A STUDY OF SEVENTH GRADE GEOMETRY POSTTEST SCORES AFTER USING THE GEOLEG MANIPULATIVE TOOL

by

Phyllis T. Pacilli

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This dissertation was prepared under the direction of the candidate's dissertation advisor, Dr. Joseph Furner, and the Department of Curriculum, Culture, and Educational Inquiry. It has been approved by the members of her advisory committee. It was submitted to the faculty of the College of Education and was accepted in partial fulfillment of the requirements for the degree of Doctor of Education.

SUPERVISORY COMMITTEE:

Joseph M. Furner, Ph.D
Dissertation Advisor

Susanne Lapp, Ed.D.

John D. Morris, Ph.D.

Angela Rhone, Ed.D.

Roberta Weber, Ed.D.

H. James McLaughlin, Ph.D.
Chair
Department of Curriculum, Culture, and Educational Inquiry

Valerie Bristor, Ph.D.
Dean, College of Education

Barry T. Rosson, Ph.D.
Dean, Graduate College

March 31, 2010
Date

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DEDICATION

God has blessed me with the best life I could ever imagine. I have an incredibly loving family, especially my two brothers Frank Pacilli and David Meyer. I am surrounded by loving and supportive friends, as well as a group of dedicated and encouraging colleagues. I work in the most noble of professions – I am a teacher. I work at the most incredible high school "South of the North Pole" – Boca Raton Community High School. The faculty and staff have been encouraging and supportive throughout my entire journey and I am extremely grateful. Each member of this community inspires me to be the best I can be.

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ABSTRACT

Author: Phyllis T. Pacilli

Title: A Study of Seventh Grade Geometry Posttest Scores After Using the GeoLeg Manipulative Tool

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The purpose of this research was to identify if 1) there is a difference in student achievement between students who use the GeoLeg manipulative tool and students who use a traditional compass, protractor, and ruler on the same geometry unit; 2) there is a difference in student achievement between the genders between those who use the GeoLeg manipulative tool and those students who do not; and 3) there is a relationship between identified learning styles and student achievement on a geometry unit posttest after using the GeoLeg manipulative tool. There were 317 students in the study. The research found that students using the GeoLeg manipulative tool produced significantly better student performance on a posttest in this particular school setting. Although these results cannot be generalized to other school sites, it is plausible that these results could generalize to school sites whose demographics are similar. The research findings revealed that there was no statistically significant difference between male and female students within the treatment group. The significant finding is that the GeoLeg manipulative tool appears to work equally well with both genders. None of the learning styles, as identified by the Honey and Mumford Learning Styles Questionnaire, were correlated with student posttest score achievement on the tested geometry unit. In addition, there was no evidence to suggest that a student's learning style moderates the effectiveness of the use of the GeoLeg manipulative tool. There is no evidence to suggest that the effectiveness of the GeoLeg manipulative tool is any different depending upon the student's gender or learning style. The results of this research provide strong support for the use of the GeoLeg manipulative tool for improving student performance. Further research is needed to confirm these results in similar and different populations.

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CHAPTER 1: Introduction

A lengthy history of research on the effects of teaching for meaning and understanding exists at all learning levels. Researchers and theoreticians have challenged and debated constructivist methods regarding student learning, based on the belief that students must comprehend what they are learning for the learning to become permanent.

Jean Piaget (1952) contended that children did not have the mental maturation to comprehend abstract mathematical concepts that are presented in words or symbols alone. He concluded that students need a variety of experiences with concrete materials and drawings for learning to occur. According to Mueller (1985), the early mathematics experiences of students should involve the use of various hands-on materials. Mueller summarized that mathematics is a “verb” for students, wherein they are actively engaged in the process of “doing” mathematics. Mueller is in favor of the use of sequential activities that utilize objects that are first concrete, then pictorial, and finally symbolic.

Piaget (1954) described four basic stages of cognitive development through which all students progress: sensory motor, preoperational, concrete operational, and formal operational. In each stage, individuals must organize information through a process called assimilation. They then adapt that information to fit into the environment through a process called accommodation. At each stage, the student has a different type of psychological structure in which to address each situation. For Piaget (1954), the concrete operational stage of development includes students between the ages of 7 and 11 years old. In this stage of development, students are able to think logically about objects and events. However, student thinking is tied to concrete activities and experiences.

With the theories of Piaget as a foundation, two basic principles of human thought could frame an analysis of the nature of students' learning of geometry.

The first principle is that the human mind constructs rather than receives meaning. The Nobel Prize winner Francis Crick stated, "Seeing is a constructive process, meaning that the brain does not passively record incoming visual information. It actively seeks to interpret it" (Crick, 1994). Psychologist, Robert Ornstein (1991) asserted, "Our experiences, perceptions, and memories are not of the world directly but are our own creation, a dream of the world, one that evolved to produce just enough information for us to adapt to local circumstances" (p. 160). As Ornstein explained his theory of human processing of information, he alluded to the concept of constructing the meaning rather than receiving the given message.

The second basic principle is that individuals construct new knowledge and understanding based on what they already know and think (Bransford, Brown, & Cocking, 1999). Students interpret their experiences based upon their current intellectual status and experiences. Students bring into the learning environment a set of experiences that could be used as resources upon which to build. As classroom activities connect with experiences, students have the opportunity to construct new knowledge and understandings.

These two basic principles frame the knowledge regarding how students construct their mathematical understanding. This understanding could have a great impact on how teachers implement national and state standards in their classrooms. With a national emphasis on assessment and achievement, it is essential for teachers to have a clear understanding of how students progress from their “starting point” of understanding, to the formal mathematical concepts that they are expected to master and apply. “There is a good deal of evidence that learning is enhanced when teachers pay attention to the knowledge and beliefs that learners bring to a learning task. Use this knowledge as a starting point for new instruction, and monitor students’ changing conceptions as instruction proceeds” (Bransford, Brown, & Cocking, 1999, p. 9).

For a number of years, concrete manipulatives such as base ten blocks, geoboards, color tiles, fraction strips, Unifix cubes, color counters, beads, and buttons have been available to assist students in constructing mathematical concepts. Many textbook companies have developed lessons that include the use of manipulatives as well as their own manipulative kits to accompany their mathematics series. The National Council of Teachers of Mathematics (NCTM) is an organization that has been very influential in mathematics reformation efforts in the United States. NCTM encourages and supports the use of manipulatives in mathematical instruction (NCTM, 2000). This organization encourages educators to teach concepts progressing from the concrete to the abstract. Teaching with concrete materials supports the constructivist theory that learning transpires when students gain an understanding of concepts through hands-on experiences.

Statement of the Problem

In the learning of mathematics, many students underachieve at levels lower than their current grade level (Schoenfeld, 2002). Teachers may need to re-evaluate the methods they use to teach mathematical concepts and skills for students to achieve at higher levels (Burns, 1996a). Manipulatives appear to be useful in teaching various concepts of mathematics, but teachers appear reluctant to use manipulatives despite being recommended as valuable instructional tools (Kober, 1991). Students' preliminary understanding of mathematical ideas and concepts seems to develop through sensory experiences. The sense of touch and handling of objects kinesthetically kindles their interest and imagination and assists in building understanding beyond any drill or stimulus-response method used (Welchman-Tischler, 1992).

Concrete objects help students make connections between ideas (Dewey, 1938). Not only do manipulatives assist students to advance deeper understanding and enhance their attitudes toward mathematics as a discipline, but concrete materials also are valuable tools for engaging learners in the language and communication of mathematical beliefs and ideas (Chester, Davis, & Reglin, 1991; Kober, 1991; Shaw, 2002). According to the National Council of Teachers of Mathematics, "students need various opportunities to test their ideas on the basis of shared knowledge in the mathematical community of the classroom to see whether they can be understood and if they are sufficiently convincing" (NCTM, 1989, p. 60).

As students progress through the elementary grades and on to middle school, it is critical for instructional materials to continue to include concrete hands-on learning material (Shaw, 2002). Students must transition from concrete manipulative to the symbolic language of mathematics. The National Council of Teachers of Mathematics Principles and Standards for School Mathematics (NCTM, 2000) advocates the significance of using manipulatives, visual representations, and mathematical modeling in each of its standards at all grade levels. Learning occurs when a student interacts with physical materials, manipulating and performing actions with various objects (Resnick & Ford, 1981).

Elementary and middle grades students who learn with concrete materials are believed to be better equipped to bridge to the abstract world of mathematical concepts and apply their knowledge to realistic situations (Kober, 1991). While students are struggling to grasp and process current information, teachers too often continue to present new information. Pressing forward with more information may be attributed to the pressures of state and national assessments. However, when students are left to struggle with concepts while lacking the depth or ability needed to continue to an abstract level, this may lead to an overall lack of confidence in mathematics as well as general dislike for the subject matter.

According to the NCTM (NCTM, 2000), students enter third grade with an interest in learning mathematics and see it as practical and important. If this positive trend is to continue, students must be encouraged to remain actively engaged and learning should not become a process of mimicking and memorization (NCTM, 2000).

The concept of geometry is part of the seventh grade curricula in most school districts in Florida. According to the National Research Council (NRC, 1989) students traditionally have found learning the concepts of geometry difficult. One reason for the difficulty with geometry may be that the application of formulas and concepts is multi-faceted, requiring knowledge of different but related concepts (Graham & Graham, 2003). Within this basic constructivist foundation in the field of mathematics, continued research regarding the use and effectiveness of manipulatives remains a critical need. While minimal research has included the GeoLeg manipulative tool, no research regarding the effectiveness of the use of the GeoLeg manipulative tool with seventh grade students is available. It is not known if the use of the GeoLeg manipulative tool benefits seventh grade student achievement while learning geometry concepts.

Purpose of the Study

The purpose of this study was to determine the effects of the use of the GeoLeg mathematical manipulative tool with seventh grade students in learning geometric mathematical concepts in an urban school setting in South Florida. Specifically, the objectives of this study are to determine: If students could make connections between concrete representations and abstract representations through the use of the GeoLeg manipulative tool; if this manipulative tool could enhance student achievement; if any effects could be generalized over gender; and if a relationship may exist between student learning styles (as identified by the Honey and Mumford Learning Styles Questionnaire, 1995) and student achievement on a geometry posttest.

Research Questions

1. Do seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on a posttest than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum?
2. Does the use of the GeoLeg manipulative tool have the same effectiveness for seventh grade males and females when learning geometry concepts?
3. Is there a relationship between the student learning style of "Reflector" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest after using the GeoLeg manipulative tool?

4. Is there a relationship between the student learning style of "Theorist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest after using the GeoLeg manipulative tool?
5. Is there a relationship between the student learning style of "Pragmatist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest after using the GeoLeg manipulative tool?
6. Is there a relationship between the student learning style of "Activist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest after using the GeoLeg manipulative tool?
7. Does the "Reflector" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg manipulative tool intervention on the geometry posttest scores after using the GeoLeg manipulative tool?
8. Does the "Theorist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores after using the GeoLeg manipulative tool?
9. Does the "Pragmatist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores after using the GeoLeg manipulative tool?
10. Does the "Activist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores after using the GeoLeg manipulative tool?

Research Hypotheses

This study was based on 10 hypotheses: First, there will be a significant difference in the posttest scores between the treatment group and control group ($p < .05$). It is expected that the treatment group students will outperform the control group upon the completion of the unit on geometry. Second, there will be a significant difference ($p < .05$) in the effectiveness of this tool between genders. Third, there will be a relationship between the "Reflector" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. Fourth, there will be a relationship between the "Theorist" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. Fifth, there will be a relationship between the "Pragmatist" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. Sixth, there will be a relationship between the "Activist" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. Seventh, a "Reflector" learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores. Eighth, a "Theorist" learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores. Ninth, a "Pragmatist" learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores. Tenth, an "Activist" learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores.

Rationale for the Study

Being able to explain and apply knowledge of mathematical concepts should be a consistent and expected feature of students' mathematical ability. Many educators place significance on the use of manipulatives to teach mathematical concepts due to the influence of behaviorist and cognitive theorists who argue that learning should begin with concrete experiences and move toward abstract symbolism (McBride & Lamb, 1986).

