or reasons to use it in her classroom. She started using GeoGebra in her classroom even before she graduated from the program: "Probably my second year in the program was when I really started using it and getting into it. A lot of it was just for class assignments, but I would make these elaborate lesson plans and use it a lot with my algebra kids." She continued using GeoGebra with her students and enjoyed the experience as much as her students did:

> I loved it, I really did ... I used them for my algebra kids, my general kids, I used them for everybody. It was exciting because I could see the excitement in the kids faces when I would take them out, and they would be engaged and I did not have to worry about them being bored or not paying attention because they wanted to do it.

Technology, in her view, helps students understand math concepts better. As she indicated, the 6th graders are not allowed to use calculators. However, some students have so many computation difficulties that the concepts are lost trying to improve their arithmetic skills:

> If the students have problems with computations, I kind of want them to get past the barrier of computation and understand the overall concepts holistically. Sometimes they miss the concepts because they are bugged out in the computation that they did not get a solid foundation of in the elementary school.

Older students are allowed to use calculators and these tools help the students to have a better conceptual understanding:

> Whereas when they get to 7th and 8th grade, they have a little more wiggle room because if they make a mistake with the computation is not a big deal because they are allowed to use the calculator. So we can look through it, and look at concepts, try out some ideas and decide what to punch in the calculator. Now estimate this, is this reasonable? Is it close to what you should be getting?

The responses on the Beliefs survey indicated that NU-09A believes that math is not a collection of rules but requires the application of formulas as well as imagination. Students can discover things by themselves without knowing the correct procedure since problems can be solved in many ways. She believes that the use of GeoGebra was helping her students discover mathematics concepts as well as fostering their imagination: “They were picking up things they had not picked up before. I would give them a set of instructions to construct things, and a lot of them were doing them at home and they were using the constructions to help them with assignments.”

Her beliefs about math teaching can be extrapolated from the Beliefs survey. She believes that all students are capable of problem solving since they need to have a math mind; however, effort is important and it can change a student’s math ability. In the interview she explained that the use of GeoGebra facilitated her teaching:

> I think GeoGebra helped me, but I was also using it to enhance what I was doing, so it worked both ways. Regardless, I feel I am a pretty strong teacher, but it helped me bring something extra to the table. It made me a little more excited, honestly.

With regards to her beliefs about math learning, she does not believe in memorizing formulas or exact procedures. Solving problems quickly does not imply that a student is good at math since they have to understand the problem first. She feels the process is more important than the answer and it is important to understand the reasons behind the answer. Also, students can discover concepts for themselves, and should figure out their own ways, even if they utilize non-standard procedures. Time spent in problem solving is important and as well as hands-on. Students should be able to figure out their own solutions and to discuss the different ways of solving. However, she thinks not all students have the same opportunities. She explained:

> I think the advanced students have more resources at home, and a lot of them are more apt to explore at home, so when they come to the classroom, they are really on and they are ready, and they understand how to use the tools. With the remedial students it wasn’t always the case. This is just a general blank statement, and of course there are cases of students who have the resources at home but they have a difficult time using them. For the most part, the advanced students took more advantage of it outside of school, so we were able to transfer that knowledge in class.

Teacher NU-09A housed a laptop cart in her own classroom, which she enjoyed because “whenever I wanted to explore a concept, I would pop up the computers.” Years later she became the math coach in her school, but since she was not teaching students, she did not have a classroom. The laptop cart went to other teachers. Lately, she has been missing being with students, and asked her administrator to let her return to the classroom. This year she is not using GeoGebra for two reasons: the computers are old and they are damaged.

> What I noticed is that the laptop carts have not changed since then [2008]. The computers are slow; a lot of the computers have been damaged by the students. Unfortunately some teachers have better classroom management when it comes to housing a laptop cart, and making sure that all the laptops are kept in order and students don’t put spyware on them or viruses, or anything like that. Now, the laptops are not in good condition and the software stalls. When you have 20 students and you have 20 computers working and you can whip them out and the students know that if they are seated at desk number 5 they have to get computer number 5. They take them out and they are all set up with folders and

> everything, it is easier to get it going. But when you have 20 students in the classroom, you might have 25 computers but only 12 of them work properly.

Her concerns about the use of GeoGebra as an instructional tool are about classroom management:

> It becomes a management issue because you hear the kids: Ms XXX, mine does not work, mine is not showing that. Honestly I just got kind of discouraged because what I would want to have in a perfect world is my own set of laptops that only I use, and that is not the case now. We are using the laptops for standardize testing now, we have computerized testing now for 6th, 7th, and 8th grade, so we literally have carts of computers that are pretty much put in a storage room until FCAT because they don't want the computers to get messed up before FCAT.

She also expressed this concern in her responses to survey 1, indicating that more computers are needed, as well as more technical support in the school.

Analyzing her testimony, it seems her TPACK is in the advancing stage of development. With respect to curriculum, she understands that sustained innovation in modifying one's curriculum to incorporate technology efficiently and effectively as a teaching and learning tool is essential. When using GeoGebra, her students "were engaged, they were excited, and they were picking up things that had not picked up before."

In the area of learning, she plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understands mathematics has to be enhanced through integration of the various technologies: "If they make a mistake with the computation is not a big deal because they are allowed to use the calculator. So we can look through it, and look at concepts, try out some ideas and decide what to punch in the calculator."

In teaching, she seems to be in the advancing stage as well; she actively and consistently accepts technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students. "When we were looking at different types of equations with my algebra kids, they were really getting into it. They were really engaged."

UN-09A permitted students to use technology in every aspect of mathematics class: "I used them for my algebra kids, my general kids, I used them for everybody." Her advanced students "took more advantage of it outside of school, so we were able to transfer that knowledge in class."

NU-09A enjoyed her time in the SMGEM program and considers herself a better teacher because of it: "There was a lot of work involved, but it also helped me to conceptualize things that I had not conceptualized before and helped me really see the proof for the math." The interview made her realize how much she learned while using GeoGebra as well as how much her students were learning from its use. She feels guilty she is not using GeoGebra with her students anymore, but her intention is to start using it again "because I am feeling guilty; one of the things I want to start doing in second semester is take my students to the lab at least twice a month so they get the opportunity to use it."

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