

ENTROPIC CONSIDERATIONS OF EFFICIENCY IN THE WEST TEXAS  
INTERMEDIATE CRUDE OIL FUTURES MARKET

By

Ryan Sagul

A Thesis Submitted to the Faculty of

The College of Business

In Partial Fulfillment of the Requirements for the

Degree of Masters of Science

Florida Atlantic University

Boca Raton, FL

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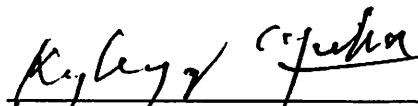
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This dissertation was prepared under the direction of the candidate's thesis advisor, Dr. Ky-Hyang Yuhn, Department of Economics, and has been approved by the members of his/her supervisory committee. It was submitted to the faculty of the College of Business and was accepted in partial fulfillment of the requirements for the degree of Master of Science.

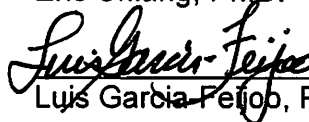
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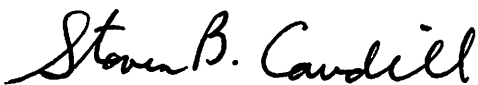
Ky-Hyang Yuhn, Ph.D.  
Thesis Advisor



Eric Chiang, Ph.D.



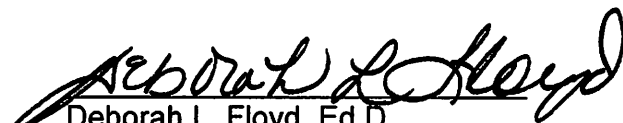
Luis Garcia-Felipe, Ph.D.



Steven Caudill, Ph.D.  
Chair, Department of Economics



Daniel Gropper, Ph.D.  
College Dean, College of Business



Deborah L. Floyd, Ed.D.  
Dean, Graduate College

August 9<sup>th</sup>, 2016  
Date

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## ABSTRACT

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For the last fifty years, the efficient market hypothesis has been the central pillar of economic thought and touted by all, despite Sanford Grossman' and Nobel prize winner Joseph Stiglitz' objection in 1980. Andrew Lo updated the efficient market hypothesis in 2004 to reconcile irrational human behavior and cold, calculating automatons. This thesis utilizes 33 years of oil futures, GARCH regressions, and the Jensen-Shannon informational criteria to provide extensive empirical objections to informational efficiency. The results demonstrate continuously inefficient oil future markets which exhibit decreased informational efficiency during recessionary periods, advocating the adaptive market hypothesis over the efficient market hypothesis.

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## INTRODUCTION

Eugene Fama's (1970) defined his so-called efficient markets hypothesis (EMH). The efficient market relies on market participants being rational actors who use all information available to make buying and selling decisions. This behavior introduces "informational efficiency" to the market place, resulting in prices which "fully reflect" all available information. Because all the information has been used, new methods of analysis cannot earn excess returns. This idea comes from John Muth (1961) Theory of Rational Expectation under which all individuals are believed to meticulously hunt down every, and any, scrap of information useful to making predictions about the future.

These assumptions lead to three forms of informational efficiency which vary in nature: weak form efficiency, in which market participants cannot make abnormal returns using only historical pricing data; semi-strong form efficiency, in which market participants cannot make abnormal returns using all available public information; and strong form efficiency, in which market participants cannot make abnormal profits using any combination of private and public information.

Before delving into the wealth of literature investigating the efficiencies of various markets and the impacts, it is interesting to gain society's viewpoint on the implications of the efficient market hypothesis. The Securities and Exchange Commission (SEC) is the United States (US) regulatory body with a mission to prevent predatory and manipulative behavior on US stock exchanges and allow

traders to participate in a secure and equitable environment. As such, the SEC enforces rules on insider trading, the practice of trading securities with private information, to promote the ideas of fair markets as spoken by Robertson (1998). Two of the most (in)famous insider trading cases involve R. Foster Winans, a reporter for the Wall Street Journal who leaked his weekly “Heard on the Street” stock advice column, and Ivan Boesky and Michael Milken, the latter a junk bond trader who provided hundreds of millions of dollars in financing to the former who was a mergers and acquisitions investor.

What, then, is learned from these two cases? In the first case, Winans did not provide any new information to the market as all of his analysis was based on public information, yet he and his partner reaped over \$600,000 in rewards over two years. The new information entering the market on his column’s release could be his opinion on stocks, but the market behavior speaks more to the irrationality of a fair share of market participants. Instead of forming their own judgements about stock prices, they rely on an aggregated source of knowledge to make decisions for them. Alternatively, the information may not have been used in market pricing as market participants offload the burden of calculations to Winans, reducing their implicit participation costs.

The second case is about fairness. The inside information held by Boesky and Milken allowed the accumulation of hundreds of millions of dollars in payouts and, according to the SEC, violates trust in the markets and drives investors and their capital from markets<sup>3</sup>. The act of using private information in trades then reduces efficiency by reducing the number of rational actors willing to participate

in the marketplace, even if the introduction of private information to pricing is technically more efficient in the short run.

From these qualitative observations, it is possible to form a reasonable layperson's view on market efficiency. Markets which resemble strong-form efficiencies are more likely to contain malevolent actors and experience a decay in efficiency over time as other rational participants choose to leave. Varying levels of market competition, obscurity, and barriers to entry, including implicit costs of investment decisions, should place most American capital markets somewhere in the weak to semi-strong range, depending on if the layperson believes in the capabilities of money-managing professionals.

Despite these rationally formed beliefs, the idea of perfection in the market places is absurd. Many barriers to entry, friction in the markets, and irrational investor group behavior pollute any theoretical notion of fully-reflective prices. The sheer impossibility of these absolute statements cannot be proven. While the assumptions made by the efficient market hypothesis simplify the real world and allow theories to become digestible and calculable, the assumptions often are violations of the realities of the world. Roman Emperor Marcus Aurelius warned against such sweeping assumptions: "treat with utmost respect your power of forming opinions, for this power alone guards you against making assumptions that are contrary to nature and judgements that overthrow the rule of reason."

The efficient market hypothesis relies on the assumption of innumerable rational actors capable of scrounging hoards information and performing

increasingly intricate calculations to provide unbiased estimations of the future with no friction. It is not reasonable to make these assumptions of normal humans as they make mistakes, take time to process information, and necessarily must be compensated for their time in order to purchase food and housing. Instead of treating the efficient market hypothesis as reality, it should be instead treated as an idealized world and real world observed market efficiencies should be compared, and expected to converge, to the theoretical maxima represented by the efficient market hypothesis. Through the use of generalized autoregressive conditional heteroscedasticity (GARCH) regressions and the adoption powerful statistical techniques from the field of information theory and statistical thermodynamics, this thesis demonstrates that the oil futures market never reaches the minimum threshold of weak market efficiency throughout its thirty three year history.

## LITERATURE REVIEW

Paul Samuelson (1965) first introduced his Random Walk theory which was the spiritual precursor to Fama's famed EMH. In his paper, Fama (1970) demonstrates that all asset returns are not associated with each other, a necessary condition for all forms of the efficient market hypothesis to hold. Even more firmly, Samuelson posits, no profit can be obtained by using any mathematical calculations or supernatural divination on prior price changes to reveal the next period's price change.

If there is no way for these speculators to make money, then why do such a large variety of firms, both those in the industry of the underlying good and those who are purely financial, participate in these futures and forwards markets, especially when counterparty default risk enters into future certainty decisions? If the markets exhibit semi-strong or strong-form efficiency, then these purely financial firms who participate are nothing more than glorified gambling operations. A better answer to this question comes from Lucas (1978) which justifies market rewards for those who undertake riskier positions. So, then, profits in these market places could be earned not from superior financial analysis, but from the insurance premiums producers or consumers of the underlying good are willing to pay to offload intertemporal risk to parties more willing to bear the uncertainties. This question also sparked Sanford Grossman's

and Joseph Stiglitz' 1980 vehement disagreement with the possibility of informational efficiency where they specifically state "prices cannot perfectly reflect the information which is available" because obtaining a better estimate of the true asset value must be rewarded.

The idea of a forward risk premium is explored by Katherine Dusak (1973) which fails to find systematic risk in wheat, corn, and soybean markets from 1952 to 1967. Later on, Froot and Frankel (1989) also fail to find any risk premiums in future exchange rate markets and, instead, attribute it to investor expected depreciation miscalculations. Robert Watts (1978) found that quarterly earnings reports cause excess returns in the stock market. The result of these, and many more papers, culminated when Michael Jensen (1978) summed the growing amount of evidence against the efficient market hypothesis, noting that the current econometric methods are not appropriate for effectively testing the efficient market hypothesis and suggests that new techniques will be available in the next five years to help prove or disprove this hypothesis.

A lot of this sophistication came in the form of the autoregressive conditional heteroscedasticity (ARCH) / generalized ARCH (GARCH) class of models, capable of detecting autoregressive conditional heteroscedasticity in time series data. These models began with Robert Engle's (1982) distinguished paper which demonstrated the effectiveness of a zero mean, "serially uncorrelated processes with nonconstant variances conditional on the past, but constant unconditional variance". In simpler words, next-period variance is linked to the current period, but the link is constant which maintains the independent

and individually distributed nature of the overall process. This ARCH model was extended with Andrew Weiss (1984) which expands the ARCH model to include an autoregressive moving average, allowing the average value of a process to fluctuate around a time-invariant mean and improved the accuracy of stock returns and measures of efficiency. This ARMA methodology was incorporated into, and ARCH improved on, Tim Bollerslev (1986) introducing GARCH methodology. This model allowed for a “much more flexible lag structure” and became the basis for an explosion of the many GARCH type models employed today.

Richard Thaler and Werner De Bondt (1985) published “Does the Stock Market Overreact?” which discovered that portfolios of “loser” stocks grossly outperformed “winners”. Thaler and De Bondt (1986) continues the research by demonstrating investor overreactions causing excess returns occurring with purchasing stocks in January. These observations violated Bayes’ rule because investors overreacted to unanticipated news and, again, brings the efficient market hypothesis into question. Lawrence Summers (1986) delves into the idea of speculation as a way to increase efficiency by introducing more rational participants, but does not find that it brings about financial efficiency. However, he also “argues that existing evidence does not establish that financial markets are efficient in the sense of rationally reflecting fundamentals. [His paper] demonstrates that the types of statistical tests which have been used to date have essentially no power against at least one interesting alternative hypothesis to market efficiency.”

David Hirshleifer (1989) advances analytical techniques by showing that output and demand shocks result in increasing risk premium using models of a common mean-variance objective function, trading opportunities, aggregate demand, and supply equilibrium for spot commodities. Despite this positive result, irregularities continue to appear in the EMH with David Laibson (1997) finding that consumers show hyperbolic discounting which is, essentially, time frame inconsistent decision making. This behavior infringes upon the rational actor hypothesis.