Long-term use of concrete material that lends meaning to mathematical theories and concepts increases student mathematical achievement and positive attitudes when utilized by teachers who are knowledgeable about their use (Sowell, 1989). Studies that examine children's comprehension as well as their computational competence show that the use of concrete material and authentic situations enable students to construct and use intuitive meanings for new mathematical concepts (Battista, 1986; Clements & McMillen, 1996; Parham, 1983; Suydam, 1986). Mathematical manipulatives are appropriate for any grade level and can be adapted to any topic, age, or ability level of students. Lessons in which teachers use manipulatives show a higher probability of achievement than do traditional lessons (Burns, 1996a; Clements & McMillen, 1996; Sowell, 1989; Suydam, 1986).

According to Grouws and Cebulla (1999), the focal point of instruction should be on enhancing important mathematical concepts that increase the level of student understanding. Research consistently shows that use of manipulatives positively influences student learning and produces better initial understanding, greater retention, and an increased probability that the concepts will be applied in new situations (Battista, 1986; Clements & McMillen, 1996; Parham, 1983; Suydam, 1986). Students who develop conceptual understanding during early learning stages perform better on procedural knowledge on more complex tasks (Battista, 1986; Clements & McMillen, 1996; Grouws & Cebulla, 1999; Grouws & Cebulla, 2000).

Instruction that primarily focuses on conceptual development, with the goal of conceptual understanding, can facilitate mathematical learning without sacrificing skill development or proficiency (Hiebert, 2003). “Students who develop conceptual understanding early perform best on procedural knowledge later” (Grouws & Cebulla, 2000). Students with lower levels of conceptual understanding require additional practice to obtain procedural knowledge (Burns, 1996b; Grouws & Cebulla, 1999; Parham, 1983; Suydam, 1986). Long-term use of manipulatives has been found to be positively related to increased student academic achievement and improved attitudes toward mathematics in general (Grouws & Cebulla, 1999). In a review of literature on activity-based learning in mathematics in kindergarten though eighth grade, Suydam and Higgins (1977) discovered that using concrete materials produced greater achievement gains. Sowell (1989) conducted a meta-analysis that compared the effects of manipulative use with the effects of more abstract instruction, and found that the long term implementation of concrete materials by teachers knowledgeable about their use improved student achievement and attitudes (Grouws & Cebulla, 1999).

NCTM emphasizes the importance of using physical models in mathematics instruction (NCTM 1989, 1991, & 2000). The NCTM Professional Standards for Teaching Mathematics document suggests that teachers need, “a rich, deep knowledge of the variety of ways mathematical concepts and procedures may be modeled, understanding both the mathematical and developmental advantages and disadvantages in making selections among the various models" (NCTM, 1991, p. 151).

High quality mathematics instruction is dependent on student interaction with meaningful learning tasks. Effective mathematical practices allow students to connect their informal knowledge and experience to mathematical abstractions. Students need to practice recognizing, understanding, and interpreting numerical information in news stories and other real-world contexts (Ginsburg & Gal, 1996). Learning tasks that use manipulative devices assist with this connection. Educators have supported using manipulatives for teaching mathematics based on theories that claim students need physical representations that reflect and develop abstract concepts. The research shows positive advantages from using physical materials (Burns, 1996a; Piaget, 1952; Sowell, 1989). Best practices in mathematics suggest that students need multiple experiences with manipulatives, visual images, and various forms of representation for effective learning to occur.

Not only do high quality instruction and manipulatives seem to make a difference in student achievement, but so does the gender of the student. When examining gender differences, male students have outscored female students in the National Assessment of Education Progress (NAEP) (2009) tests in the majority of the states and jurisdictions from 1990 to the present. The 2009 gender gap performance in the State of Florida has remained at 2 points as both males and females have improved at the same rate since 2007 (p. 192). The bulk of the evidence in the past 50 years suggests that the gender gap in mathematics does not exist before children enter school, but is large and significant in the middle school years and beyond (Fennema, 1974; Hyde et al, 1990; Maccoby, 1966; Maccoby & Jacklin, 1974).

A wide range of theories has been explored to explain the gender gap in mathematics. One strand of the literature examines biological differences. Those who have studied the biological theories argue that innate differences in spatial ability, higher order thinking, and or brain development produce a gap in achievement (Wilder & Powell, 1989). Another group of researchers emphasize societal factors as the cause of the gap. Societal explanations focus on how girls are socialized into believing that math is not important, useful, doable, or part of the identity of a girl (Wilder & Powell, 1989).

Significance of the Problem

It is a well-documented fact that manipulatives benefit students' mathematical learning (Burns, 1996b; Clements & McMillen, 1996; Sowell, 1989; Suydam, 1986). In spite of this information, teachers implement manipulative tools sparingly because of time constraints, classroom management issues, and a lack of knowledge regarding their usage (Gilbert, 1988). Incorporating manipulative tools in early instruction can enable students to master mathematical concepts during primary education, which gives students a future advantage in taking rigorous high school mathematics and science courses.

The results of the Third International Mathematics and Science Study (TIMSS) (1996) established that many students began their high school careers without a firm foundation in mathematics, closing doors prematurely for further education and better careers (Riley, 1997). Elementary and middle school teachers have the critical task and responsibility of teaching the mathematical curricula that prepares students for success in rigorous high school courses. It is imperative that elementary and middle school teachers incorporate a wide variety of manipulatives to ensure a solid understanding of mathematical concepts. For students to have the basic foundation for the advanced mathematical curricula that they will face in the upper level classes, teachers must provide early experiences with manipulatives so that students master the four basic mathematic concepts: addition, subtraction, multiplication, and division.

Teachers must build on basic mathematical concepts when introducing geometry. Many school districts spend substantial revenue each year to purchase manipulative materials, and teachers spend a great deal of time in training and staff development learning to implement hands-on material while teaching concepts. This study is significant because it explores the use of the GeoLeg manipulative as an effective and useful learning tool. An understanding of how this manipulative affects students' academic achievement will provide invaluable information that can build upon current knowledge already known about the positive use of manipulatives.

According to Peggy Cannon, the developer of the GeoLeg manipulative tool, over one million students in the United States, Canada, and Korea are currently using this tool. To date, there has not been any published research performed on the effectiveness of this geometry tool. The only available information is an unpublished data analysis provided to the Shawnee, Oklahoma school district (Paul, 2006).

The GeoLeg manipulative tool described in this study may provide teachers and administrators with valuable information regarding the effectiveness of the GeoLeg manipulative tool. This study may also shed light on the use of this tool by males and females, and show how students with different learning styles perform academically. This research may provide decision makers in schools with valuable information regarding the allocation of their funding. Teachers may be able to use the findings from this study for collaborative staff development and training, parental education, and effective program planning. Finally, improving mathematical instruction through the use of the GeoLeg manipulative tool may assist many students in the future by an improved method for learning geometry.

Definition of Terms

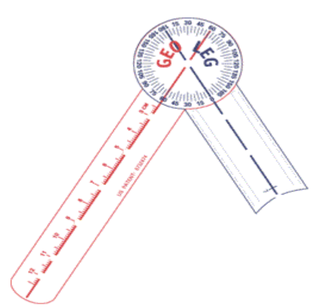
The following terms and their definitions are presented to assist the reader's understanding of the content described in this report.

Concrete – Objects that are tangible, visible, real.

Control group – A group of untreated subjects used as a test benchmark. The group of test subjects left untreated or unexposed to a procedure and then compared with treated subjects in order to validate the results of the test.

GeoLeg – “An innovative and quantitative geometry tool designed to assist students in developing visual and spatial sense in geometry and measurement” (www.geoleg.com, retrieved January 22, 2009).

Figure 1. GeoLeg Manipulative Tool



Learning Styles – “Self-consistent, enduring individual differences in cognitive organization and functioning” (Ausubel, Hanesian, Hanesian Novak, & Novak, 1978, p. 203).

Manipulative – “Any of the various objects designed to be moved or arranged by hand as a means of introducing, developing, or reinforcing understanding abstractions, especially in mathematics. They are concrete objects that can be touched and moved by students to introduce or reinforce a mathematical concept” (NCTM, 1989).

Next Generation Sunshine State Mathematics Standards - The Sunshine State Standards were first approved by the State Board of Education in 1996 to provide for student achievement in Florida. The “Next Generation” was passed by the Florida State legislature in 2007. For grades K-8 it includes Big Ideas, standards, and supporting ideas. For high school, the standards include specific standards per course taught (Florida Department of Education, 2007a).

Treatment group – the group of participants in a research study who receive the treatment being studied.

T-Test – “A statistical test involving means of normal populations with unknown standard deviations; small samples are used, based on a variable *t* equal to the difference between the mean of the sample and the mean of the population divided by a result obtained by dividing the standard deviation of the sample by the square root of the number of individuals in the sample.” (Sci-Tech Dictionary retrieved November 20, 2009 from http://www.answers.com/topic/t-test)

History of the GeoLeg Manipulative Tool

The following is a summary of an interview conducted by the researcher and the developer of the GeoLeg manipulative tool, Peggy Cannon (personal communication, September 24, 2008). During the late 1990s, the developer worked with a National Science Foundation Project, the Delta Rural Systemic Initiative, in a project with school districts, communities, and educators concerning a systemic reform of the mathematical and science education program in rural school districts in Arkansas, Mississippi, and Louisiana.

The first TIMSS (Trends in International Mathematics and Science Study) report, released in 1995, revealed reliable and timely data regarding the mathematics and science achievement of students in the United States as compared to students in other countries. Cannon views the TIMSS report as the framework for the pedagogical reform techniques that she was developing.

The 1995 TIMSS report inspired Cannon to find a way to make geometry more accessible to all students. An elementary classroom teacher who became certified in secondary mathematics, Cannon used her background experience as the foundation for her quest to find a method wherein geometry students could physically "see" geometric theorems and angle-side relationships, rather than simply memorize them.

She contended that geometry, as it was being taught, kept students working at an abstract, theoretical level rather than moving from a concrete experience to the abstract thought. The TIMSS report confirmed for her that many students were unsuccessful in the areas of measurement and geometry. Cannon's educational focus shifted and she set out to change the way that geometry was taught to students in the United States.

Cannon knew from her classroom teaching experience that many students exhibited a greater depth of understanding when the instructional methods included sketches and other visuals as students solved problems and proofs. It was this pedagogical experience that stirred her curiosity to develop a tool that would allow students to "see and manipulate polygons." Cannon wanted a tool that would allow students to see how a change in one measurement affected the geometric relationships of other measurements. Her basic premise was that if students could explore and see geometric relationships for themselves, mathematical achievement would increase.

Cannon began by building a triangle with 3 drinking straws and added a protractor to each vertex, which was the beginning of a very exciting exploration for this developer. She had discovered a method that gave triangles fluidity so that a student could see triangles quantitatively. With this simple model, she felt that she was on the way to the developing of a tool with which students could *see measures* and *changes in measures*.

Cannon then extended her exploration by building a similar quadrilateral and then parallel lines with a transversal. She came to realize that with a single tool, any polygon could be built that would give students the quantitative information necessary to see and explore geometric concepts and relationships. She believed that the memorization of theorems could be replaced with a true understanding of geometry as students *experienced* Geometry.

With her prototype, Cannon began working with engineers to develop a tool that would meet the needs of students. She asked the engineers key questions such as, "What size would best fit the hands of a student? What is the most durable and safe material for this product? What is the best length relationship of the two GeoLeg legs for maximizing polygon sizes? What is the best way to provide for continuous length measures and give equal importance to angles?" Over the years, Cannon responded to feedback and made minor printing changes to make the GeoLeg manipulative tool more usable as a ruler, and more user friendly. According to Cannon, "the single GeoLeg piece was in itself a major improvement over the classroom protractor because the average secondary geometry student was generally confused about how to use a protractor" (Cannon personal communication, September 24, 2008).

When Cannon first developed the GeoLeg prototype, she presented it at the 1998 NCTM Annual Conference in Washington, D.C. and to the1999 NCTM Annual Conference in San Francisco. She received feedback from teachers and administrators who had piloted the prototype. In response to the feedback, she added drawing points so that the measurement tool could also be used as a construction tool. After incorporating changes, Cannon stated her belief that, "now (she) had a tool that could do everything" (Cannon personal communication, September 24, 2008).

