A COMPUTATIONAL STUDY ON DIFFERENT PENALTY APPROACHES FOR CONSTRAINED OPTIMIZATION IN RADIATION THERAPY TREATMENT PLANNING WITH A SIMULATED ANNEALING ALGORITHM

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1. Introduction

- In intensity modulated radiation therapy (IMRT) a treatment plan is a high dimensionality optimization problem with the goal to give the prescribed radiation dose to the Planning Target Volume (PTV) while sparing critical organs. The radiation dose is given to the cancer patients using linear accelerator (LINAC) which delivers high-energy x-rays to the tumor regions of patients.

2. Method

- Each treatment plan has several beam angles
- Each beam is divided into a number of smaller rectangular beams which are called pencil beams (PBs).
- Patient CT scan is subdivided into cubic voxels
- Next figure shows 3D rendering of PTV (yellow), bladder (red) and rectum (blue) when one beam is used for a prostate patient. The fluence map (map of PB intensities) is plotted in gray coloring scale.
- Inverse planning optimization algorithm along with dose calculation engine must be utilized to optimize treatment objective function.

3. Objective function

- *Objective function for the PTV*

$$OF_{PTV} = \frac{1}{N_{PTV}} \sum_{i=1}^{N_{PTV}} (D_i - D_{prescribed})^2$$

D_i : dose to voxel i of the PTV
$D_{prescribed}$:prescription dose
N_{PTV} :total number of voxels of the PTV.

- *dose-volume constraints (DVC)*

$$V_{D_{max}} \leq V_{max}$$

"No more than V_{max}% of the volume should receive more than D_{max}"

- *Final Objective function for optimization*

$$f = OF_{PTV}(\boldsymbol{w}) + \sum_{i=1}^{OAR\#} \alpha_i^t \, H_i[cOAR_i(\boldsymbol{w})] cOAR_i^2(\boldsymbol{w})$$

$\boldsymbol{w}$: pencil beam weights α_i^t : penalty coefficient
$cOAR_i$: DVC for the OAR "i". H : Heaviside function

4. Optimization method

Simulated annealing (SA) algorithm

- Global optimizer
- mimics the physical annealing process in metallurgy
- SA starts from a high initial temperature (T_0) which is reduced gradually to a low temperature.

$$T_{new} = \alpha . T_{old} \, , \; 0 < \alpha < 1$$

- In each temperature pencil beam weights ($\boldsymbol{w}$) undergo several random walk transitions

$\boldsymbol{w_{new}} = \boldsymbol{w_{old}} + stepsize * r$, r is a random number.

- Acceptance criteria

if ($\Delta f < 0$) change is accepted with probability 1
if ($\Delta f \geq 0$) change maybe accepted according to the Boltzmann probability $P(\Delta f, T) = e^{\frac{-\Delta f}{kT}} > R$
R is a random number from a uniform distribution U~[0,1].

5. Purpose

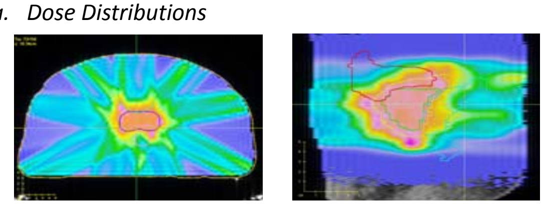
How each penalty method affects the SA algorithm?

1. Static penalty $\alpha_i^t = \sigma$
2. Dynamic penalty $\alpha_i^{t+1} = \alpha_i^t + C . |h_i^t(x)|$
3. Adaptive penalty

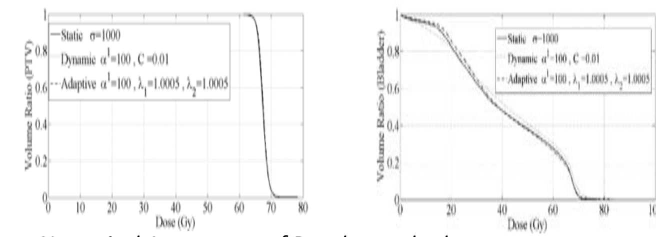
$$\alpha_i^{t+1} = \begin{cases} \frac{1}{\lambda_1} \alpha_i^t & case\ A \\ \lambda_2 \alpha_i^t & case\ B \\ \alpha_i^t & case\ C \end{cases} \qquad \lambda_1, \lambda_2 > 1$$

6. Results for 5 prostate cases with 2 OARs

a. Dose Distributions



b. Dose Volume Histograms



c. Numerical Assessment of Penalty methods

Method/PTV	D95	D5	Dmin	Dmax
Static	66±0.2	73.8±0.3	54.6±2.7	79.3±2.4
Dynamic	65.9±0.2	73.8±0.3	54.4±2.6	78.5±1.6
Adaptive	66.0±0.2	73.8±0.3	54.8±2.8	78.4±1.6

Method/Bladder	V40	V50	V60	Dmax
Static	52.5±2.3	38.6±3.1	27.5±3.8	80.3±5.8
Dynamic	52.2±2.6	38.3±3.2	27.1±3.4	79.8±5.9
Adaptive	51.7±2.7	38.0±3.1	27.1±3.3	81.3±6.3

Method/Rectum	V40	V50	V60	Dmax
Static	57.2±2.8	37.1±2.3	23.1±4.6	77.8±6.9
Dynamic	57.5±2.9	36.6±2.6	21.9±5.0	76.3±5.8
Adaptive	57.1±2.5	36.8±2.0	22.6±4.2	79.2±6.8

d. Sensitivity analysis