With the inconsistencies piling up, in Burton Malkiel (2005) was compelled to defend the efficient market hypothesis with his “Reflections on the Efficient Market Hypothesis: 30 Years Later”. His evidence, that markets behaved efficiently, was the lack of investors who made excess returns, citing the S&P 500’s returns dominated the average of all actively managed mutual funds by 200 basis points. Furthermore, it is impossible to determine which mutual fund managers would be “winners” in the next term. Warren Buffett (1996), the titan of investment managers, agrees with this sentiment, advocating the choice of index funds because they outperform most investment professionals. Shortly afterwards, the infamous 2008 recession occurred and Malkiel (2011) doubled down on the efficient market hypothesis, unequivocally stating that “prices are always wrong [and] no one knows for sure if they are too high or too low”. Nonetheless, irrational investors will always be preyed upon by rational ones, bringing the prices to an equilibrium.



Samuelson points out in his thesis that his methods “[are] essentially the method of thermodynamics, which can be regarded as a purely deductive science based upon certain postulates (notably the First and Second Laws of Thermodynamics)”. The clock is turned back to 1948 to the foundations of information theory where the modern processes used to measure informational efficiency were conceived. Claude Shannon (1948) published his famous “A Mathematical Theory of Communication” in which he investigated the fundamental limits of signal processing and data compression, but is now the foundation of information theory. In this paper, Shannon analyzes signals in wires, calculating the information stored in various bits of “noise” and quantifies them with his information criteria. Shannon’s information criteria is a measure of saturation, reaching a maximum when no further information can be contained in a given channel. His research was quickly incorporated into statistical mechanics, ecology, cryptography, linguistics, computer science, and neurobiology where it remains in wide use today.

Shannon’s information criteria can be used to test the weak form of the efficient market hypothesis as prices, theoretically, should be saturated with information. Lopez-Ruiz et al (1995) expanded Shannon’s work to account for disequilibrium within studied systems of bonds. They followed up with several papers utilizing this methodology, with Zunino (2012) proving greater information loss in corporate bond markets than sovereign bond market during the 2007 recession, linking economic growth and market size to the entropy measures,

and Bariviera et al (2013) presenting country-specific information loss in bond prices during the 2008 recession.

To expand on the literature presented, this thesis uses a comprehensive WTI oil future data set which restricts market participants to, likely, a homogenous set of investors yielding many snapshots of efficiency's evolution. Monthly maturation dates allow long time series for greater predictive power of these econometric techniques without sacrificing any time between observational periods. The new entropy technique introduced to the bond markets by Lopez-Ruiz et al is applied to a larger data set, allowing more generalized economic implications to be discerned with confirmation from proven GARCH methodology.

## DATA AND METHODOLOGY

Daily price history for US WTI Crude Oil and each crude oil future contract from June 1983, CLN83 contract, until February 2016, CLM6 contract, was obtained through a Bloomberg terminal. Every day on which these contracts were not traded was removed from the data set, resulting in a final data set of 282,427 unique daily observations. Each individual future contract has between 57 and 2244 daily observations.

Because this time series data is expected to be heteroscedastic and nonstationary, the daily price was converted to log daily returns:

$$r_t = \log(p_t) - \log(p_{t-1}) \quad (1)$$

for  $t > 0$

This manipulation necessarily removed the first observation from each future data set.

To define the GARCH( $q,p$ ) process, first  $Z_n$  is defined as a series of iid random normal variables with a mean of zero and an standard deviation of one. If the process  $\epsilon_t$  can be described by:

$$\epsilon_t = \sigma_t Z_t \quad (2)$$

$$r_t = \mu + \epsilon_t \quad (3)$$

for any positive integer  $t$ , and  $\sigma_t$  is a nonnegative such that:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \quad (4)$$

for  $\alpha_0 > 0, \alpha_i \geq 0$  for  $i = 1..q, \beta_i \geq 0$  for  $i = 1..p$

The remaining proof of the GARCH process is omitted, but it is assumed that the data follows a GARCH(1,1) model, yielding:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (5)$$

The values of  $\omega, \beta, \alpha$ , and  $\mu$  for the GARCH(1, 1) model, implemented with the rugarch R package, are estimated for each future contract using the maximum likelihood estimator. The presence (lack) of  $\beta$  is evidence against (for) the efficient market hypothesis, as it (contra)indicates the influence of the previous period's price on the current period. In order for the process to be stationary, the summation of  $\alpha$  and  $\beta$  must be less than one.

Utilizing the more recent information theory approach, the calculation of the entropy offset begins by manipulating the initial price vector by adding a small, normally distributed random error term to each value in the price vector. This thesis uses an error with mean zero and standard deviation of 0.0001, such that even a  $10\sigma$  deviation will not affect the ordering of real prices. The purpose of adding this random error term is to allow entropy calculations on a two-state system wherein prices can be greater than or less than other prices, instead of a

three-state system, wherein prices are less than, equal to, or greater than other prices. This significantly simplifies entropy calculations. In order to account for the potential change in entropy due to the addition of these random numbers over potential periods of non-movement of prices, the average of an arbitrarily large number of trials, in this paper 250, are chosen in order to meet the law of large numbers to ensure sufficient accuracy on these entropy calculations.

It is important to note that a major drawback of choosing a two-state model instead of a three-state model is an absolute overstatement of entropy, as the randomness terms included will arbitrarily add entropy where it would not otherwise would be found. Large periods of static prices should yield a low entropy state, but determining a true and fair estimate of the expected probability of prices moving upwards, downwards, or remaining the same is well beyond the scope of existing economic or financial literature. This absolute overstatement of entropy, however, makes statistical significance tests redundant in the case of two or more same prices observed sequentially, as the odds of any continuous distribution causing no observed return is exactly zero.

After adding these minor perturbations, the data is segmented into chunks of length  $D$ . The literature suggests that good values of  $D$  range between three and seven for time series. If the data is truly independent and individually distributed with equal probability of an observation above or below the current point, no matter if the distribution is normal, fat tailed, or otherwise, it is expected that for each set of  $D$  observations, there is an equal chance that sorting those

observations by size will result in each of  $D!$  possible permutations. The expected probability for each sorted observation is  $P = \frac{1}{D!}$ .

Imagine the following vector of price observations:

PRICE = (\$4.50, \$4.20, \$4.35, \$4.36, \$4.70)

For  $D = 3$ , the set of all possible observation orderings are (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), and (3,2,1). When  $D = 3$ , the first three observations from PRICE, \$4.50, \$4.20, and \$4.35, are chosen and sorted giving \$4.20, \$4.35, and \$4.50. This is an observation of the permutation (2,3,1). The next set of three is chosen, \$4.20, \$4.35, and \$4.36 and chosen, leading to an ordering observation (1,2,3). This cycle is repeated down the length of the vector of observed prices.

The results of sorting are stored in vector which is converted to a probability vector by dividing each element by the number of observations in the initial price vector minus  $D$  plus one. This probability vector is a store of information of which C.E. Shannon's Logarithmic Measure is a numeric measure:

$$S(OBS_D) = -K \sum_{i=1}^m P_i \ln(P_i) \quad (6)$$

For simplicity, this paper chooses  $K = 1$  as subsequent normalization will remove the effects of this constant. If the probability of an event is 0, the natural logarithm is undefined. This event may occur under nonrandom conditions and, as such, an assumption is made that L'Hôpital's approximation of  $\lim_{x \rightarrow 0^+} x \ln(x) = 0$  is a suitable evaluation of  $0 \ln(0)$ . It is also suitable, then, for an entropy of 0 to

exist for a perfectly in order system where the probability of an event occurring is 100%.

It is an interesting note that when  $K = K_B$ , the Boltzmann constant, under an equiprobability distribution then this Shannon Logarithmic Information is the same as the formula for the entropy definition which is the foundation of statistical thermodynamics.

In order to compare entropies from different systems, the normalized Shannon entropy is calculated:

$$H_S = \frac{S(\text{Price})}{S_{\max}} \quad (7)$$

The state of maximum entropy is achieved with the equal probability distribution, true randomness, and it is evident that  $S_{\max} = \ln(m)$ . This normalization also causes the cancellation of the constant  $K$  in Shannon's Logarithmic Measure, validating the simple choice of  $K = 1$ .

This  $H_S$  is then fed into the Jensen-Shannon Divergence equation, a measure of the disequilibrium present in the system:

$$J(P_{\text{obs}}, P_{\text{eq}}) = S\left(\frac{P_{\text{obs}} + P_{\text{eq}}}{2}\right) - \frac{S(P_{\text{obs}})}{2} - \frac{S(P_{\text{eq}})}{2} \quad (8)$$

$$Q_J(P_{\text{obs}}, P_{\text{eq}}) = Q_0 J(P_{\text{obs}}, P_{\text{eq}}) \quad (9)$$

$$Q_0 = \text{Max}[J(P_{\text{obs}}, P_{\text{eq}})] \quad (10)$$

This Jensen-Shannon Divergence is a measure of the disequilibrium in the system, having minimums of 0 at  $H = 0$  and  $H = 1$ .

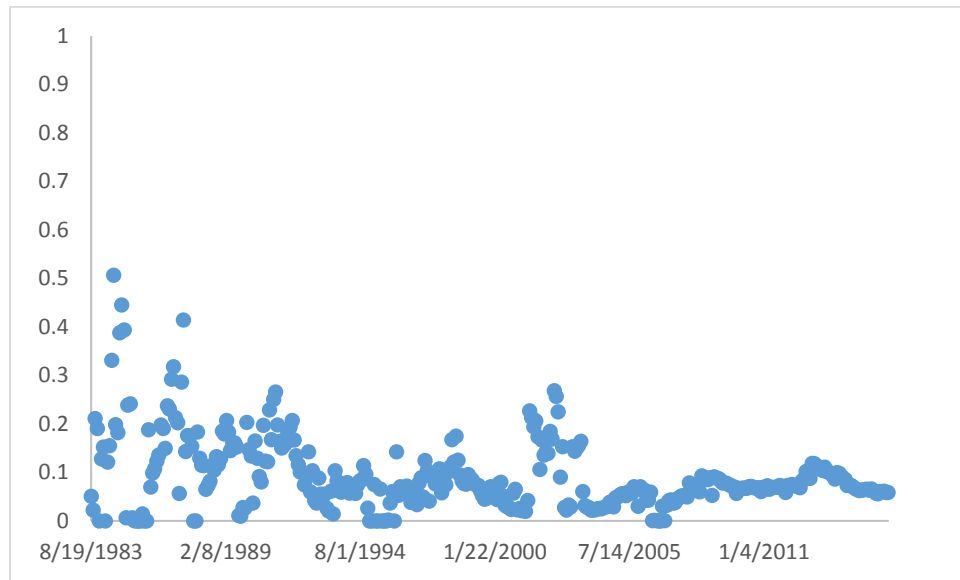
The complexity of the information stored is given by:

$$C_{JS} = H_s Q_J \quad (11)$$

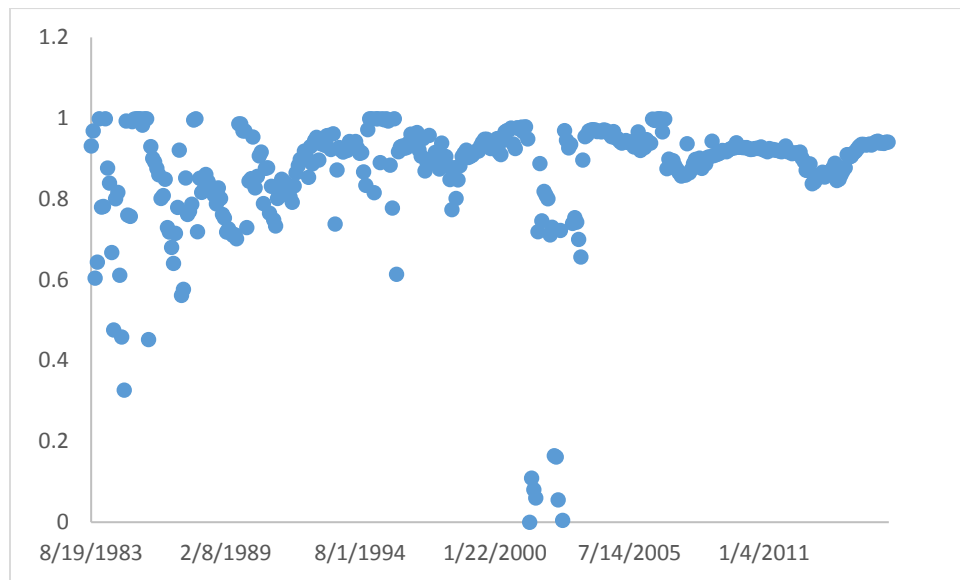
By plotting  $H_s$  versus  $C_{JS}$ , the complexity of the information stored in the price vector, roughly the exploitability of economic opportunities, can be discerned.