The GeoLeg was a more efficient manipulative tool than the protractor and the compass and could uniquely form quantitative polygons. It also served as a centimeter ruler and straight edge. Cannon succeeded in replacing two cumbersome tools (the protractor and the compass) with one streamlined manipulative. Cannon is the patent holder of an innovative concept, as well as the developer and marketer of an educational product with the potential of changing the manner in which students learn measurement and geometry.

At the 2001 Orlando NCTM Annual Conference, the GeoLeg Company received greater attention from educators as standardized state testing began to dictate greater rigor in the curriculum. At that time, teachers began requesting teaching materials to accompany the GeoLeg manipulative tool. GeoLeg began to be the focus in a geometry program that identified critical, vertical objectives and pedagogical methods that would address geometry objectives, while incorporating the use of the GeoLeg tool.

Since the GeoLeg manipulative tool was introduced to the marketplace, districts in all 50 states and throughout Canada have purchased the tool. It is estimated that approximately one million students are currently using this tool to support their geometric learning experiences. No significant educational research has been done on the GeoLeg manipulative tool. Therefore, this research will add to the body of knowledge in mathematics education.

Assumptions and Limitations

Several assumptions were associated with the study. It was assumed that each of the seventh grade teachers would participate in the study as instructed, that the teachers would diligently implement the professional development training regarding the use of the GeoLeg manipulative tool, and that the students in the study had limited or no prior exposure to the GeoLeg manipulative tool. Finally, the study assumed the honesty of participants' answers to the online survey questions.

Several conditions may have manifested during the study that the researcher was unable to control. The researcher had no control over students' home environments, where manipulatives might have been used by parents or guardians to reinforce the students' understanding of the concepts. The generalizability of the findings of this study was limited by the sample size of 317 students from 14, seventh grade classroom sections in a single urban school district in southern Florida, United States.

Additionally, participants in the study were permanently grouped in intact classes where randomization of participants was not possible. However, the researcher randomly chose the particular sections that participated as either a control group or as a treatment group. This methodology was implemented to reduce potential bias. A further limitation was that the study only measured the GeoLeg tool manipulative usage during a three-week unit on geometry. This was a single group study based on students in seventh grade math classes for a period of one month.

Delimitations

Three possible delimitations may impact the findings of this study:

1. This study focused on use of traditional geometry tools (compass, ruler, and protractor) versus the use of the GeoLeg tool to measure academic achievement in seventh grade geometry. However, no other manipulative tools were considered.

2. Although there are many aspects of geometry, only a portion of the curriculum was examined. Therefore, more study is necessary to examine the effectiveness of the GeoLeg manipulative tool with the entire scope and sequence of the curriculum.

3. This study focused on seventh grade students. In order to gain a broader picture of the effectiveness of this tool, it would be necessary to conduct further study with students in other grade levels.

Nature of the Study

This was a mixed-methods study that attempted to determine the significance of the GeoLeg manipulative as a learning and teaching tool. Fourteen seventh grade classes with approximate 20 students in each section participated in the study. Ten of the classes used the GeoLeg manipulative tool while learning geometric concepts (treatment group) and four classes (control group) used traditional tools while completing the exact same activities. Each class met daily for a 55 minute period. The geometry unit was taught for 15 school days.

Teachers took students to a computer lab where the students completed the Honey Mumford Learning Style Questionnaire on-line (Honey & Mumford, 1995). Field notes were taken, coded, and analyzed for themes. At the completion of the unit, all participating teachers were interviewed. Additionally, randomly chosen students from both the treatment (seven students) and control groups (seven students) were interviewed at the conclusion of the unit of instruction.

Summary of the Chapter

Chapter 1 presented the problem to be investigated and noted areas of concern to be analyzed. Also included were the assumptions of the study, limitations of the study, and definition of terms. The remainder of this report will be divided as follows: Chapter 2 of this report will discuss literature relevant to the impact and influence of manipulatives on student achievement as well as learning styles and how they influence student achievement. Chapter 3 will describe and discuss the research methodology used, including treatment and control groups, and data collection procedures. Chapter 4 will present and analyze the data collected using the methodology described in Chapter 3. Chapter 5 will present a summary of the conclusions and implications drawn from the information presented in Chapter 4. It will also present recommendations for future research studies.

Chapter 2: Review of the Literature

This study's research questions determined the focus of this literature review. This review provides background information related to this study of mathematics manipulatives and their possible relationship to academic achievement. The findings of educational philosophers and researchers regarding various issues related to the use and effect of manipulatives have been examined. A historical perspective of current educational policies is provided in this chapter in order to gain a deeper perspective into the issue of manipulative use in the classroom.

The overarching research question is "Do seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on a posttest than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum?" This chapter provides a look at current research related to how the usage of geometry manipulatives affects academic achievement in geometry.

Specifically, the proposed study will investigate whether or not seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on the district approved geometry curriculum tests than those students who do not use the GeoLeg manipulative while learning geometry concepts with the same curriculum. As a secondary investigation, this study will explore whether there is any relationship between how students perform on a geometry unit posttest and their gender, and whether there is any relationship between student-identified learning styles on the Honey and Mumford Learning Styles Questionnaire and student achievement on a geometry posttest. This literature review will focus on three components: manipulatives in mathematics and learning, constructivism, and learning styles as related to achievement in mathematics.

National and State of Florida Mathematics Achievement

The National Assessment of Educational Progress (2009) is known as "The Nation's Report Card." NAEP's two major goals are to measure and compare student achievement in each of the 50 states and jurisdictions. NAEP reports changes in achievement over time for student enrolled in 4^{th}, 8^{th}, and 10^{th} grades. The *No Child Left Behind Act of 2001* provided a new context for NAEP. Section 11(c)(d) of NCLB requires states/districts that receive Title I funding to participate in biennial State NAEP assessments for 4^{th} and 8^{th} graders in Reading and Mathematics, beginning with the 2002-2003 school year. Florida State Statute 1008.22(2) Student Assessment Program for Public Schools (2008), states that "the Commissioner of Education shall direct Florida School Districts to participate in the administration of NAEP both for the national sample and for any state-by-state comparison programs" (*The 2009 Florida Statutes* retrieved November 20, 2009 from www.leg.state.fl.us/statutes/index.cfm?App_mode=Display_Statute&Search_String=&URL=Ch1008/Sec22.HTM).

The NAEP mathematics framework requires testing in number properties and operation, measurement, geometry, data analysis, probability, and algebra. Students are graded in mathematics on a scale from 0 to 500. The national trend for United States students has been an increase in mathematical achievement since 1990 (NAEP, 2009).

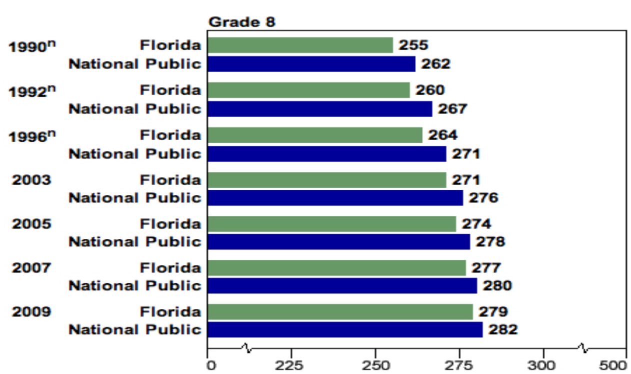
Figure 2 shows that the national average for 4th grade students has improved from 220 in 1992 to 240 in 2009 (a 20-point gain), while 8th national student scores have improved from 263 to 283 (a 20-point gain).

Figure 2. National trends in 4th and 8th grade NAEP mathematics average scores

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), various years, 1990–2009 Mathematics Assessments.

During this same period of time, Florida 4th grade student scores have improved from a score of 214 to 242 (a 28-point gain), and 8th grade students have improved from 260 to 279 (a 19-point gain).

Figure 3. Scale scores for math, Florida vs. national public



n Accommodations were not permitted for this assessment
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), various years, 1990–2009 Mathematics Assessments.

When examining 2009 average gender gap scores between selected groups, male students in Florida had an average score that was not significantly different from that of female students. This performance gap was not significantly different from that in 1990 (3 points).

Figure 4. Trends in 8th grade NAEP mathematics average scores and score gaps, by gender

* Significantly different (p < .05) from 2009. NOTE: Score gaps are calculated based on differences between unrounded average scores. Score differences were not found to be statistically significant in 1990, 1992, 1996, and 2000. Score gaps reflect the average scores for male students minus the scores for female students.

Learning Theories

An examination of learning theories of teaching mathematics is the cornerstone to understand how students learn mathematical concepts. This study will be framed by a discussion of constructivism. It will include theories of two of the most prominent constructivist theorists: Piaget and Bruner.

Constructivism. A Constructivist approach to teaching mathematics is a radical departure from the traditional model of teaching that has dominated American education. Rather than passively sitting and being the recipients of knowledge dispensed by a teacher, students are active participants in the acquisition of their knowledge. Students bring experiences into the learning environment. The students build upon these experiences by connecting new experiences with their life experiences. Students are then able to assimilate their experience to construct their understanding about theories and concepts. Ultimately, the experience for the student is an intimate relationship among their experiences, the meanings that they extrapolate, and the language they use to describe the connections. The ultimate goal of constructivism is for the student to be an active participant in his or her own learning experience.

During the first half of the 20th Century, William Brownell (1935) advanced ideas that became forerunners of constructivism. Brownell envisioned mathematics as a closely-knit system of ideas, principles, and processes. Brownell believed that "arithmetic should be less a challenge to a pupil's memory and more a challenge to his intelligence" (p. 20). Traditional teaching of mathematics has at its core memorization of facts and practicing algorithms to develop abstract thought through repetition. Although this model may show results in greater student's knowledge and achievement, researchers such as Gardner (1987) hold the position that most students learn through experiences such as thinking, feeling, doing, and watching.

The result of Kolb's (1986) research indicates that students tend to have greater success when they learn in a concrete manner. Kolb contends that when students have concrete experiences, new knowledge is introduced cogently. As the students rely on their experiences, the students will continue to make connections from one experience to another and thereby build upon their knowledge base. Students can then more easily transition from the concrete representation of mathematics to the symbolic stage. Kolb has extended his original research to explore different ways in which students learn. His work on learning styles has been used by many researchers and institutions including Polytech of Hong Kong, and Honey and Mumford in Great Britain.

Cobb, Wood, and Yackel (1992) describe the constructivism as an active, constructive process in which students solve and resolve problems as they actively participate in mathematical experiences in the classroom. In addition to Brownell (1935), Jean Piaget (1954), Jerome Bruner (1966), Carpenter et al. (1994), and Cobb (2003, 2004) have each contributed significantly to the idea that a constructivist style of teaching enhances student achievement.

Many of the recommendations for teaching mathematics advocated by the National Council of Teachers of Mathematics (NCTM) in the *Professional Standards for Teaching Mathematics* (NCTM, 1991) and *Principles and Standards for School Mathematics* (NCTM, 2000) are based on popular theories of how children learn mathematics. Both of these documents provide a strong support for changing from a traditional approach to teaching mathematics to constructivist approach. For example, the National Research Council (1989) states, “educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding” (p. 164).

What does it mean for students to construct their own mathematical understanding? This concept has different meanings to different researchers and theorists. However, for the purposes of this research, the following two basic viewpoints of constructivism will be the basis of discussion.

1. Knowledge is not passively received. Knowledge is actively created or invented (constructed) by students. Piaget (1973) suggests that children construct mathematical understanding.
2. Learning reflects a social process in which children engage in self-reflection and dialogue and discussion with others as they develop intellectually (Bruner, 1986). Bruner suggests that students are involved with manipulating materials, discovering patterns, and generating solutions, but they are also sharing their observations, describing the relationships, explaining their procedures, and defending the processes they followed to obtain them.

