# NONLINEAR DYNAMICS OF AUTONOMOUS UNDERWATER VEHICLES IN OCEAN CURRENTS

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## INTRODUCTION

When operating in strong ocean currents, autonomous underwater vehicles (AUVs) may experience navigation drift and "crabbing" due to hydrodynamic loads exerted by the currents, which can risk the success of the AUVs' mission [1]. Therefore, AUVs should be capable of path keeping and heading adjustment in environments featuring strong ocean currents.

Conventional AUVs use a stern mounted thruster and active control surfaces to steer their motions. However, control surfaces work depending on relative flows. Therefore, they may not be as effective as expected in strong ocean currents.

In the present research, a thorough analysis of nonlinear dynamics of a REMUS AUV in ocean currents is conducted. The stability of the AUV's horizontal motion is studied in order to propose a method to allow AUVs the capability of path keeping in strong ocean currents without using active control surfaces.

## HYDRODYNAMIC LOADS

The hydrodynamic loads acting on a vehicle in an unbounded fluid can be derived by both Newtonian and Lagrangian methods,

$$\begin{bmatrix} \boldsymbol{F}_0 \\ \boldsymbol{M}_0 \end{bmatrix} = \bar{\boldsymbol{M}}\dot{\boldsymbol{\nu}}_c + \bar{\boldsymbol{C}}(\boldsymbol{\nu})\boldsymbol{\nu}_c - \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r - \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \begin{bmatrix} -\boldsymbol{\Phi} & \boldsymbol{O}_{3\times3} \\ \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \end{bmatrix} (\boldsymbol{M}_A + \bar{\boldsymbol{M}})\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r \tag{1}$$

in which $\boldsymbol{\nu}_c$ is the velocity vector of the current flow, $\boldsymbol{\nu}_r$ is the relative velocity vector of the vehicle with respect to the current, $\bar{\boldsymbol{M}}$ is the mass matrix of the fluid displaced by the vehicle, $\bar{\boldsymbol{C}}$ is the Coriolis-centripetal matrix of the displaced fluid, $\boldsymbol{M}_A$ is the added mass matrix, $\boldsymbol{C}_A$ is the Coriolis-centripetal matrix of the added mass, $\boldsymbol{D}$ is the damping matrix and $\boldsymbol{\Phi}$ is the symmetric velocity gradient of the undisturbed incident flow expressed in a body-fixed reference frame as [2]

$$\Phi_{ij} = \frac{\partial^2 \phi_o}{\partial x_i \partial x_j}$$

## REMUS AUV

A Remus 100 AUV is studied in the present research due to its extensive use in ocean engineering.

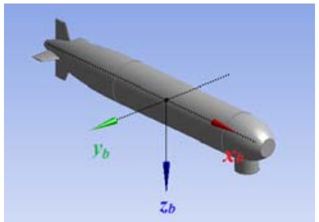

**Figure 1:** Remus 100 AUV

**Table 1:** Main parameter of the AUV

| Name | Value | Units | Description |
|---|---|---|---|
| $l$ | $+1.33\text{e}{+}000$ | m | Total Length |
| $d$ | $+1.91\text{e}{-}001$ | m | Max. Diameter |
| $\nabla$ | $+3.15\text{e}{-}002$ | $\text{m}^3$ | Hull Volume |

The hydrodynamic coefficients are calculated by using empirical formulas and CFD methods.

## MODELING OF THE VEHICLE-FLUID DYNAMICAL SYSTEM

The nonlinear dynamical equations governing the horizontal motion of the AUV in steady and uniform flow can be written as follows:

$$\begin{cases} \dot{u} = Avr + Dr^2 + PU_x r\sin\psi + \alpha(u - U_x\cos\psi)|u - U_x\cos\psi| + T' \\ \dot{v} = G(u - U_x\cos\psi)(v + U_x\sin\psi) + Hur + K(v + U_x\sin\psi)|v + U_x\sin\psi| + Mr|r| + RU_x r\cos\psi \\ \dot{r} = I(u - U_x\cos\psi)(v + U_x\sin\psi) + Jur + L(v + U_x\sin\psi)|v + U_x\sin\psi| + Nr|r| + SU_x r\cos\psi \\ \dot{\psi} = r \end{cases} \tag{2}$$

in which $u$, $v$ and $r$ denote linear and angular velocities of the vehicle, and the constants depend on its parameters and hydrodynamic coefficients. $U_x$ is the velocity of the ocean current.

## RESULTS

The nonlinear dynamical system given by Eq. 2 has a unique equilibrium point as:

$$\boldsymbol{x}_0 = [U_x\cos\psi_0 + U_0, -U_x\sin\psi_0, 0, \psi_0]^T \tag{3}$$

The stability of the equilibrium point can be analyzed first by using linearization method. Then the stability criterion of the dynamical system in its neighborhood can be determined.

If the stability criterion is not satisfied, the projection of the phase portrait of the dynamical system into $Ouvr$ space is given by

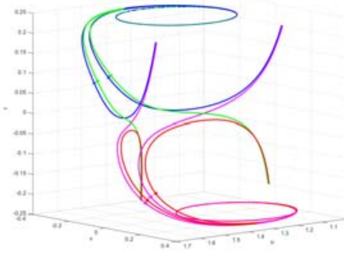

**Figure 2:** The phase portrait in the unstable case

In this case, two stable limit circles exist.

If the stability criterion is satisfied, the projection of the phase portrait is illustrated as follows

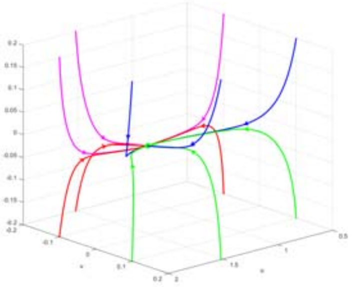

**Figure 3:** The phase portrait in the stable case

In this case, two stable limit circles converge to be the stable equilibrium point.

## CONCLUSION

- Given the data of a Remus 100 AUV, it can be found that its uncontrolled dynamical system is unstable.
- If the dynamical system is not stable, the vehicle will move in a circular path and drift away with ocean currents.
- If the uncontrolled dynamics of AUVs can achieve stability around the equilibrium point, the path of the vehicle is a straight line although the vehicle will experience "crabbing" in the presence of ocean currents.

## FUTURE RESEARCH

The degree of "crabbing" depends on the steady-state heading angle of the underwater vehicle. Although the "crabbing" cannot be eliminated entirely, the navigation drift can be minimized as small as possible by carefully adjusting the heading angle and forward speed of the autonomous underwater vehicle in ocean current.

Internal actuators are promising complement to control surfaces considering operations in strong ocean current, since they operate independently of the relative flow and are less prone to the corrosion and damage than external actuators.

## REFERENCES

[1] Russell B. Wynn, Veerle AI Huvenne, Timothy P. Le Bas, Bramley J. Murton, Douglas P. Connelly, Brian J. Bett, Henry A. Ruhl, Kirsty J. Morris, Jeffrey Peakall, Daniel R. Parsons, et al. Autonomous underwater vehicles (auvs): Their past, present and future contributions to the advancement of marine geoscience. *Marine Geology*, 352:451–468, 2014.

[2] A. Galper and T. Miloh. Generalized kirchhoff equations for a deformable body moving in a weakly non-uniform flow field. *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, 446(1926):169–193, 1994.