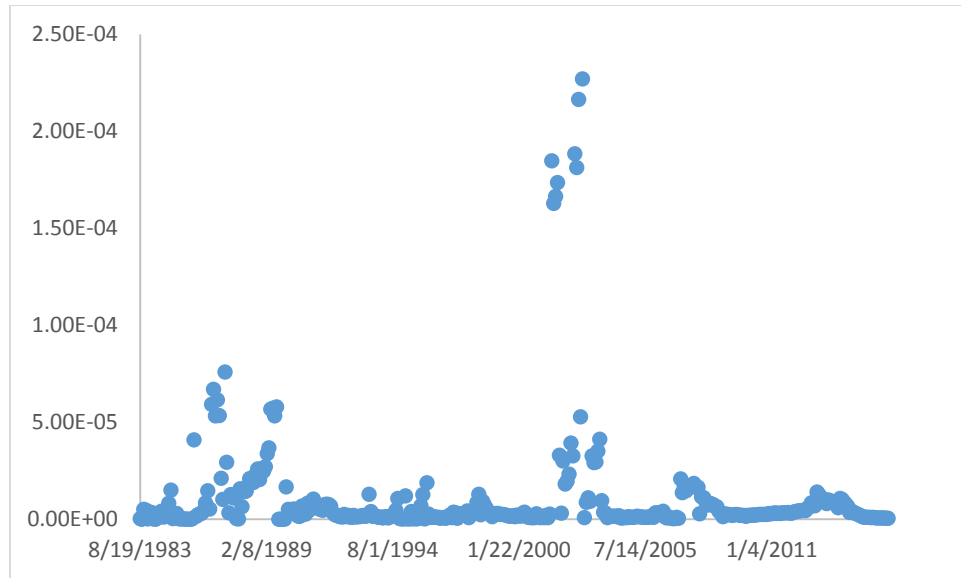
## RESULTS AND DISCUSSION



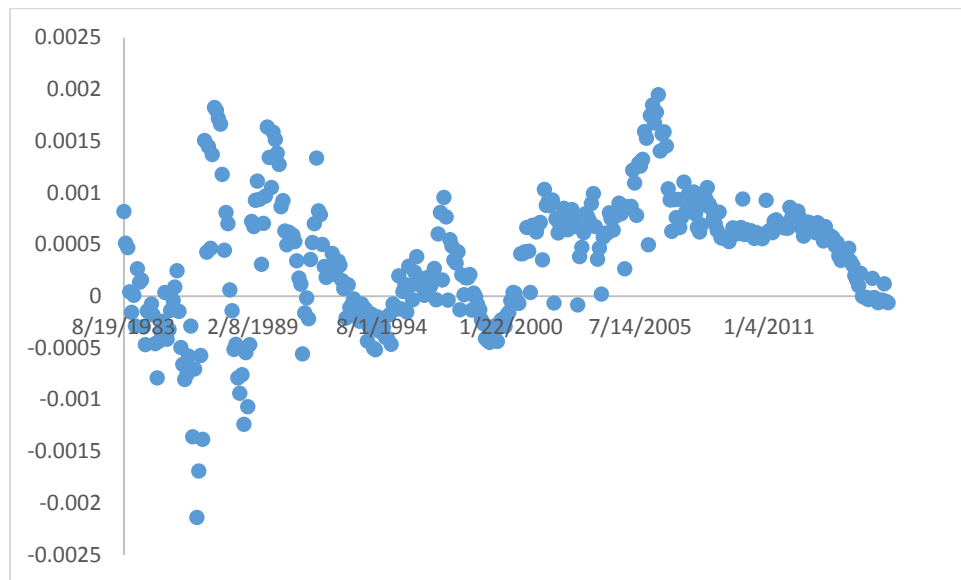
**Figure 1.** A plot of GARCH  $\alpha$ 's plotted at the maturity date of its respective contract.



**Figure 2.** A plot of GARCH  $\beta$ 's plotted at the maturity date of its respective contract.



**Figure 3.** A plot of GARCH  $\omega$ 's plotted at the maturity date of its respective contract.



**Figure 4.** A plot of GARCH  $\mu$ 's significant at the 95% confidence level plotted at the maturity date of its respective contract.

$\alpha$ Value	Amount
> 0.1	121
> 0.25	13
> 0.5	0

$\beta$ Value	Amount
> 0.80	329
> 0.90	225
> 0.95	71

**Table 1** (left). GARCH  $\alpha$  coefficients which exceed different thresholds. **Table 2** (right). GARCH  $\beta$  coefficients which exceed different thresholds.

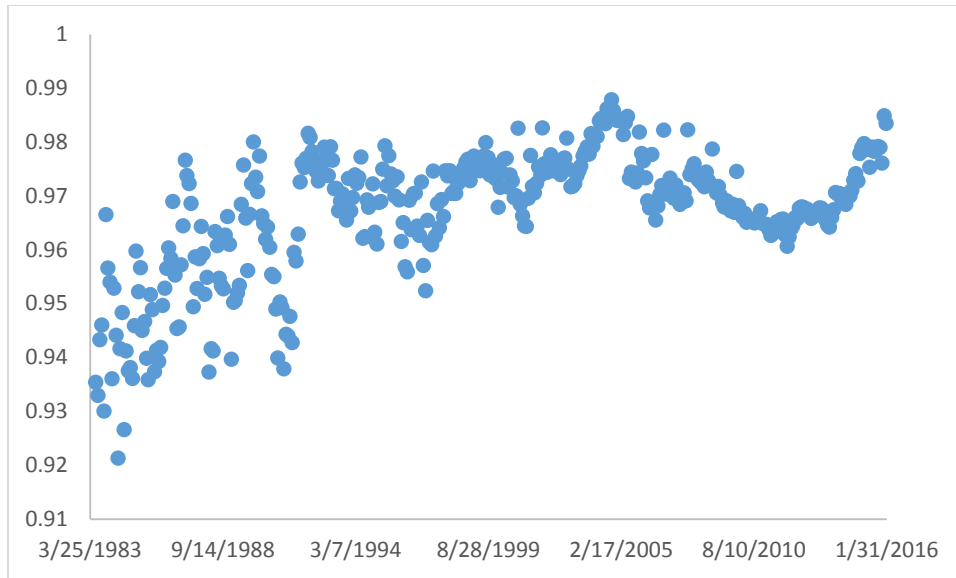
Significance Level $\alpha$	Amount	Significance Level $\beta$	Amount
90%	145	90%	336
95%	127	95%	328
99%	88	99%	312

**Table 3** (left). The number of GARCH  $\alpha$  coefficients which are statistically significant at each confidence level. **Table 4** (right). The number of GARCH  $\beta$  coefficients which are statistically significant at each confidence level.

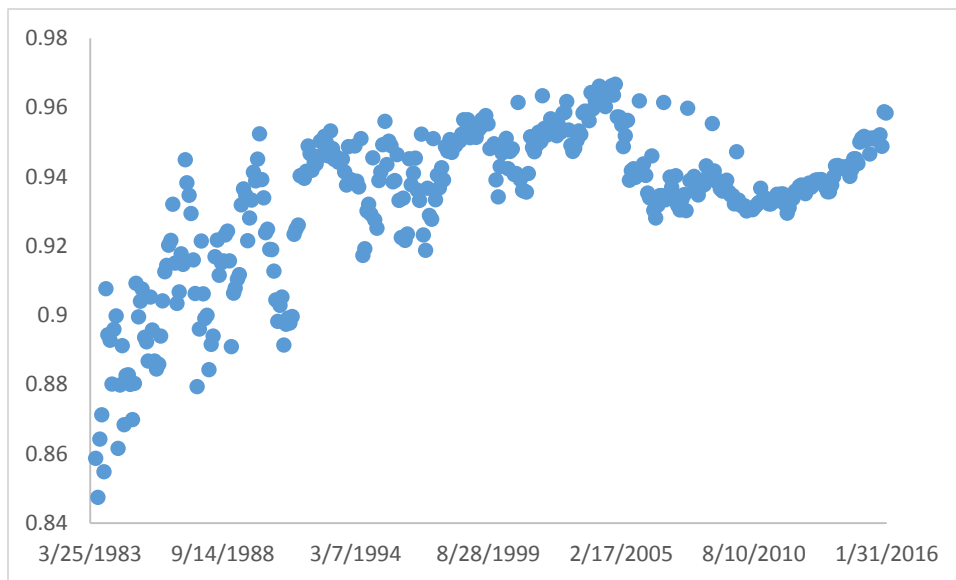
Significance Level $\mu$	Amount
90%	77
95%	48
99%	8