Piaget. Schoenfeld (1987) refers to Piaget as "the most famous constructivist" (p. 20). Von Glaserfeld (1990) calls Piaget "the greatest pioneer of the constructivist theory of knowing" (p.20). Noddings (1990) describes Piaget's theories as thoroughly constructivist in the sense that they imply that "not only are intellectual processes themselves constructive, but cognitive structures themselves are products of continued construction" (p. 9).

Why is Piaget referred to as a constructivist? In one of his many statements, Piaget (1973) claimed:

> To understand is to discover. A student who achieves a certain knowledge through free investigations and spontaneous effort will later be able to retain it: he will have acquired a methodology that can serve him for the rest of his life, which will stimulate his curiosity without the risk of exhausting it. At the very least, instead of having his memory take priority over his reasoning power. He will learn to make his reason function by himself and will build

> his own ideas freely. The goal of intellectual education is not to know how to repeat or retain ready-made truths. It is in the learning to master the truth by oneself at the risk of losing a lot of time and of going through all of the roundabout ways that are inherent to real activity. (p.106)

According to Copeland (1979), Piaget's definition of knowledge is "a spontaneous process of total development involving the physiological, emotional, and mental systems" (p. 36). For Piaget (1970), knowledge has three forms. The first is instinctive knowledge, which is present in animals and very limited in human beings. The second, physical experience knowledge, is more advanced and depends upon memories such as wood floats and water freezes. The third is logico-mathematical knowledge; this is an extension of the second as it is derived from actions performed on objects rather than the objects themselves. For example, the actions of touching, throwing, pushing, and rearranging may lead to simple abstractions.

Piaget (1973) emphasized that children attempt to create a balance between assimilation and accommodation using a mechanism known as equilibrium. As students have new experiences, they actively try to make sense of the new ideas presented to them in relation to the previous experiences and ideas. Piaget refers to this disequilibrium as a state of conflict between new and old ideas. Piaget contends that the student will reach a state of equilibrium when the conflict between new and old ideas has been resolved. Piaget views this state of disequilibrium as essential in the learning process.

Piaget is perhaps best known for his extensive work regarding the development of children's thinking. He asserted that children go through stages as they develop intellectually. He identified four stages, each representing distinctly different ways of understanding the world. The first stage is the sensorimotor stage of cognitive development. This stage lasts from birth to age 2. During this stage of development, infants and children develop concepts primarily though interactions with the physical world. The second stage (pre-operational stage of cognitive development) begins at around age 2 and ends at about age 7. During this stage of development, children begin using language to express their ideas. This stage is the beginning of symbolic thinking. The third stage is the concrete operational stage, which lasts from age 7 until about age 12. This stage marks the beginning of logico-mathematical thought. It is during this stage of cognitive development that children develop concepts using concrete objects to explore relationships, model abstract ideas, and use language as an integral component for expressing and remembering concepts. Finally, Piaget's last stage is the formal operational stage of cognitive development. This stage represents thinking from about age 12 through adulthood. At this stage, "the child now reasons or hypothesizes with symbols or ideas rather than needing objects in the physical world" (Copeland, 1979, p. 25).

The influence of Piaget's theory of how children learn can be seen in today's mathematics classrooms. "Today mathematics programs are strongly influenced by the constructivist paradigm and cognitive science" (English & Halford, 1995 p. 21). The emphasis can be seen with the importance of hands-on manipulatives used in real-world contexts and in the call for the need to develop students' understanding of mathematical structures.

Bruner. A major theme in the theoretical framework of Bruner is that learning is an active process in which students construct new ideas or concepts based upon their current/past knowledge. The learner selects and transforms information, constructs hypotheses, and makes decisions, relying on a cognitive structure to do so. Cognitive structure provides meaning and organization to experiences and allows the student to "go beyond the information given" (Bruner, 1966, p. 225).

Bruner's view of teacher instruction is that the teacher should try to encourage students to discover principles by themselves. The teacher and student should engage in an active dialogue. The task of the teacher is to translate information to be learned into a format appropriate for the student to understand. Curriculum should be organized in a spiral manner so that the students continually build upon what they have already learned.

Bruner (1966) states that a theory of instruction should address four major aspects: (1) predisposition toward learning; (2) the ways in which a body of knowledge can be structured so that it can be most readily grasped by the learner; (3) the most effective sequences in which to present material; and (4) the nature and pacing of rewards and punishments. Good methods for structuring knowledge should result in simplifying, generating new propositions, and increasing the manipulation of information. Many theorists have adopted Bruner's approach of teaching (Watson, 1993).

Bruner's belief in the constructivist view and his interest in cognitive processes are clearly seen in his four theorems of mathematics learning. The construction theorem states that the best way for children to learn a concept is by allowing them to make their own representations of the concepts through manipulation of materials. The notion theorem states that early construction can be simply understood by the student if the constructions contain ideas that are appropriate for the student's level of intellectual development. The contrast and variation theorem attests that when students move from the concrete to the abstract, processes of contrast and variation are involved. Bruner contends that students need experiences in which different concepts are compared and contrasted to each other. For example, prime numbers can be defined in contrast to composite numbers. Bruner also believes that students need to be made aware of connections between mathematical concepts when learning mathematics. The connectivity theorem emphasizes the need to connect each concept, principle, and skill to other concepts, principles, and skills.

Manipulatives in Mathematics and Learning

Definition of Manipulatives. Manipulative materials are "objects that can be put in place or moved to represent a mathematical situation" (Grossnicle & Reckzeh, 1990, p. 33). For Kennedy and Tipps (1998), manipulative materials include a wide variety of teaching-learning aids that model and demonstrate mathematical concepts and processes. For example, a manipulative can vary from a folded piece of paper that demonstrates a fractional part of a whole to commercially produced components that can be used to measure length, volume, and mass. Papert (1980) refers to manipulatives as "objects-to-think-with" (p. 15). Marshall (1997) defines manipulatives as materials that embody a mathematical concept in such a way that the learner mentally constructs concepts through manipulating materials.

For the purpose of this study, manipulatives will be defined as materials that are concrete and designed to make sense of a mathematical concept (NCTM, 1989). They should assist students in making connections from concrete to abstract concepts.

Use of Manipulatives as Related to Student Achievement. In each decade since 1940, The National Council of Teachers of Mathematics (NCTM) has encouraged the use of manipulatives. In a review of activity-based mathematics learning, Suydam and Higgins (1977) determined that achievement in mathematics increased when students used manipulatives. Sowell (1989) performed a meta-analysis of 60 studies to examine the effectiveness of manipulatives used by students in kindergarten through post-secondary. Sowell determined that manipulatives could be effective in student learning and achievement. However, it was also found that many teachers did not use manipulatives. Sowell found that long-term use of manipulatives was more effective than short-term usage. However, even when manipulatives were used over an extended period of time, teachers' level of training critically influenced the effectiveness of the manipulatives.

Clements (1996) asserts that concrete knowledge can be one of two types. He describes the first as sensory-concrete which is demonstrated when students use sensory material to make sense of an idea. The second is described as integrated concrete - which is built through learning. According to Clements, "Integrated-concrete thinking derives its strength from the combination of many separate ideas in an interconnected structure of knowledge" (p. 59). When students are able to interconnect ideas and concepts to the physical objects and actions performed, students possess a strong cognitive mathematical foundation. This aligns with the constructivist belief that students build their own knowledge; they do not receive knowledge pre-packaged from others (Clements, 1996).

Szendrei (1996) lists four concerns concerning the use of manipulatives: How can teachers learn the proper use of materials? Is the academic instructional time that is invested regained? Does the transfer effect exist and will the knowledge gained by students be effective in real-world situations? The use of manipulatives does not guarantee student understanding of concepts and skills. For Szendrei, mathematics is a value added to the concrete materials. Teachers must possess the organizational skills and understanding of how to properly combine the mathematics with the materials.

In a paper presented at the Annual Meeting of the Midwestern Education Research Association, Sharp (1995) expressed similar concerns regarding the usage of manipulatives. She contended that concrete representations of abstract ideas, such as algebra tiles, wherein students manipulate algebraic expressions and equations with concrete manipulatives, provide the necessary foundation for concept development. She maintained that the use of manipulatives should be accompanied by a specifically developed plan in which the teacher creates opportunities for students to transfer the mathematical concepts from the concrete manipulation to the symbolic representations.

In her article *Magical hopes: Manipulatives and the reform of math education,* Deborah Ball (1992) addressed concerns with concrete objects when teaching mathematics. She stated that teachers cannot assume that students are automatically drawing mathematical conclusions simply because students are using and interacting with manipulatives. Ball contended that for manipulatives to be used effectively, the manipulatives must represent identifiable components of problems. The usage of manipulatives should add to and not distract the student from the intended mathematical learning experience. Ball stated, "Manipulatives alone cannot—and should not—be expected to carry the burden of the many problems we face in improving mathematics education in this country" (p. 47).

Another concern with manipulatives is classroom management. It is imperative that lessons are well planned to help eliminate any behavioral issues before they become a reality in the classroom (Johnson & Norris, 2006). It is suggested that students work in small groups so that the members of the group become the focus of managing the manipulatives. This may facilitate a more timely management of distribution and collection of materials.

Scott (1983) conducted a survey to compare teacher usage and availability of manipulative materials. He surveyed 75 elementary schools in a large school district. Responses were received from 88% of the schools and from over 60% of the teachers. He reported that most teachers used few materials other than textbooks in mathematics instruction and that the majority of the surveyed teachers reported that they did not need more materials. He also found that there was no significant correlation between material usage and achievement at the fifth grade level.

Research findings (Suydam,1986) and theoretical considerations (Hiebert, 1984, 1988) have supported the use of manipulatives in classroom instruction. For example, when physical three-dimensional objects are available, students experience mathematics as a process that demands thought, creativity, and understanding (Davis, 1984). This concept is in sharp contrast to the findings of limited learning that occurs when students participate in seatwork on algorithms or procedures (Carpenter & Lehrer, 1999).

Raphael and Wahlstrom's (1989) study of 103 Ontario eighth grade mathematics teachers surveyed from a random sample consisting of 120 private, public, and parochial schools. Students were tested at both the beginning and at the end of the school year. The teachers completed questionnaires that provided the researchers with detailed information about their backgrounds and teaching methods. Researchers reported in their findings that the more experienced teachers might have had more valid grounds for their selection of instructional aids and tools than the less experienced teachers. The researchers also reported that the effective use of instructional aides resulted in increased student achievement. Additionally, they found that greater use of manipulatives was related to more comprehensive topic coverage. They found that the teachers who used the manipulatives were more likely to explore topics more thoroughly. Specifically in geometry, Raphael and Wahlstrom (1989) found that manipulative use was directly correlated to higher achievement when used in combination with in-depth curriculum coverage.

The same results were found by Beattys (1986), who carried out research with students in grades 5 through 7. She divided students into groups: One group used manipulatives to learn measurement while the other group used a traditional textbook approach of instruction. When the pretest and posttest of this research was compared and analyzed, it was determined by Beattys that the students who used the manipulatives were able to solve problems on area measurement that they were unable to solve prior to the use of manipulatives. This finding indicated that the use of manipulatives at least contributed to an increased level of student achievement.

Chester et al. (1991) used a non-equivalent pretest-posttest control group design to investigate the effects of a teaching method that emphasizes the use of manipulatives in the mathematics achievement of third grade students. Two third-grade classes with 26 students in each class participated in the study. Both classes were taught a geometry unit from the Silver Burdett textbook for a period of 2 weeks. The teacher used mathematics manipulatives with the experimental group in order to teach the concepts included in the unit of study. The teacher used only drawings and diagrams with the control group to teach the same concepts to the control group. Chester et al. claimed that an analysis of covariance showed that the experimental group using manipulatives scored significantly higher in mathematics achievement on the posttest of the control group.

Howley and Boren (1993) provided a description of the development and evaluation of mathematics activities manuals for grades K-8. According to the authors, 17 elementary teachers and 7 secondary teachers wrote draft revisions of the manuals and demonstrated activity-based mathematics and manipulatives during the first half of 1989. Howley and Boren reported that initially 113 teachers from the Tennessee school systems completed the pilot program during the 1989-1990 school year. Later, the study culminated with more than 4,000 teachers receiving in-service training.

After completing a statistical analysis that examined the changes in student achievement and effective response to mathematics, a 12-item instrument was used to measure students' attitudes towards mathematics and the Comprehensive Test of Basic Skills was administered to the students. According to Howley and Boren (1993), their findings indicated that the use of mathematics activities (including those that involved the use of manipulatives) assisted in improving student achievement and created a positive effect on participant attitude towards mathematics as well as reduced the negative influences of gender (principally female).

Raymond and Leinenbach (2000) conducted a 2-year collaborative action-research study to investigate the effects of the use of "Hands-On Equations" mathematics manipulatives. They studied five classes of inner-city middle school students enrolled in eighth grade pre-algebra. They studied students' confidence, interest, and ability to solve and retain understanding of algebraic equations. During the first phase of the study, researchers documented and compared 120 urban students' reactions and achievement during both manipulative and textbook approaches to learning algebra in their eighth grade classrooms. During the second phase of the research, the researchers examined surveys from 19 of the original participants, who were then in the ninth grade in high school, and conducted one-on-one interviews with eight of those students.

According to Raymond and Leinenbach (2000), data from the surveys, interviews, student reflections, work samples, and test scores suggest that most of the eighth grade students performed better academically and expressed more positive attitudes about algebra when working with manipulatives as opposed to working only with the textbook. Interestingly, this result occurred at the same time as students performing satisfactorily on standardized algebra testing, which exceeded the expectations of teachers and administrators.

Witzel, Mercer, and Miller (2003) conducted a study of 34 matched pairs of sixth and seventh graders "in a comparison of an explicit concrete-to-representational-to-abstract (CRA) sequence of instruction with traditional instruction for teaching algebraic transformation equations" (p. 121). Those students who were identified as learning disabled or at risk for algebra difficulties were mainstreamed into regular classrooms. The students who were in the CRA group used manipulatives as part of the regular instructional practice. The CRA students outperformed those instructed with traditional abstract methods on both the posttests and the follow-up tests. The data analysis revealed that student errors matched the type of instruction they received. The treatment group demonstrated a significant improvement in the ability of student to solve single variable and multi-step equations.

Olkun's (2003) study compared computer virtual manipulatives to concrete manipulatives during the instruction of two-dimensional geometry. The participants in the study were 93 fourth and fifth grade students. A pretest and posttest experimental design were employed. The pretest and posttest were paper and pencil tests designed by the researcher and consisted of 24 two-dimensional geometry questions. The treatment groups consisted of students using the computer virtual manipulatives, concrete manipulatives and the control group. All groups solved problems involving tangrams. The findings showed significant academic achievement in the groups using the virtual manipulatives and the concrete manipulatives. The fourth graders made greater gains with the concrete manipulatives while the fifth grade students gained more with the virtual manipulatives.

Research Related Specifically to the GeoLeg Tool

The Oklahoma School Testing Program evaluated four school districts (Shawnee, Tecumseh, Bowlegs, and Bethel) that participated in a 2-year project entitled "Oklahoma Einstein Project" (Paul, 2006). The project focused on 1522 students and 60 teachers in grades 3, 4 and 5, including special education teachers who used GeoLeg Mathematics during the 2004 and 2005 school years. Paul used the Evaluation of Academic Performance Index (API) to evaluate student performance.

From the Arkansas River Education Service Cooperative (2006), Paul reported in an unpublished document that during the 2004-2005 school year, between 47% and 74% of 3rd grade students scored at a proficient level on the Oklahoma's School Testing

Program. After one year of instruction (2005-2006) in mathematics using GeoLegs, students scored between 55% and 87% on the Oklahoma's School Testing Program. This score increased from 2% to 13% on the API after using GeoLegs in their mathematics instruction.

For 4th grade students, Paul reported that 62% to 76% scored as proficient on the Oklahoma's School Testing Program (2006). After 1 year of instruction (2005-2006) in mathematics using GeoLegs, students scored between 72% and 80%. Paul reported the results as an increase of between 4% and 11% on the API after using GeoLegs in their mathematics instruction.

For the students in the 5th grade, Paul reported that 41% to 88% of the students scored as proficient on the Oklahoma's School Testing Program (2006). After 1 year of instruction (2005-2006) in mathematics using GeoLegs, students scored between 48% and 92%. Paul reported that this was an increase of between 4% and 29% on the API after using GeoLegs in their mathematics instruction.

Paul (2006) reported that the variance between 2% to 29% was due to the implementation of the GeoLeg program. Paul's evaluation concluded that teachers who fully implemented the GeoLeg program in their mathematics classrooms showed an average student gain of at least 12% (2006). Additionally, Paul also found that teachers who did not fully implement the GeoLeg program showed an average student gain of a maximum of 3% to 4%. Paul concluded that the student usage of the GeoLeg tool was a primary contributor to the improved mathematics scores, because it was the major change in the mathematics curriculum within the schools during the 2005-2006 school year. Paul concluded that the GeoLeg mathematics program enhanced elementary teachers' content knowledge, improved instructional strategies, was favorably perceived by teachers, and provided an essential learning tool for students whose learning styles included a tactile preference. The Oklahoma Shawnee School District was able to make Adequate Yearly Progress (AYP) following the 2006 state testing, partly because of the increase in the students' mathematics test scores. As a result of this accomplishment, Oklahoma's Shawnee School District was removed from the "District in Need of Improvement" category because of its AYP achievement.

Summary as Related to Manipulatives

A review of literature in the use of manipulatives in mathematics instruction and the relationship to student achievement is mixed. Researchers found that the use of manipulatives has varied results, ranging from no impact on student achievement to a significant impact on student achievement. The overall findings of the studies reviewed are antithetical and suggest that the use of manipulatives does not seem to be detrimental to the learning of mathematical concepts. Most of the findings support the theoretical foundation that mathematics can and should be taught experientially. This methodology may or may not provide immediate improvement in student achievement. However, the results in student achievement using the GeoLeg manipulative tool are impressive and warrant further study.

Student Achievement Related to Gender

Studies comparing mathematical achievement and gender have yielded mixed results. In cases where gender differences in mathematics are found to exist, the magnitude of these differences correlates with problem difficulty (Bielinski & Davison, 2001; Penner, 2003) and appear more pronounced among groups of low- or high-scoring individuals (Swiatek, Lupkowski-Shoplik, & O'Donoghue , 2000). According to Penner (2003), boys have a slight advantage on easy math problems, and a greater advantage on more difficult math problems, with problem difficulty defined by the percentage of incorrect answers across the population (i.e., easy problems are those with a low percentage of incorrect answers).

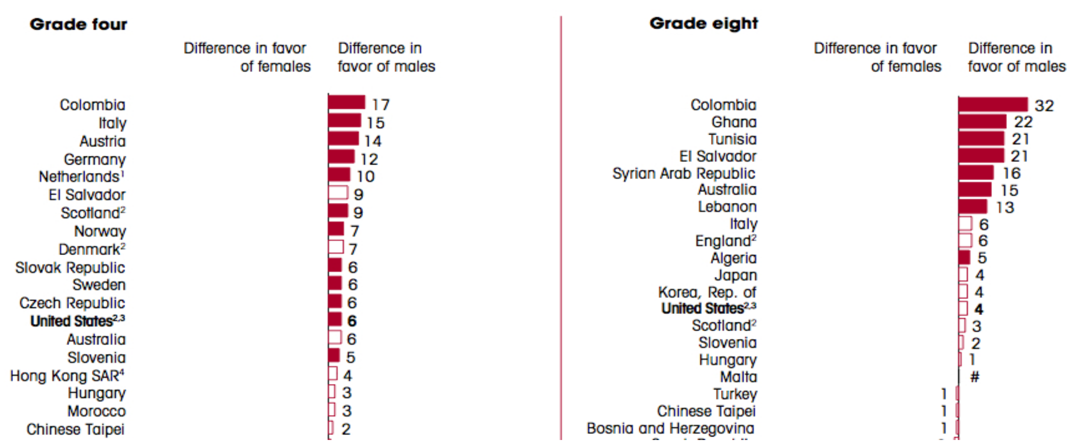
One consistency in the literature is the claim that the magnitude of differences varies with age (Hyde, Fennema, & Lamon, 1990; Leahey & Guo, 2001; Voyer D., Voyer S., & Bryden, 1995). For example, true gender difference findings are nonexistent in infancy, minimal in early and later elementary school years, emerging at adolescence, and more consistently reported through adulthood. Studies of infants' mathematical skills typically fail to show sex differences in early skills, such as the ability to discriminate between small sets of numbers (Antell & Keating, 1983; Starkey, Spelke, & Gelman, 1990; Strauss & Curtis, 1981), and in how quantity is affected by addition and subtraction (Starkey, 1992). Few or no sex differences in quantifiable math abilities (i.e., counting skills, conceptual knowledge, or simple arithmetic skills) have been found before first grade (Lummis & Stevenson, 1990). Four- and five-year old girls outperform boys, to minimal degrees, on simple arithmetic skills (Ginsburg & Russell, 1981), but in general there is no clear advantage of one sex over the other at this age.

During the early school years, reports of gender differences in mathematics continue to be inconsistent. Differences are either reported as nonexistent (Geary, 1994), slightly favoring girls (Ginsburg & Russell, 1981), or favoring boys on specific tasks (Lummis & Stevenson, 1990). Research findings on the late elementary and middle school years are inconsistent.

The Trends in International Mathematics and Science Study (1996) is an international comparison of student mathematics and science knowledge and skills of fourth- and eighth-graders. This comparison study has been administered since 1995. TIMSS is designed to align broadly with mathematics and science curricula in the participating countries.

In 2007, U.S. fourth-grade males outperformed females by 6 score points on average in mathematics (Figure 5). In addition to the United States, of the 35 other countries participating at grade four, 20 showed a significant difference in the average mathematics scores of males and females: 12 in favor of males and 8 in favor of females. The difference in average scores between males and females ranged from 37 score points in Kuwait (in favor of females) to 17 score points in Colombia (in favor of males).

Figure 5. Differences in average mathematics scores of fourth and eighth grade students, by sex and country: TIMMS 2007



Male-female difference in average mathematics scores favors males and is statistically significant ($p < .05$)
Male-female difference in average mathematics scores is not measurably different ($p < .05$)
Male-female difference in average mathematics scores favors females and is statistically significant ($p < .05$)

Rounds to zero.
1Nearly satisfied guidelines for sample participation rates only after substitute schools were included.
2Met guidelines for sample participation rates only after substitute schools were included.
3National Defined Population covers 90 percent to 95 percent of National Target Population.
4Hong Kong is a Special Administrative Region (SAR) of the People's Republic of China.

In a longitudinal study of over 7000 children, Marshall & Smith (1987) found no sex differences in third to sixth graders' basic conceptual understanding on arithmetic problems, although the girls outperformed boys, due primarily to boys' procedural errors. Yet, between elementary and junior high school years, boys surpass girls in their math skills (Leahey & Guo, 2001; Maccoby & Jacklin, 1974).

In a 2008 study out of the University of Wisconsin-Madison, Hyde, Lindberg, Ellis, and Williams stated, "there just are not gender differences anymore in math performance" (p. 260). Hyde et al. investigated SAT results and mathematics scores from 7 million students. The researchers examined average performance, scores of the most gifted, and the ability to solve complex mathematical problems, gender, grade level, and ethnicity in 10 states. The findings of this study revealed that there was no statistical difference between male and female performance on the SAT.

In a longitudinal study analyzing mathematical achievement, math ability, and math-related tasks, 200 primary school children were studied from Kindergarten through the third grade (LaChance & Mazzocco, 2005). The findings support the idea that gender in mathematics has a minimal or nonexistent effect on standardized psychometric tests routinely given in assessments of primary school age children. Their research indicated that there was no gender advantage in math performance overall, during any single year of the study, or in any one area of math or spatial skills. Growth rates for all skills were comparable for boys and girls. The findings fail to support either persistent or emerging gender differences for students during their primary school age years.

Learning Styles

Being aware of how students learn and attempting to utilize instructional methods and strategies that maximize pedagogical skills to promote student learning has been in existence since as far back as Hippocrates, c.370BC. Credited as the "father of medicine," he founded the Hippocratic School of Medicine. Hippocrates identified Four Temperaments of learners (cheerful, somber, enthusiastic, and calm).