**Table 5.** The number of GARCH  $\mu$  coefficients which are statistically significant at each confidence level.

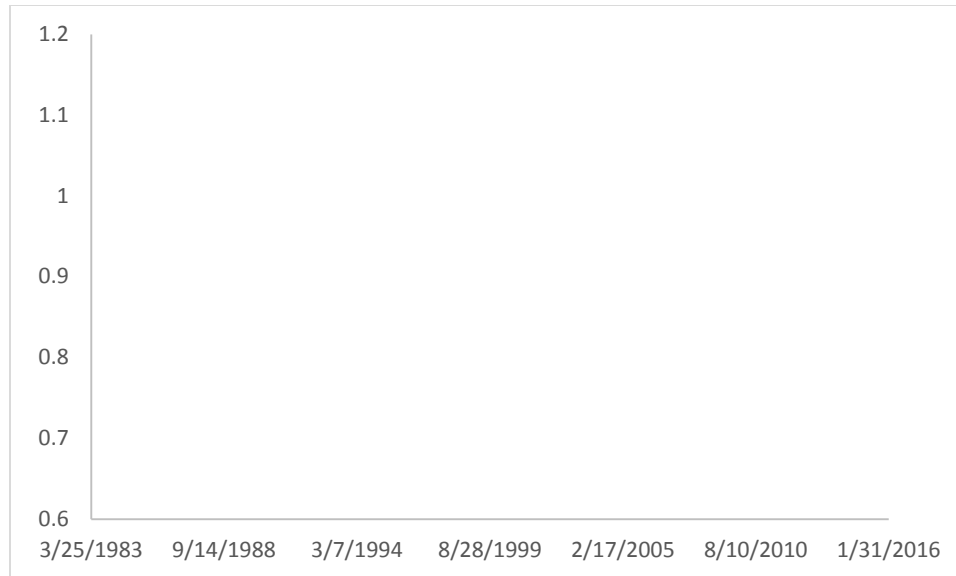
The results of the GARCH models are cut and dry. More than 86% of the  $\beta$  coefficients are significant at the 90% confidence level and there is only a 3.6% drop off when increasing the threshold to the 99% confidence level. This is strongly indicative of previous period volatility affecting current period volatility. The maximum value for  $\mu$  is  $1.9 \times 10^{-3}$  and most  $\mu$ s are statistically insignificant, indicating that there are few, if any, risk premiums being paid on the contracts as the average return is expected to be 0. In fact, the mean absolute return is a measly  $\$0.155 \pm \$0.114$  per year, negligible to even a \$10 oil contract. The GARCH models for CLN83 and CLQ83 were not run as the number of daily observations, under 100, were not enough for the solver to converge and is not expected to alter the results presented.



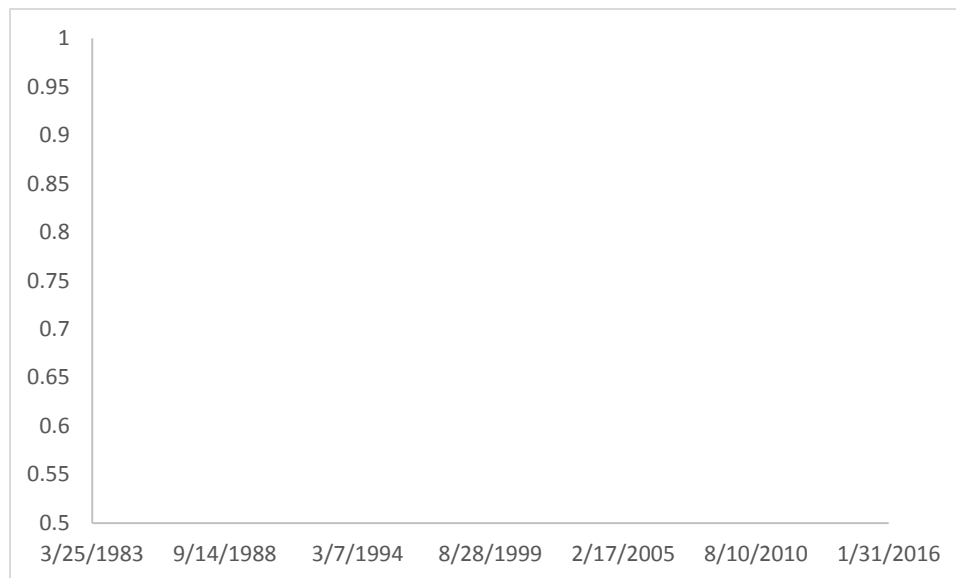
**Figure 5.** The normalized Shannon's Logarithmic Information in the plane  $D = 3$  for each futures contract plotted on its maturation date.



**Figure 6.** The normalized Shannon's Logarithmic Information in the plane  $D = 4$  for each futures contract plotted on its maturation date.



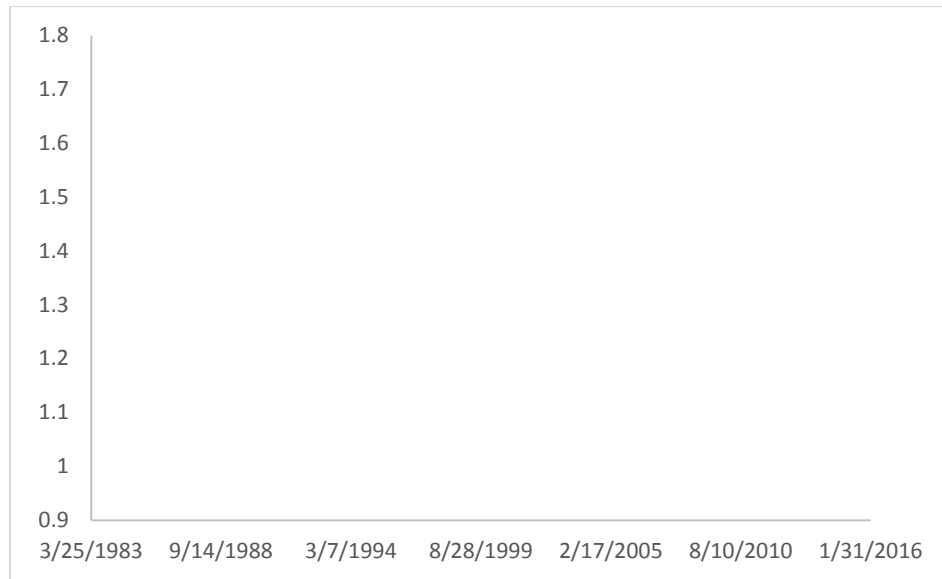
**Figure 7.** The normalized Shannon's Logarithmic Information in the plane  $D = 5$  for each futures contract plotted on its maturation date.



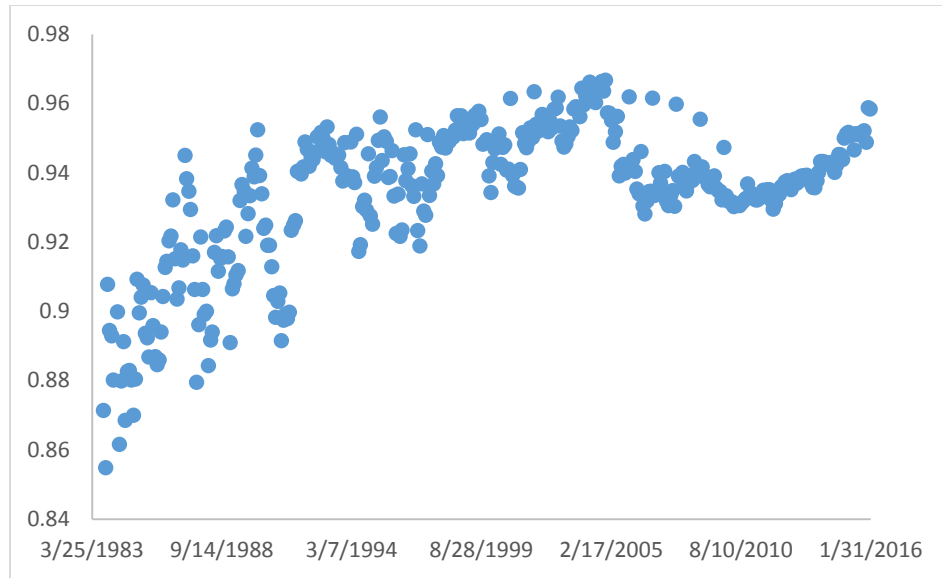
**Figure 8.** The normalized Shannon's Logarithmic Information in the plane  $D = 6$  for each futures contract plotted on its maturation date.

As is evident by a simple glance, the overarching trend of entropy in the oil market is increasing over time, as measured through price. However, the entropy is not at unity which would indicate complete randomness of the price distribution. An important note is that these entropy calculations rely on

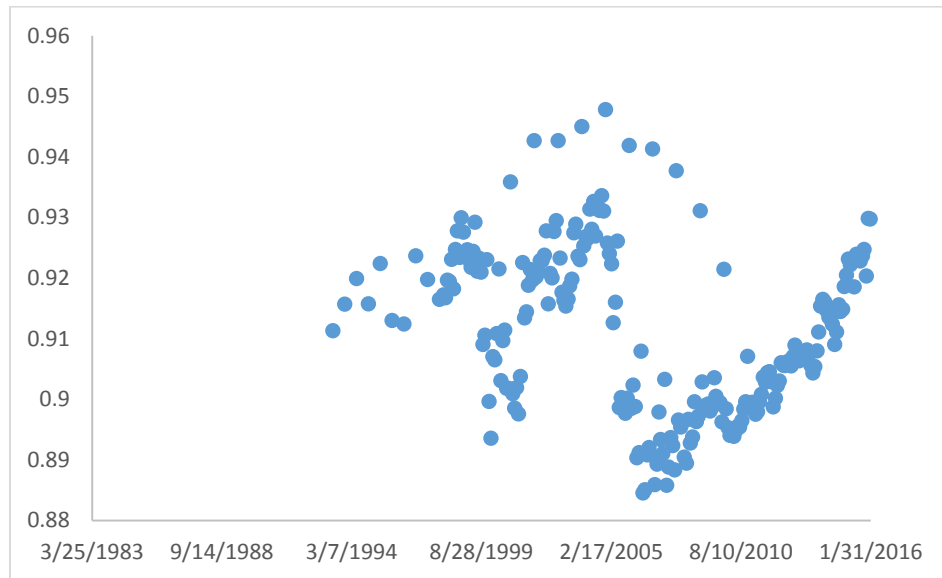
asymptotic efficiency because, as mentioned in the methodology,  $N \gg D!$  for the law of large numbers to take effect. Arbitrarily choosing  $5 D!$  as the cutoff point should allow sufficient observations for the multinomial distribution to have at least some predictive power.



**Figure 9.** The normalized Shannon's Logarithmic Information in the plane  $D = 3$  for each futures contract plotted on its maturation date. Contracts with less than  $5 D! = 30$  observations have been omitted.



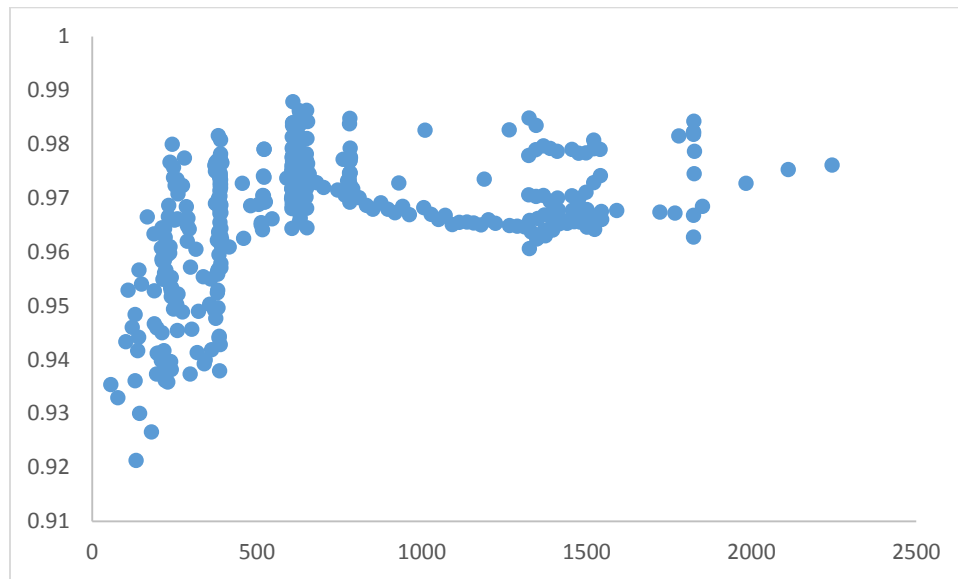
**Figure 10.** The normalized Shannon's Logarithmic Information in the plane  $D = 4$  for each futures contract plotted on its maturation date. Contracts with less than  $5 D! = 120$  observations have been omitted.