Even at this early date, researchers have attempted to explain and categorize the different manners in which people learn, attain, and retain information and concepts. "Cognitive styles," or "learning styles" have been defined as "self-consistent, enduring individual differences in cognitive organization and functioning" (Ausubel, Hanesian, Hanesian, Novak, & Novak, 1978, p. 203); a "composite of characteristic cognitive, affective, and physiological factors that serve as relatively stable indicators of how a learner perceives, interacts with, and responds to the learning environment" (Keefe, 1982, p. 44); "distinctive behaviors which serve as indicators of how a person learns from and adapts to his environment" (Gregore, 1979, p. 234); and "educational conditions under which a student is most likely to learn" (Stewart & Felicetti, 1992, p. 64).

Theoretical frameworks have included the categorization of learning styles as concrete-abstract and random-sequential (Gregore, 1979) and field-dependent/global or field/independent/analytic (Ramírez, Castañeda, & Castañeda, 1974; Witkin, 1976). Myers (1987) delineated 16 personality types, each with implications for learning. Dunn and Dunn (1979) promoted four groups of elements in which they believe learners have distinct preferences when learning: environmental, emotional, sociological, and physical. Barbe and Milone (1981) focused on modality strengths rather than modality preferences.

Witkin. Witkin, an American psychologist with interests in cognitive psychology and learning psychology, developed the Embedded Figures Test in the late 1960s. This test required individuals to find a simple figure embedded within a complex design. The individual responses determined “field dependence” or “field independence.” A field dependent person has difficulty finding the embedded or “hidden” geometric shapes. Generally, those who are field-dependent perceive things as a whole, think globally, make broad general distinctions among concepts, and learn material and concepts in a social context. In contrast, a person who is field-independent has the ability to readily identify embedded shapes regardless of the embedded background. Those who are field independent perceive things in parts rather than as a whole unit. They prefer autonomy when working in a social setting and are often perceived as task-oriented and analytical thinkers.

Gregore. Gregore (1979) is well known for his theory on Mind Styles Model and is associated with their Style Delineator. The Gregore Style Delineator is a self-scoring instrument that elicits responses to a set of 40 specific words. The responses are given a value for a model that is set within two axes – abstract as related to concrete and sequential as related to random. These two sets of dualities result in the participant being categorized in one of four learning preferences within the following model (Gregore, 1979).

According to Gregore, the abstract sequential learner is easily able to decode written, verbal, and image symbols. For this type of learner, symbols and pictures are important. They prefer presentations that are rational, substantive, and well organized.

According to Gregore, the abstract random learner is skilled in sensing and interpreting the atmosphere and mood of a situation. This learner associates the medium with the message. The presenter's manner, mood, delivery, and personality are as important as the message itself. The information is gathered in an unstructured manner, reflected upon, and then organized in a manner that best makes sense to the learner.

As referenced by Gregore, the concrete sequential learner prefers hands-on experiences that use all five senses. It is important for this learner to have step-by-step instruction that is well organized and he or she will defer to the presenter of guidance and direction in the learning environment.

Gregores final category is the concrete random learner. This type of learner prefers to learn through experimentation and uses intuition when drawing conclusions. They prefer trial and error when gathering information and prefer autonomy rather than teacher intervention.

Gregore (1979) proposed that both major and minor learning style tendencies emerge from innate predispositions and that those styles a student has as a minor proclivity should be developed. He feels that the most successful students are those whose learning style matches that of the teacher (Gregore, 1979). In a study of learning styles among a group of community college students, O'Brien and Thompson (1994) found that this claim was only true for those students whose learning style was categorized as concrete random.

Myers and Briggs. The Myers-Briggs Type Indicator (MBTI) emanated from the theories of Carl Jung in 1921 and is a psychometric questionnaire assessment designed to measure psychological preferences. These dichotomies are divided into four pairs according to how people view their environment (sensing vs. intuition), make decisions (thinking vs. feeling), focus on ideas and concepts on the outer world of people and things (introvert vs. extrovert), and respond to situations with acceptance or a judgmental attitude (perception vs. judgment). The sixteen possible combination types are as follows (Myers, 1987):

ISTJ Serious, quiet, thoughtful

ISFJ Quiet, friendly, responsible and conscientious

ISTP Quiet, reserved, observes life with curiosity and humor

ISFP Retiring, quietly friendly, sensitive, kind and modest

INFJ Succeeds through perseverance, originality, desire to do what is needed

INTJ Original mind, great drive for own ideas and purpose, independent

INFP Enthusiastic, loyal and often absorbed in projects

INTP Quiet, reserved, enjoys problem solving

ESTP Adaptable, tolerant, conservative, problem-solver

ESFP Outgoing, easygoing, accepting and friendly

ESTJ Practical and realistic

ESFJ Warm-hearted, talkative, popular, conscientious and active

ENFP Warm, enthusiastic, high-spirited, ingenious and imaginative

ENTP Quick, ingenious, alert and outspoken

ENFJ Responsive, considerate of others, and sociable

ENTJ Decisive, confident, well informed and frank, has leadership qualities

This instrument has been used for more than 50 years, primarily in career counseling, employee training, marketing, leadership training, life coaching, and personal development. Since this instrument is intended for use by adults, most educational research involving MBTI focuses on teacher and pre-service teacher behavior.

Dunn and Dunn. The Dunns (1979) have been actively promoting learning-style instruction for more than 30 years. Their Learning Style Inventory (LSI) is an instrument used to identify student preferences. This instruments measures 21 elements in 5 categories:

1. Environmental: sound, light, temperature, and design
2. Emotional: motivation, persistence, responsibility, and structure
3. Sociological: self, pair peers, team, adult, and varied
4. Physical: perceptual, intake, time, and mobility
5. Psychological: global/analytic, hemispheric, impulsive/reflective (Dunn & Dunn, 1979; Dunn, Dunn, & Perrin, 1994)

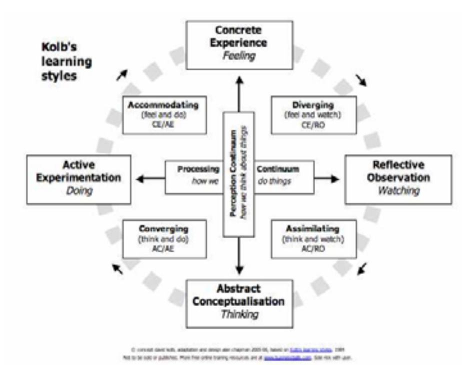
Dunn and Dunn contend that teachers do not necessarily teach as they were instructed to teach, but rather in the manner in which they learn. They believe that modifications to teaching styles are often difficult. However, Dunn and Dunn promote the idea that elements of teaching style can be adapted to student preferences, including room arrangement and design, teaching environment, teaching methods, and evaluation techniques.

According to learning-style theory, learners' cognitive, affective, and physiological patterns determine their academic outcomes. These patterns are relatively stable indicators of how individuals perceive, interact with, and respond to their instructional environment. Research conducted over more than three decades shows that understanding the multi-dimensional aspects of learning is one of the few known ways of helping learners to improve their capacity to concentrate, process information, and remember new and difficult academic information (Dunn & Dunn, 1979; Dunn, Dunn, & Perrin, 1994).

Kolb. Kolb published his learning styles model in 1984, which gave rise to such terms as Kolb's experimental learning theory (ELT) and learning styles inventory (LSI). Kolb (1986) acknowledged and built upon the earlier works of Piaget, Rogers, and Jung. Kolb's theory describes four distinct learning styles based upon a four-stage learning cycle. Kolb's theory (1986) is unique in that it affords a method with which to understand individual learning styles while offering an explanation of a cycle of experimental learning that applies universally.

This "cycle of learning" is the four stages on which concrete experiences provide a basis for observations and reflections. These observations and reflections are then processed into abstract concepts that produce new implications for action that can then be tested and treated as a new experience.

Figure 6. Kolb's learning style model



Kolb's model, therefore, has two levels in a four-stage cycle:

1. Concrete Experience – feeling

2. Reflective Observation – watching

3. Abstract Conceptualization – thinking

4. Active Experimentation – doing

It also has four types of learning styles (Diverging, Assimilating, Converging, and Accommodating). Kolb's learning styles could be read along two continuums – an east-west axis called the Processing Continuum (how a task is approached) and the north-south axis called the Perception Continuum (the emotional response to a task).

Honey and Mumford. Honey and Mumford developed their learning styles system as a variation on the Kolb Model during the 1970s. Honey and Mumford credit Kolb's theory as the basis of their work. However, their Learning Style Questionnaire (LSQ) is a less direct approach than that of Kolb. Rather than asking people how they learn, Honey and Mumford's questionnaire presented questions that probed general behavioral tendencies. The reasoning behind this method was driven by the researchers' assumption that most people do not consciously think about how they learn.

While very similar to Kolb's Model, they substituted the words "reflector" for reflective observation, "theorist" for abstract conceptualization, "pragmatist" for concrete experience, and "activist" for active participation. Additionally, these new labels have slightly different meaning than those proposed by Kolb. Honey and Mumford also contended that learning styles are on a continuum and that individuals prefer different learning styles depending upon the situation they are presented, rather than being dominated by one particular style in all situations.

Honey and Mumford's learning cycle also differs slightly from Kolb in that they contended that a person will have an experience, reflect upon it, draw their own conclusions (theorize), and then put their theory into practice to see what happens. Based upon the results of their actions, a person can move through the cycle again, enter any part of the cycle, and then quit the cycle whenever they feel that they have learned a task or material.

1. Reflector – Learners prefer to learn from activities that allow them to watch, think, and review what is happening. A learner who is a reflector prefers brainstorming. Lectures are helpful if they provide expert explanations and analysis.

2. Theorist – The learner prefers to think problems through in a step-by-step method. These learners prefer lectures, analogies, models, and readings.

3. Pragmatist – The learner prefers to apply new learning to actual practice to see the application of knowledge. These learners prefer lab work, field experience, and observations.

4. Activist – The learner prefers the challenges of new experiences. They prefer involvement with others, and role-playing. This type of learner prefers new experiences such as problem solving and group discussions. (Honey and Mumford, 1992).

Learning Styles and Student Achievement

The idea that people may have different abilities to learn, depending on the modality of instruction, has been around for more than 100 years (Willingham, 2005). When applied to an educational setting, the fundamental nature of this viewpoint is that students learn differently, so that instruction should incorporate multiple techniques in order for all learning styles to be included. Thus, if multiple learning styles are in the instruction, then the likelihood of all students learning the desired concepts is presumed to increase. If this logic were to be substantiated by research, then it would be especially helpful in today's classroom that face the many pressures of performance on national and state standardized tests. However, the results of research concerning learning styles is mixed.

Fine's (2003) study compared the learning styles of high school students in special education with those in regular education. There were 214 regular education students and 208 special education students in the study. In the study, Fine identified the learning styles of all participants and then compared the student test scores with the identified learning styles. When examining the short-term achievement of students, Fine found that the total mean posttest scores were significantly better when taught through learning-style approaches than with traditional instructional methods. Overall, Fine reported that the achievement of previously underachieving and failing special education students improved significantly when students were taught using their learning style strengths.

Hillis (2009) investigated the effects of using Learning Profile Assessments (LPAs) to differentiate instruction for elementary students. The study included six groups of fourth and fifth grade students. Students in the experimental group were given LPA's to determine learning styles and interests of students. The teachers then used this information to differentiate their instruction. This study examined whether or not the students who received instruction specifically tailored to learning styles performed better on posttest assessments. The results of the study revealed that there was no significant difference on student achievement between the two groups.

Yushau's (2005) study examined 120 students sampled from the population of students enrolled in the second pre-calculus algebra course at the preparatory year program of King Fahd University of Petroleum and Minerals during the 2003/2004 academic session. The study examined the extent to which selected variables (mathematics attitude, mathematics aptitude, computer attitude, computer prior experience, computer ownership, proficiency in language of instruction, and learning style) contribute to students' achievements in pre-calculus algebra classes that are supplemented with a computer lab program.