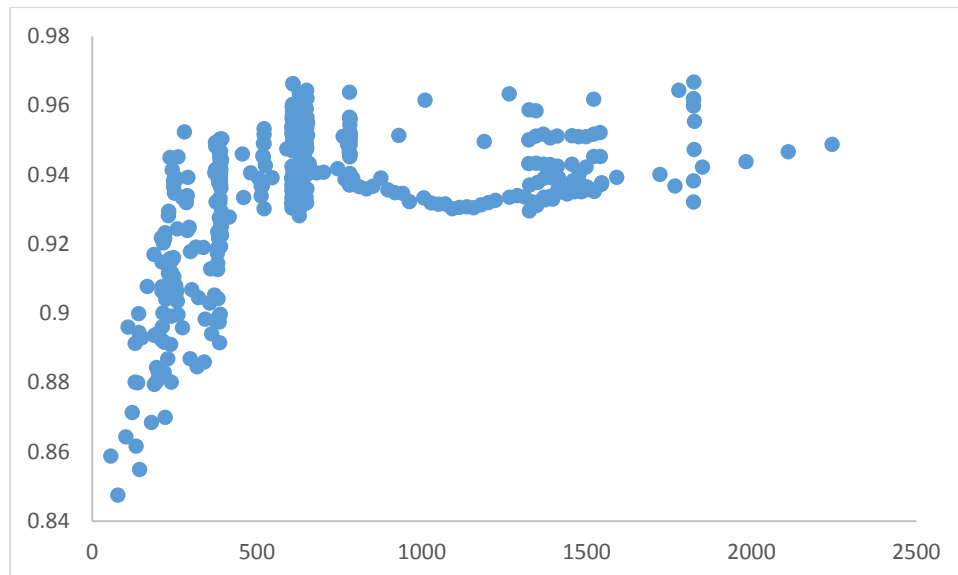
**Figure 11.** The normalized Shannon's Logarithmic Information in the plane  $D = 5$  for each futures contract plotted on its maturation date. Contracts with less than  $5 D! = 600$  observations have been omitted.

Again, an increase in entropy is seen over time, showing that markets are gaining efficiency over time but still they are not reaching the theoretical maximum of one which would indicate that weak form of the efficient market

hypothesis is in effect. The price movements of the oil futures are not entirely random. However, in order to better judge where this analysis technique “falls off” in predictive power the entropy is plotted against length. To determine if the length of data in each future sample is large enough to contain predictive power, charts of length of sample versus entropy are created.

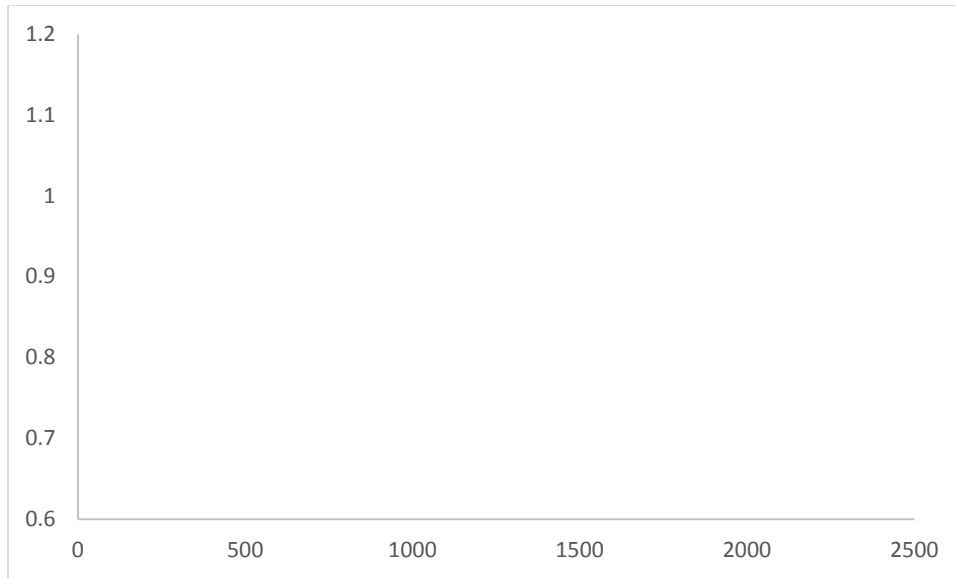


**Figure 12.** Length of future contract versus observed entropy for  $D = 3$ .

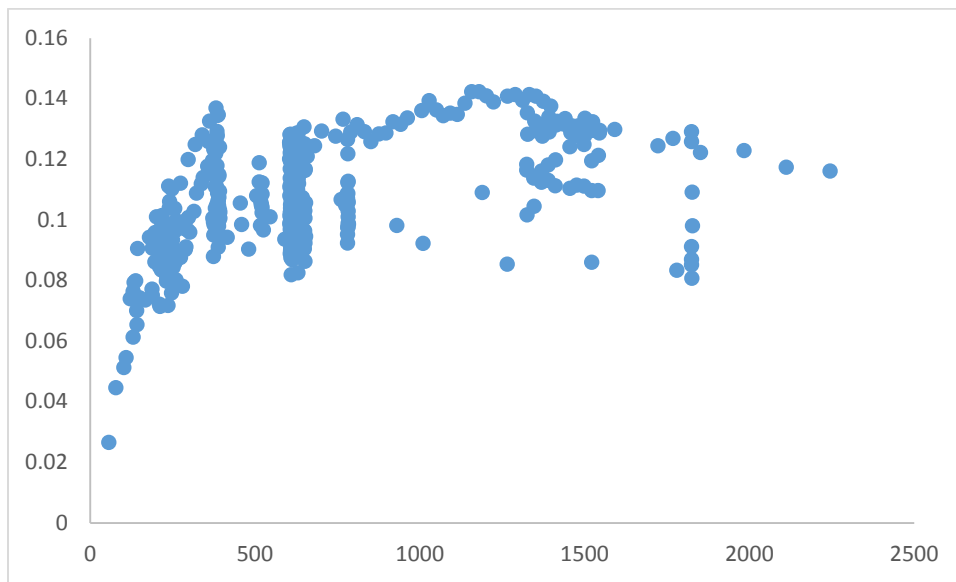


**Figure 13.** Length of future contract versus observed entropy for  $D = 4$ .

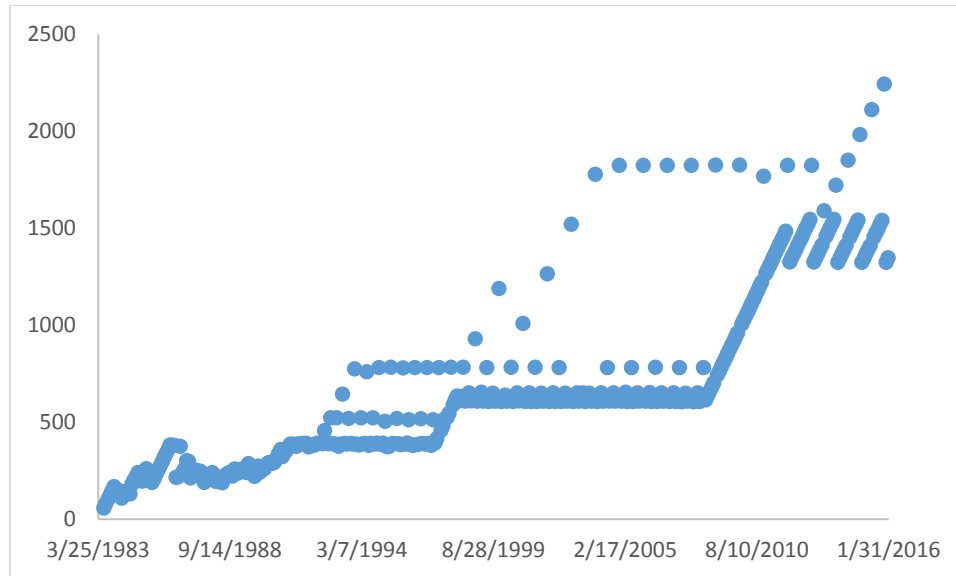




**Figure 14.** Length of future contract versus observed entropy for  $D = 5$ .



**Figure 15.** Length of future contract versus observed entropy for  $D = 6$ .

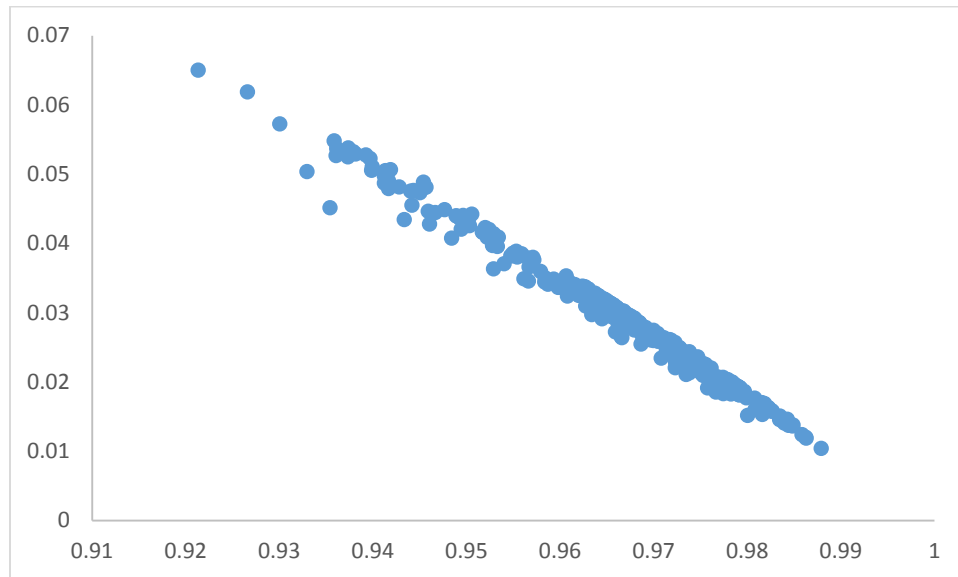


**Figure 16.** Number of daily observations for each future contract.

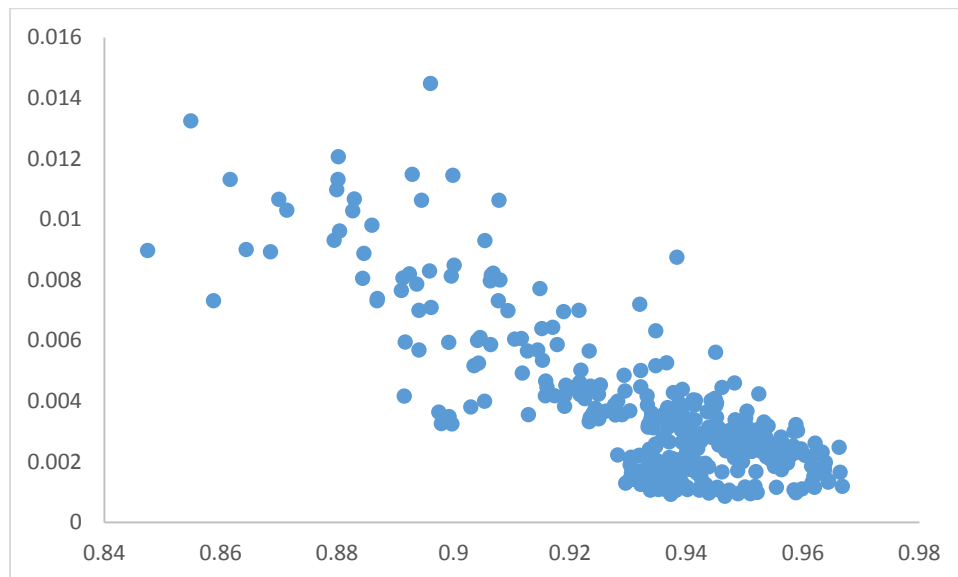
A brief glance at the four entropy planes, Figures 12-15, is very startling: asymptotic efficiency begins at around the 500 observation mark for each entropy plane. Because each entropy plane is based on a different number of permutations,  $3!$ ,  $4!$ ,  $5!$ , and  $6!$ , it should be expected that  $N \gg D!$  should occur at different  $N$ 's for each value of  $D$ . The odds of asymptotic efficiency occurring at  $N \cong 500$  is more than just a coincidence. In the  $D = 6$  plane  $D!$  is 720 implying  $N$  should be much larger, but the observed convergence is occurring before  $D!$  observations are seen! The  $D = 6$  plane does, however, continue to exhibit a slight growth as  $N$  increases and, likely, the plane does not contain statistically significant information.

Looking at Figure 16 paints another picture: all of the data shorter than length 500 comes from the earliest oil futures. From a purely theoretical perspective, it would make sense for efficiency to be increasing over time. The

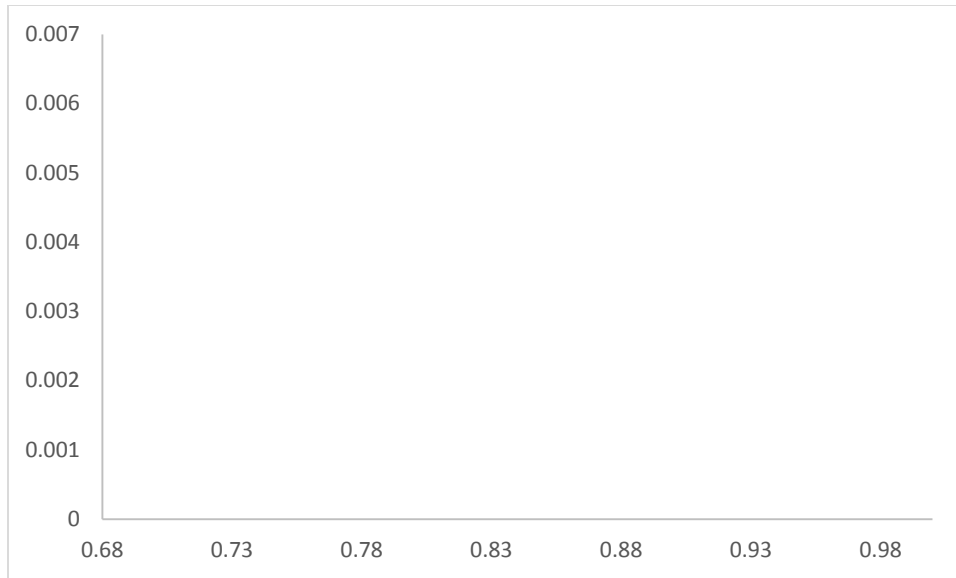
asymptotic efficiency seen at the length of 500 may be because time, length, and efficiency within the oil futures data set are inextricable.



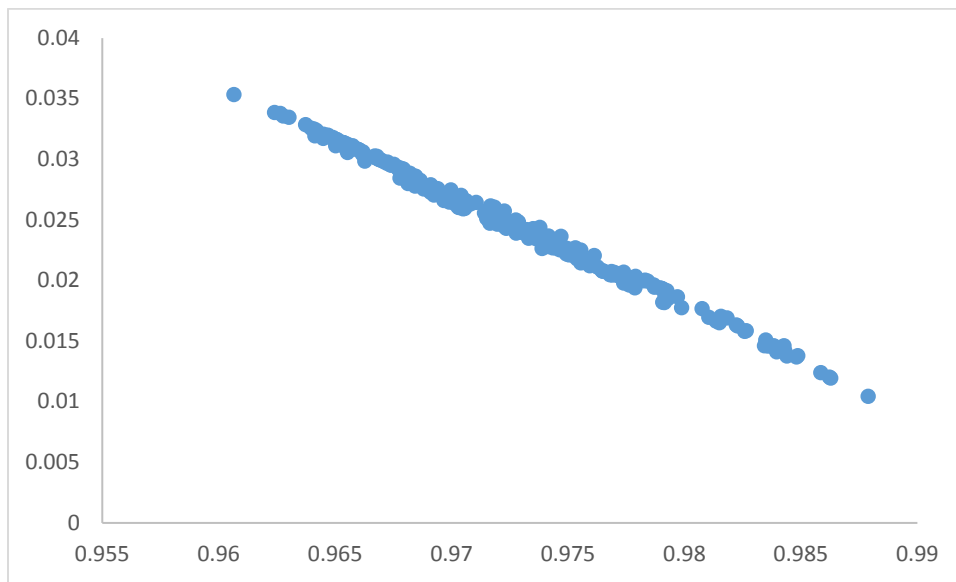
**Figure 17.** Graph of the complexity entropy plane  $D = 3$  for all oil futures.



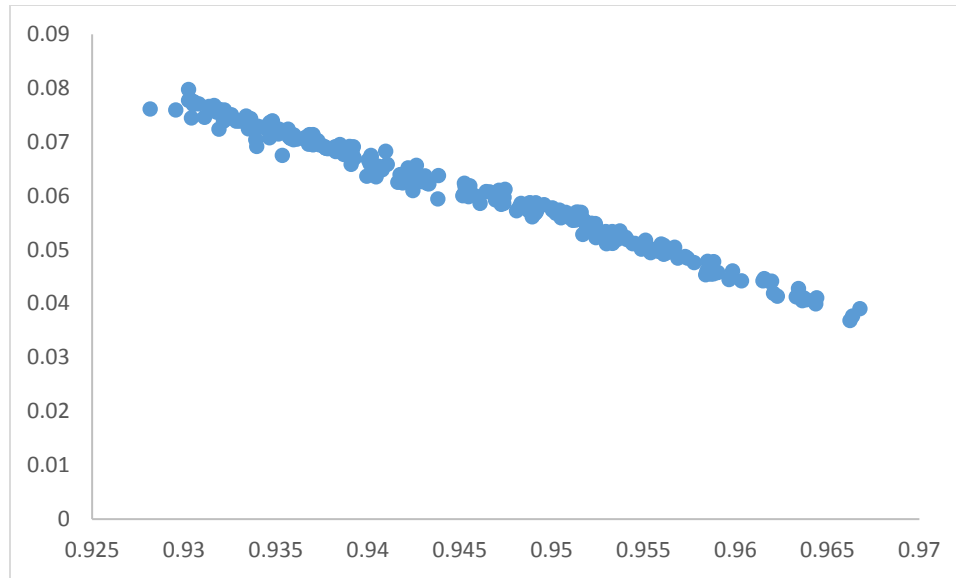
**Figure 18.** Graph of the complexity entropy plane for  $D = 4$  for all oil futures.



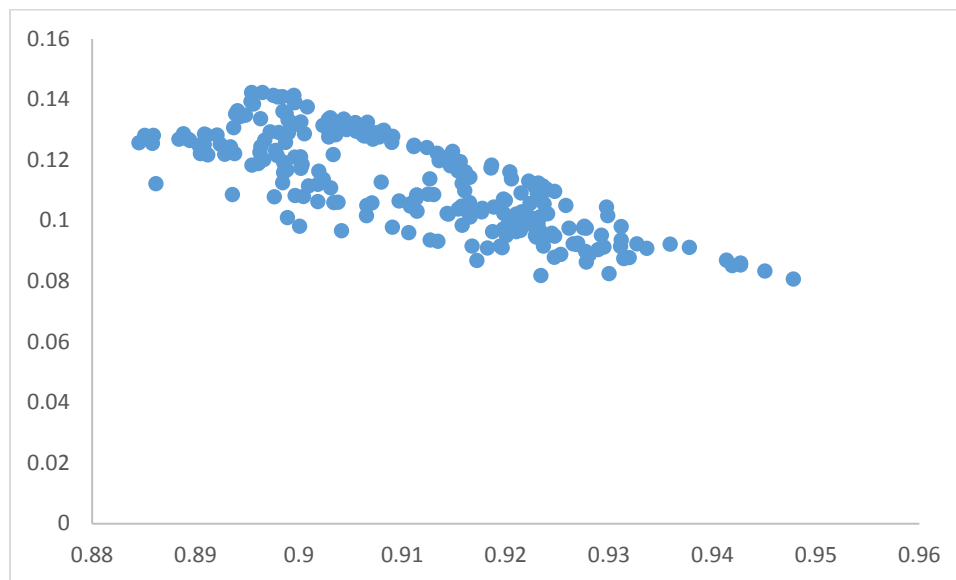
**Figure 19.** Graph of the complexity entropy plane  $D = 5$  for all oil futures.



**Figure 20.** Graph of the complexity entropy plane for  $D = 3$  filtered for all futures with lengths greater than 500.