The instruments used were the mathematics attitude scale (Aiken, 1979), the computer attitudes scale (Lloyd & Gressard, 1984), and the learning styles questionnaire (Honey & Mumford, 1992). The results of the study were that mathematics aptitudes and English language proficiency were the most significant contributors to students' mathematics achievement. Yushua (2009) found that no other variables revealed statistically significant effects on students' achievement.

In a relational study examining learning styles and teaching performance at the University of Missouri, Garton, Spain, Lamberson, and Spiers (1999) studied college students enrolled in their first semester as Animal Science majors. The study contained 187 students. The purpose of the study was to describe relationships between students, learning styles, instructor's teaching performance, and student achievement in an introductory animal science course. The Group Embedded Figures Test (GEFT) was used to assess students preferred learning styles. Student achievement in the course was found to have a low positive relationship with their preferred way of learning. The diversity of learning styles was found to have little to no influence on student achievement in the course or their perceptions of the instructors' teaching performance.

In a study that examined the learning style preferences of 44 second-year Japanese college students pursuing an undergraduate degree and learning English as a Second Language at a New Zealand college, Thomas, Cox and Kojima (2000) found no relationship between learning style and student achievement. The study examined learning styles and achievement of this unique group of students. Achievement was measured by grades as well results on the Test of English for International Communication (TOEIC). Learning style preferences were measured on the Perceptual Learning Style (PSLP) and the Style Analysis Survey (SAS). The results of the study showed that there was not a statistically significant relationship between learning style and TOEIC scores.

Kopsovich's (2001) doctoral study from the University of North Texas examined whether learning styles of students affect their math achievement scores on the Texas Assessment of Academic Skills Test. The study included 500 fifth grade students attending a North Texas Intermediate school. Learning styles were determined by student responses to the Learning Style Inventory by Dunn, Dunn, and Price. Kopsovich concluded that a correlation existed between fifth grade students learning styles and their standardized math test scores for all students, with the learning style preference of a high level of persistence to complete difficult learning task or assignment and their math achievement scores.

Summary of Learning Styles

Many researchers have debated the interpretation, usefulness, and delineation of learning styles. Researchers have promoted elements such as environmental, emotional, sociological, physical, logical thinking, abstract thinking, psychological preferences, concrete representations, active learning, field work, and role playing (Dunn & Dunn, 1979; Gregore, 1979; Honey & Mumford, 1992; Kolb, 1986; Myers, 1987; Ramirez, Castañeda, & Castañeda, 1974; Witkin, 1976). It is agreed that students learn in different manners and that instruction should be differentiated in order to equitably reach the learning potential of all learners. However, research results examining learning styles and their relationship to student achievement have had mixed results.

Summary of the Chapter

The purpose of a literature review is to gather accumulated knowledge from the past as a portal for directing future research. The theoretical foundation that has been presented will guide this research as well as determine the statistical relationships investigated.

The reviews presented have established the foundation for the use of mathematical manipulatives when teaching mathematics. Though findings concerning the use of manipulatives have been mixed, a substantial body of knowledge confirms the idea that the use of manipulative materials enhances student achievement. Paul's (2006) findings are particularly significant to this study of the GeoLeg manipulative tool and whether or not its use will enhance student achievement in students in the middle grades.

Chapter 3: Methods and Procedures

The information contained in this chapter highlights the methodology used to answer the research questions posed in this study as well as the methods used to test the hypotheses. Once the procedures were approved, a written application was submitted to the Institutional Review Board at Florida Atlantic University. Upon IRB approval, the study was initiated. This chapter includes sections devoted to this study's (a) Purpose, (b) Research Design, (c) Criterion and Predictor Variables, (d) Research Questions, (e) Hypothesis, (f) Setting and Participants, (g) Procedure, (h) Data Collection, and (i) Data Analysis.

Purpose of This Study

This study focused on the relative effectiveness of using the GeoLeg mathematical manipulative tool with seventh grade students in a geometry unit. More specifically, this treatment investigated effectiveness of student connections between concrete representations and abstract representations through the use of the GeoLeg manipulative tool. In doing so, the goal of the research was to determine if this treatment is more effective than using traditional geometry tools such as a protractor, compass, and ruler when teaching seventh grade geometry. The study further explored the question as to whether or not gender moderates the effect of the treatment. Finally, the study investigated whether or not there is a relationship between student learning styles as identified by the Honey and Mumford Learning Style Questionnaire and seventh grade student achievement on a geometry posttest. The methodology utilized in this study was a mixture of quantitative and qualitative analysis. This methodology was chosen obtain to the broadest spectrum of data in order to examine the relative effectiveness of this treatment in general and more specifically as it relates to student achievement. The methodology assisted in the examination of student achievement on a geometry unit posttest, whether or not gender is a factor in achievement, and relationships between learning styles and student achievement on a geometry unit posttest for seventh grade students learning geometry.

Research Design

This research project used a randomized controlled experimental study design. All statistical analyses were performed using PASW (formerly SPSS) for Windows (PASW 18.0, SPSS Inc., Chicago, IL). All of the analyses were two-sided with a 5% alpha level. Demographic characteristics of the study sample will be described using the mean, standard deviation, and range for continuous scaled variables and frequency and percent for categorical scaled variables.

Hypotheses 1 and 2 were tested using two-tailed t-tests. The two-sample t-test produced a probability value (p-value) with a specified number of degrees of freedom. If the p-value was less than 0.05, then the null hypothesis was rejected. For hypothesis 1, the posttest score was compared between the control and treatment groups. For hypothesis 2, the posttest score was compared between males and females within the treatment group.

Hypotheses 3 through 6 were tested using Pearson's correlation coefficient. A two-sample t-test was performed to test the null hypothesis that the correlation coefficient was 0, versus the alternative hypothesis that the correlation coefficient was not 0. If the p-value was less than 0.05, then the null hypothesis was rejected. The posttest score was correlated with the Reflector learning style for hypothesis 3, the Theorist learning style for hypothesis 4, the Pragmatist learning style for hypothesis 5, and the Activist learning style for hypothesis 6.

Hypotheses 7 through 10 were tested using multiple linear regression analysis. The dependent variable in the regression model was the posttest score. The independent variable was the instruction given to the treatment group. The moderator variable was the Reflector learning style score for hypothesis 7, Theorist learning style for hypothesis 8, Pragmatist learning style for hypothesis 9, and Activist learning style hypothesis 10. The learning style scores were centered prior to conducting the analysis. For example, to center the Reflector learning style score, the average Reflector learning style score for the entire sample was subtracted from each student's individual Reflector learning style score. The benefit of centering is that it reduced the effect of multi-collinearity between the Reflector learning style score, and the interaction between the Reflector learning style score and treatment group. The interaction between the Reflector learning style score and treatment group is of primary importance in the analysis. The interaction term was not statistically significant so the null hypothesis was not rejected, and it was concluded that a Reflector learning style did not moderate the effect of the intervention on the posttest score. The equation of the model was reported and statistically significant regression coefficients were interpreted. The R-square for the final model was also presented and interpreted.

Criterion and Predictor Variables

The criterion variable was the student posttest scores. The predictor variables were treatment (control group related to treatment group), gender, and student learning styles. The student learning style was determined through an on-line questionnaire.

Research Questions

1. Do seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on a posttest than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum?

2. Does the use of the GeoLeg manipulative tool have the same effectiveness for seventh grade males and females when learning geometry concepts?

3. Is there a relationship between the student learning style of "Reflector" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

4. Is there a relationship between the student learning style of "Theorist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

5. Is there a relationship between the student learning style of "Pragmatist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

6. Is there a relationship between the student learning style of "Activist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

7. Does the “Reflector” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg manipulative tool intervention on the geometry posttest scores?

8. Does the “Theorist” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

9. Does the “Pragmatist” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

10. Does the “Activist” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

Research Hypotheses and Analysis

This study was based on 10 hypotheses:

1. Hypothesis: There is a difference in the average posttest score between the control and treatment group.

Null hypothesis: There is no difference in the average posttest score between the control and treatment group.

Method of Analysis: A two-sample t-test was used to test the hypothesis. If the p-value was less than 0.05 then the null hypothesis was rejected.

2. Hypothesis: There is no difference in the average posttest score between males and females in the treatment group.

Null hypothesis: There is a difference in the average posttest score between males and females in the treatment group.

Method of Analysis: A two-sample t-test was used to test the hypothesis. If the p-value was less than 0.05 then the null hypothesis was rejected.

3. Hypothesis: There is no correlation between the posttest score and the Reflector learning style score.

Null Hypothesis: There is a correlation between the posttest score and the Reflector learning style score.

Method of Analysis: A Pearson correlation statistic was calculated and a two-sample t-test was used to test the null hypothesis that the correlation statistic was 0, versus the alternative hypothesis that the correlation statistic was not 0. If the p-value was less than 0.05, then the null hypothesis was rejected.

4. Hypothesis: There is no correlation between the posttest score and the Theorist learning style score.

Null Hypothesis: There is a correlation between the posttest score and the Theorist learning style score.

Method of Analysis: A Pearson correlation statistic was calculated and a two-sample t-test was used to test the null hypothesis that the correlation statistic was 0, versus the alternative hypothesis that the correlation statistic was not 0. If the p-value was less than 0.05, then the null hypothesis was rejected.

5. Hypothesis: There is no correlation between the posttest score and the Pragmatist learning style score.

Null Hypothesis: There is a correlation between the posttest score and the Pragmatist learning style score.

Method of Analysis: A Pearson correlation statistic was calculated and a two-sample t-test was used to test the null hypothesis that the correlation statistic was 0, versus the alternative hypothesis that the correlation statistic was not 0. If the p-value was less than 0.05, then the null hypothesis was rejected.

6. Hypothesis: There is no correlation between the posttest score and an Activist learning style score.

Null Hypothesis: There is a correlation between the posttest score and an Activist learning style score.

Method of Analysis: A Pearson correlation statistic was calculated and a two-sample t-test was used to test the null hypothesis that the correlation statistic was 0, versus the alternative hypothesis that the correlation statistic was not 0. If the p-value was less than 0.05, then the null hypothesis was rejected.

7. Hypothesis: A Reflector learning style does not moderate the effect of the intervention on posttest scores.

Null Hypothesis: A Reflector learning style moderates the effect of the intervention on posttest scores.

Method of Analysis: Multiple regression analysis was used to test the hypothesis. If the p-value for the interaction between a Reflector learning style and group was less than 0.05, then the null hypothesis was rejected.

8. Hypothesis: A Theorist learning style does not moderate the effect of the intervention on posttest scores.

Null Hypothesis: A Theorist learning style moderates the effect of the intervention on posttest scores.

Method of Analysis: Multiple regression analysis was used to test the hypothesis. If the p-value for the interaction was between a Theorist learning style and group was less than 0.05, then the null hypothesis was rejected.

9. Hypothesis: A Pragmatist learning style does not moderate the effect of the intervention on posttest scores.

Null Hypothesis: A Pragmatist learning style moderates the effect of the intervention on posttest scores.

Method of Analysis: Multiple regression analysis was used to test the hypothesis. If the p-value for the interaction between a Pragmatist learning style and group was less than 0.05, then the null hypothesis was rejected.

10. Hypothesis: An Activist learning style does not moderate the effect of the intervention on posttest scores.

Null Hypothesis: An Activist learning style moderates the effect of the intervention on posttest scores.

Method of Analysis: Multiple regression analysis was used to test the hypothesis. If the p-value for the interaction between an Activist learning style and group was less than 0.05, then the null hypothesis was rejected.

Setting and Participants

This study investigated whether seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on district-approved geometry curriculum tests than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum. The study was conducted in an urban middle school located in South Florida. There are 1,211 students enrolled, with the following ethnic breakdown: 57% White, 12% Black, and 19% Hispanic. The gender breakdown was 55% males and 45% females. Twenty-one percent of the students received free and reduced lunch.