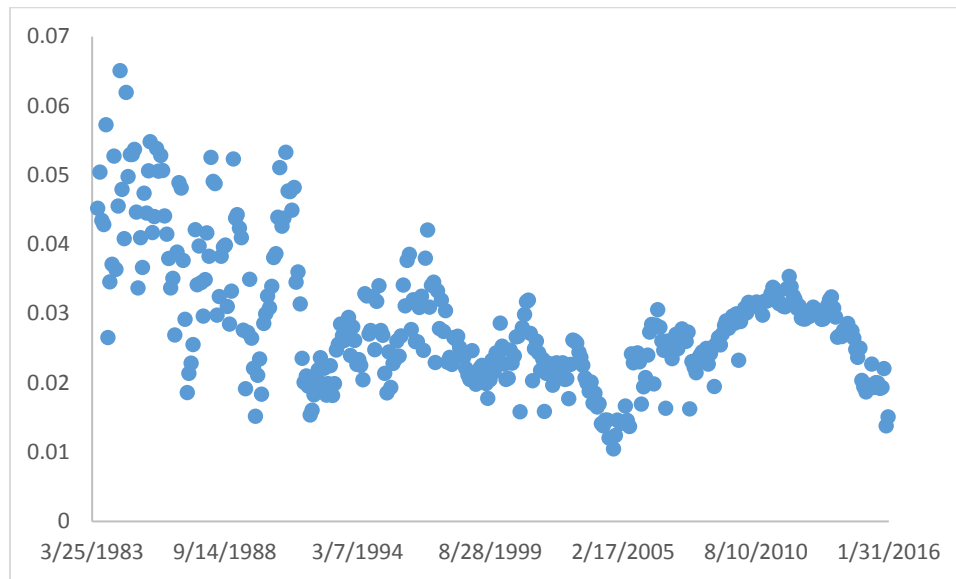
**Figure 21.** Graph of the complexity entropy plane for  $D = 4$  filtered for all futures with lengths greater than 500.



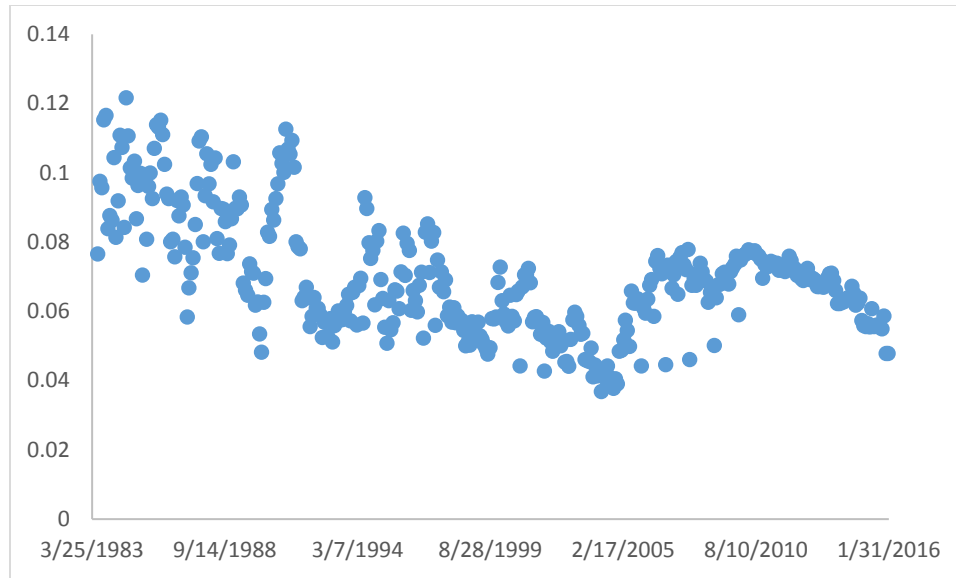
**Figure 22.** Graph of the complexity entropy plane for  $D = 5$  filtered for all futures with lengths greater than 500.

The complexity entropy planes, Figures 19-24, demonstrate that the decrease in entropy is related to the information that market prices don't reflect. The futures contracts that are below 500 observations in length store less information and, thus, demonstrate lower complexity than those with larger

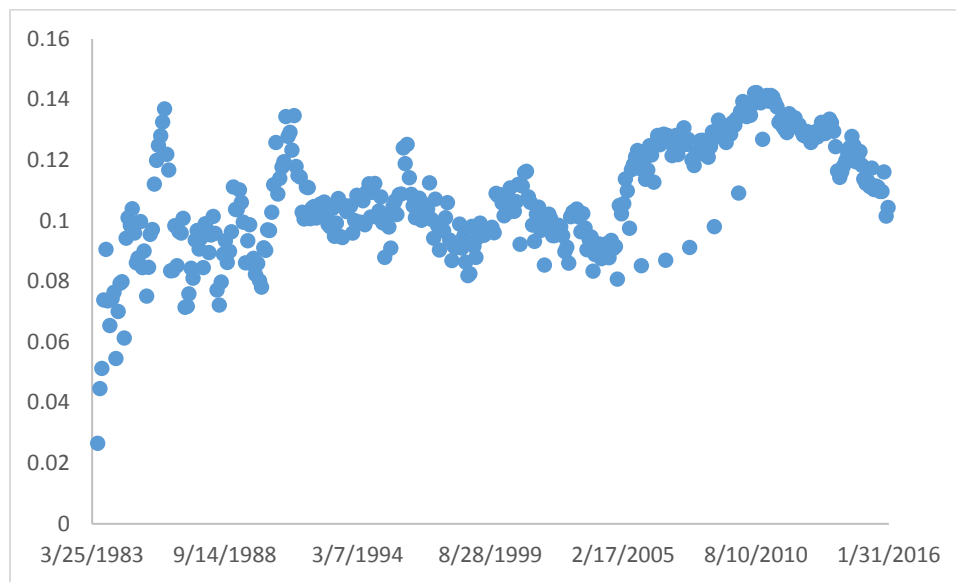
lengths, implying that their reported low entropies are more likely due to the length of the sample size instead of worse market efficiencies. The tightness of the complexity also degrades with each increasing plane, possibly due to sample size limitations. All of these planes show that complete price information has never been present in the oil futures market.



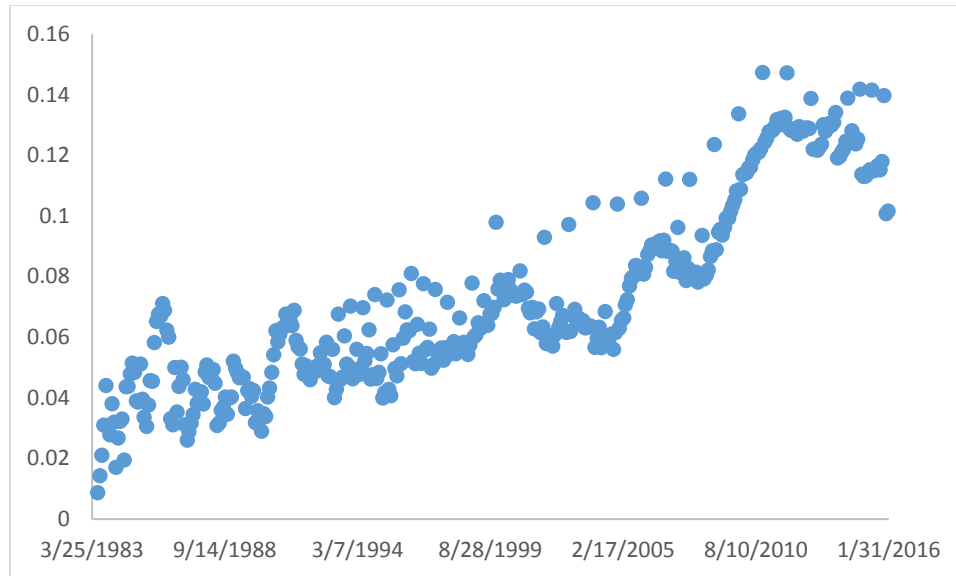
**Figure 23.** Graph of complexity versus time for  $D = 3$  for all oil futures.



**Figure 24.** Graph of complexity versus time for  $D = 4$  for all oil futures.



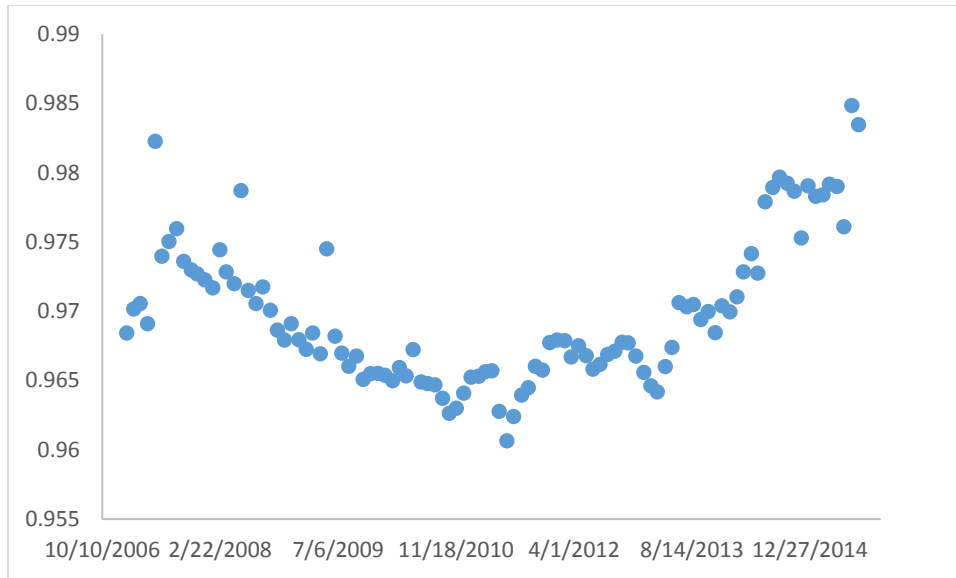
**Figure 25.** Graph of complexity versus time for  $D = 5$  for all oil futures



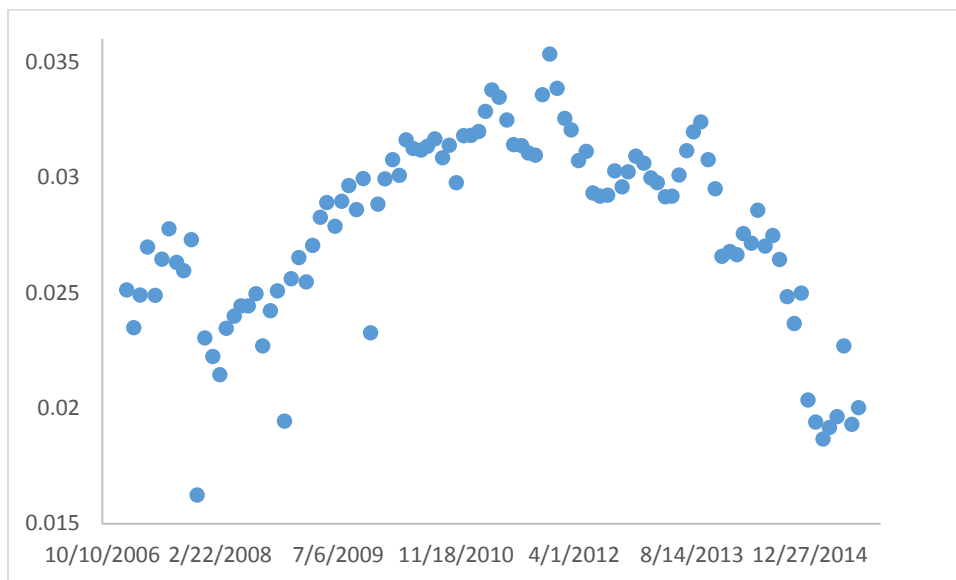
**Figure 26.** Graph of complexity versus time for  $D = 6$  for all oil futures.

As mentioned previously, complexity is a rough measure of the strength of conclusions able to be drawn from the patterns of randomness. The complexity versus time graphs, Figures 23-26, support the conclusions drawn by the previous entropy versus time graphs. The planes for  $D = 3$  and  $D = 4$  show decreasing complexity over time and, thus, greater inclusion of previous period's prices over time as there is less ability for the information to be incorporated into models. The  $D = 5$  and  $D = 6$  planes show an increased complexity over time, likely due to the longer price histories giving more statistically valid results instead of a contradiction showing worsening market efficiency over time.

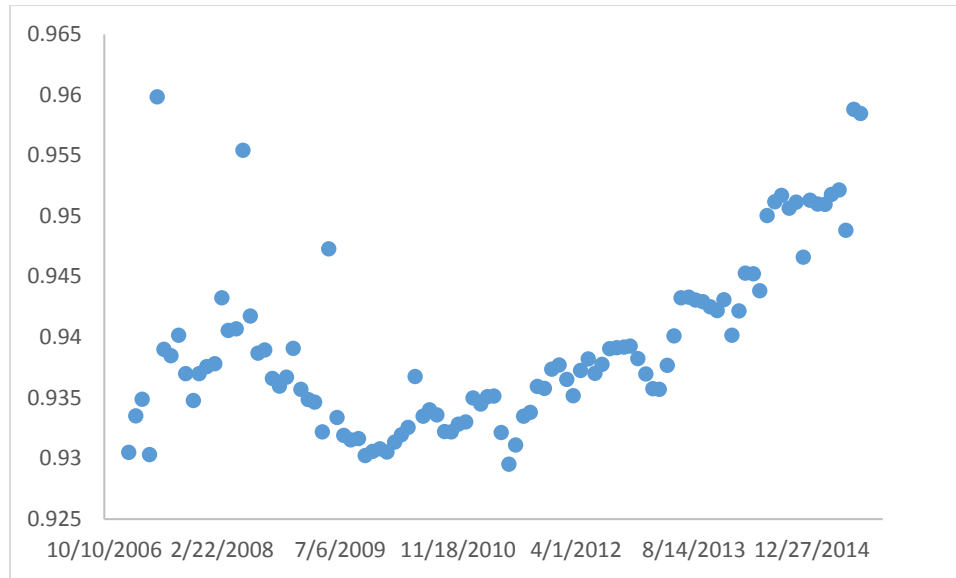




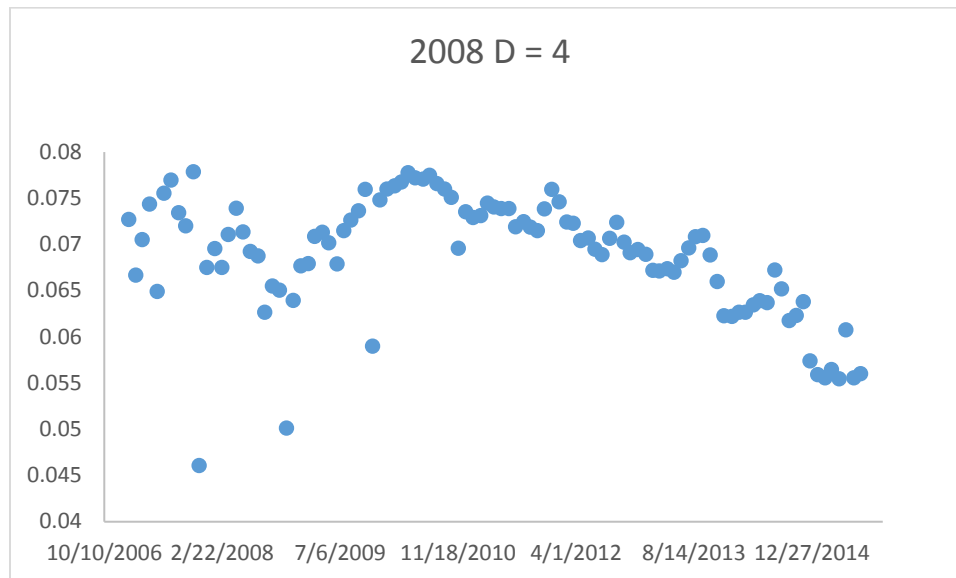
**Figure 27.** Zoomed in section of the  $D = 3$  entropy plane during the 2008 recession.



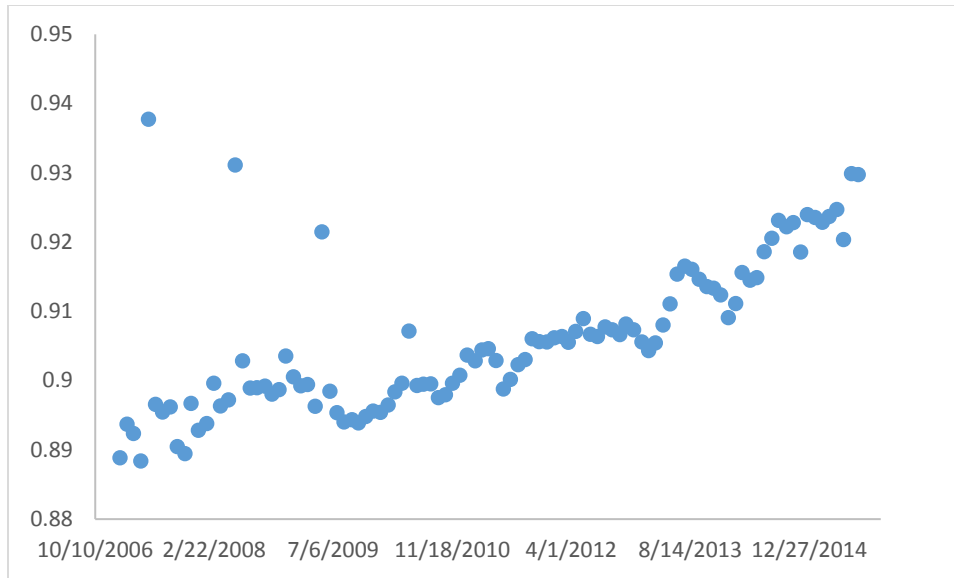
**Figure 28.** Zoomed in section of the  $D = 3$  complexity plane during the 2008 recession.



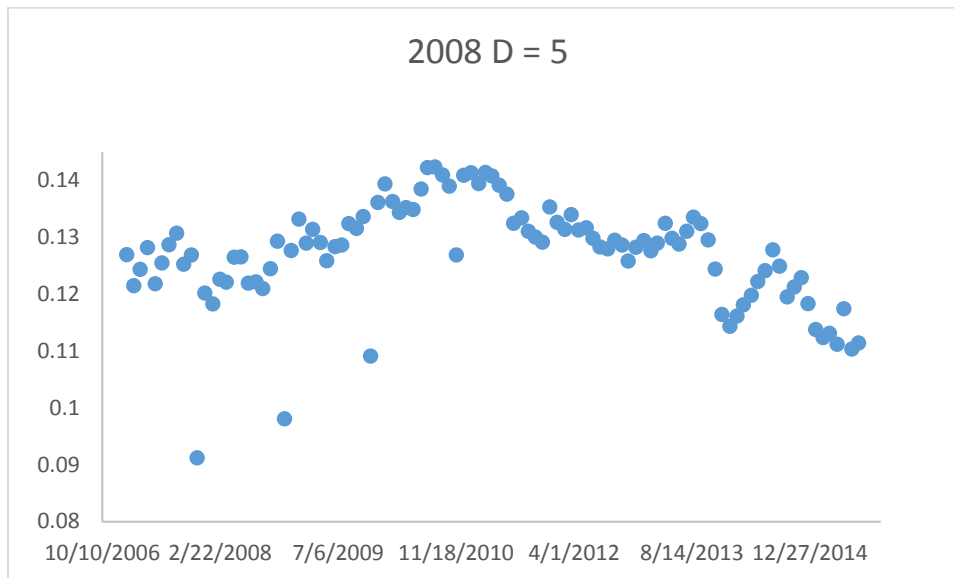
**Figure 29.** Zoomed in section of the  $D = 4$  entropy plane during the 2008 recession.



**Figure 30.** Zoomed in section of the  $D = 4$  complexity plane during the 2008 recession.



**Figure 31.** Zoomed in section of the  $D = 5$  entropy plane during the 2008 recession.



**Figure 32.** Zoomed in section of the  $D = 4$  complexity plane during the 2008 recession.

Another interesting feature of these entropy and complexity planes is documented in Figures 27-32. During the 2008 recession, a notable loss of entropy occurs followed by an increase as the recession ends, mirrored by an

increase then decrease in complexity. This recessionary informational efficiency dip also occurs in tandem with the 1989 and 2000 recessions, showing synchronized movements down then up when compared with the random spattering of nonrecessionary data points. This change in information in the market, which is measurably, but not majorly, below weak form efficiency, is in contradiction with the efficient market hypothesis.

Another notable discrepancy in informational efficiency within the oil futures market is seen in the November oil contracts from 2000-2010, the very obvious arch in the  $D = 4$  and  $D = 5$  planes, and the May 2007-2010 contracts during the 2008 recession. The November oil contracts have approximately 1700 daily observations associated with them instead of the approximately 600 associated with nearby contracts. The May contracts, however, are nearly exactly the same length as nearby contracts. Looking at the length vs entropy graphs shows that the November contracts are outliers as well, having entropies well above contracts similar in length. Thus, the cause of increased entropy could be due to increased market participation and more robust competition due to the popularity of the contracts' maturation.

## CONCLUSIONS

The evidence presented confirms Sanford Grossman', Joseph Stiglitz', and Andrew Lo's, amongst others, views that the efficient market hypothesis and informational efficiency cannot exist in real markets. Evidence of information loss during recessionary periods can be attributed to investors with poor decision making capabilities being weeded out of the market. The subsequent increase in entropy their replacement by more savvy investors as larger rewards can be reaped, consistent with these economists' beliefs.

For exactly zero of the three hundred and ninety two futures contracts observed was weak form efficiency found as the normalized Shannon efficiency demonstrated that prices could be used as predictions of future price movements better than random chance. However, the oil futures demonstrate the market is very close to theoretical maximum efficiency predicted under the EMH. The GARCH models showed many linkages of previous period returns reflected in the subsequent period. The information presented is inconsistent with the theoretical efficient market hypothesis and the constant reflection of complete price information in the markets. This suggests that a price must be paid in order to make "good" investment decisions with the market destroying participants with incorrect rationales.

Future work needs to be done on larger markets to better determine the predictive power of Shannon's information criteria and the subsequent complexity calculations. More securities, stocks, bonds, and futures, can be analyzed to see if the patterns detected in this study are ubiquitous market phenomenon and potentially provide further evidence for the irrationality of man. Other branches of information theory and thermodynamics could have analogous equivalents in the realm of economics and their importation would provide powerful tools to quantify phenomenon beyond rhetorical arguments and strengthen empirical tools presently employed.

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