Three hundred and seventeen seventh grade students participated in this study. All students were currently enrolled in 7th grade mathematics, in one of 14 sections, with one of 7 teachers. Four teachers only taught one section of seventh grade mathematics. Each of these teachers had at least one treatment group who used the GeoLeg manipulative tool with their students. Three teachers taught multiple sections of seventh grade mathematics. Each teacher who taught multiple sections of seventh grade mathematics had at least one section of students who were a control group (where students used a protractor, compass, and ruler). The control and treatment group sections were assigned arbitrarily to each teacher by using a random number generator. All teachers used the same instructional materials doing the same activities with both the control and treatment groups. The only difference between the two groups was the use of the GeoLeg manipulative tool.

Procedure

All seventh grade teachers were willing to participate in this research study; all agreed to participate. All teachers accepted the invitation to participate. Four teachers (Albert, Brenda, Charles, and Deborah – identified by pseudonyms) who taught only one section of seventh graders, and three teachers (Eddie, Francis, George – identified by pseudonyms) taught multiple sections. The four teachers who only taught one section of seventh grade mathematics were assigned treatment groups. Two of the teachers taught three sections of seventh grade mathematics – Eddie and Francis. From their three sections, one section was chosen using a random number generator to be a control group, while the other two sections were treatment groups. Finally, one teacher (George) taught four sections of seventh grade mathematics. Two sections were chosen using a random number generator to be control groups and the other two sections became treatment groups. This procedure was chosen to reduce bias.

The researcher met with the teachers for one and one-half hours and discussed the geometry unit to be taught. Teachers worked with the researcher and agreed upon a commonly planned unit of instruction with common assignments and tasks. The participating teachers had all necessary materials on hand to teach the unit, including the GeoLeg manipulative tool. This tool would be provided to the teachers during their professional development training.

Prior to the commencement of this study, all teachers participated in a 3-hour professional development training provided by the researcher, a representative from the GeoLeg company, and the South Florida representative for the district approved text. The focus of the training was the usage of the tool as it pertained to the geometry unit to be studied. This professional development session did not seek to add to the geometry knowledge of the teachers or discuss best practices and methodology of instruction, but was restricted to instruction regarding the usage of the manipulative tools as they pertained to the specific activities in the study.

None of the participating teachers had prior experience using the GeoLeg manipulative tool. However, all of the teachers had used protractors, rulers, and compasses in their geometry instruction. Therefore, the use of the GeoLeg manipulative tool should not have confounded or influenced the instruction of sections of students who were not being taught with the GeoLeg tool. Teachers worked through each activity that was included in the instructional unit. All teachers with a treatment group were supplied with sufficient GeoLeg manipulatives for each student.

After teacher professional development training was completed, a timeline was agreed upon with the teachers. All teachers agreed to take their students to the school computer lab during the week prior to the commencement of the geometry unit instruction. Teachers instructed students to take an online learning styles survey. Teachers read the following statement to students:

> The survey you are about to take is an important learning tool. This survey will provide valuable information as to how you think and learn. The survey will take you about 20 minutes. Please carefully read each question and answer each question as honestly as you can. Please do not talk to or disturb others while they are taking their survey.

Students had the option to choose not to participate in the survey. Students were also given the option to get the results of the survey. Results would be delivered to the teacher to dispense. There were no negative implications for not participating, should a student choose that route. To maintain an equitable environment for all students, teachers monitored the students as they took their surveys. Monitoring consisted of making certain that students were able to complete their survey without being disrupted by others talking or distracting them.

The control group and treatment groups used commonly planned materials agreed upon by the teachers prior to the commencement of the unit. The only difference between the treatment group and the control group was the use of the GeoLeg tool by the treatment group to complete tasks and assignments. The GeoLeg tool contains a ruler, a protractor, and a compass. All teachers with students in treatment groups were provided with sufficient GeoLeg tools so that all students had their own tools for all activities. All teachers with control group students were provided with sufficient rulers, protractors, and compasses so that all students had their own tools for all activities.

Each class section met each day for 50 minutes. Each classroom was equipped with a whiteboard, LCD projector connected to the teacher computer station, an overhead projector, and a document camera. The method of instruction in each of these classes was a mixture of lecture and hands-on activities. The instructional plan utilized by each teacher involved similar strategies, the same instructional textbook, resources, student handouts, and supplemental materials that were developed and agreed upon during a common planning meeting.

During the instructional process, I observed each teacher twice during instruction. Observations included both treatment and control groups and were pre-arranged with the teacher and the school administration. Each observation lasted 15 minutes and was carefully recorded in great detail as to whether or not the teachers were following the prescribed curriculum. Student engagement and student on-task behaviors were also noted. These field notes were used during the interpretation of the results section of the study to possibly assist in answering the research questions.

I interviewed individual teacher following the completion of the student posttest to add depth and breath to the study. Interview protocols were developed for post-unit teacher debriefing interviews (Appendix D). These interviews add depth to the quantitative data and clarify teachers' perceptions of the geometry unit, student engagement, and the usefulness of the GeoLeg manipulative tool in this geometry unit.

Student interviews were also conducted following the posttest. I used a Random Number Generator to select students from each participating section (pseudonyms ST1, ST2, ST3, ST4, ST5, ST6, ST7, ST8, ST9, ST10, ST11, ST12, ST13, and ST14). Interview protocols were developed and used in these interviews (Appendix E). These interviews add depth to the quantitative data and clarify students' perceptions of the geometry unit.

Data Collection

I followed the IRB protocol of notifying the parents of all students involved in the study (Appendix A). All students completed a child assent form (Appendix C), and teachers signed a consent form (Appendix B). Data collection began with the student learning style survey taken on-line at the school's computer lab (Appendix D). Students were given the option not to participate, but all students chose to participate.

The qualitative observation data were collected during the instruction of the unit. All observations were pre-arranged with the teacher and the school administration. The researcher had copies of the pre-approved lesson plans and ancillary materials during observations. Each classroom observation lasted 15 minutes. Detailed field notes were taken in order to ensure that the agreed-upon curriculum was being taught equitably to all students by all teachers to all students.

Student interviews were conducted (see Appendix F) after the students completed the posttest. Students were arbitrarily chosen from each section using a random integer generator. Interview protocols, approved during the FAU IRB process, were used during the interviews. Student interview times were arranged in cooperation with the school administration to minimize disruption of student and teacher schedules.

Teacher interviews (see Appendix E) were scheduled after the completion of the student posttests but prior to the scoring of the posttests. FAU IRB-approved interview protocols were used during the interviews. Teachers were informed that their interview would last between 20 and 35 minutes and they chose the most convenient time for them to meet. This researcher accommodated all teacher requests.

Instruments

Honey and Mumford Learning Style Questionnaire

The Manual of Learning Styles was published in 1982, revised in 1992, and then replaced in 2000 by *The Learning Styles Helper's Guide* and the Learning Style Questionnaire (LSQ). According to Honey and Mumford, their learning styles "have been translated into dozens of languages, are now used throughout the world, in all sectors of commerce and education, and enjoy high face validity" (Honey and Mumford, 2000, foreword). The current version of the LSQ consists of 80 items that probe preferences for four learning styles with 20 items for each style. Reliability and validity in *The Learning Styles Helper's Guide* (2000) provides statistical data on the LSQ. With regard to reliability, a test–retest study of 50 people, with an interval of 2 weeks between tests, provided a correlation of 0.89. The correlations for the four styles are: Reflectors, 0.92; Theorist 0.95; Pragmatist, 0.87; Activist, 0.81.

The on-line survey used in this study was re-written, keeping the original ideas in language that seventh grade students would understand. The reading lexile measurement was checked using Metametrics lexile analyzer to ensure that the reading level of the survey was appropriate. Metametrics is an educational measurement and research organization that specializes in psychometric measurement. The lexile for this survey was

800L. According to Barton and Coley (2009), reader levels for mid-year seventh grade students between the 25th and 75th percentiles should be between 735L and 1035L. A reading level of 800L was therefore deemed an appropriate reading level for seventh grade students.

Posttest Assessment The posttest assessment used in this study was extracted from 7th grade State of Florida and District of Palm Beach County approved curriculum. The material was specifically taken from the testing materials provided by the Glencoe *Mathematics: Applications and Concepts Course 3* textbook series. The items tested meet the State of Florida Sunshine State Standard benchmark requirements for seventh grade.

Qualitative Data Analysis

The main purpose of the student interviews was to explore the results of the data in further depth and better answer the research questions, rather than to generalize the findings. Interviews were scheduled following the completion of the unit with teachers and students. Scheduling for interviews was coordinated with the school's administration. All of the qualitative data, including observations and interviews, was transcribed, coded, and analyzed for themes.

The analysis of the qualitative data was based upon a constant comparison and grounded theory. The interviews were separated into student and teacher groups. The interviews were transcribed and given to the teachers to check for accuracy. Teachers were given the opportunity to edit and approve the final transcript. Students were not given the opportunity to check their interview transcripts because they were not in school or available to contact after the interviews were concluded. The indicators of categories in responses were coded in the transcripts. The codes were compared to find consistencies and differences. Consistencies within the codes that pointed to the same basic ideas were grouped into categories. Categories were examined for emerging themes.

Summary of the Chapter

Once the research focus was determined and the supporting research information was gathered, research questions were developed. A method for testing these research questions and hypotheses was crafted. Arrangements were made with a school site and instruments were developed and selected in order to adequately complete the research. The statistical analysis methods that were most effective for answering the research questions were chosen. Based upon this decision, statistical software was chosen. Once the basic structure of the study was in place, the appropriate institutions were notified; permissions for research were applied for, and were subsequently granted. At that juncture, the research study commenced.

Chapter 4: Analysis of Data

The objective of this study was to determine whether or not the use of the GeoLeg manipulative tool affects student achievement with seventh grade students. An analysis was completed comparing students who used the GeoLeg manipulative tool and those in the control group (who use the traditional geometry tools of a protractor, compass, and ruler), student posttest scores, and the student's gender. Additionally, an analysis of student learning styles, as determined by the Honey and Mumford Learning Styles Questionnaire (Honey & Mumford, 1995), from an on-line student survey prior to the commencement of geometry instruction was performed. The results of this survey were compared to student posttest scores.

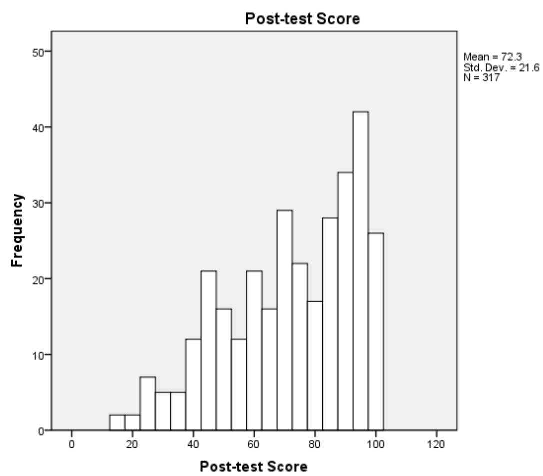
A three-tiered, mixed-methods approach was utilized (a student posttest, teacher and student interviews, and an on-line survey) to answer the research questions. This analysis will be divided into two main sections: the reporting of the findings of the quantitative data, followed by a report of the findings of the qualitative data.

Participants

This study investigated whether seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on a district approved geometry curriculum test than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum. The study was conducted in an urban middle school located in South Florida.

Three hundred and seventeen seventh grade students participated in this study. The control group (students used a protractor, compass, and ruler) was comprised of 91 students (28.7% of the total) and the treatment group (students used the GeoLeg manipulative tool) contained and 226 students (71.3%). All students were currently enrolled in 7th grade mathematics, in 1 of 14 sections, with one of 7 teachers. The combined group averaged 72.3% on the posttest with a standard deviation of 21.6 and a range from 15 to 100.

Figure 7 Posttest Score Histogram



The total group of 317 students was comprised of 163 males (51.4%) and 154 females (48.6%). There were 225 White students (71%), 11 Asian (3.5%), 18 Multiracial (5.7%), 13 Black (4.1%), and 50 Hispanic (15.8%). 58 (18.3%) of the students were on free or reduced lunch status. After examining the data results from the student Learning Styles Questionnaire that the students completed on-line, the Activist Learning style yielded a mean of 12.94 with a standard deviation of 2.914 and a minimum score of 0 and high of 19; the Reflector Learning style yielded a mean of 11.88 with a standard deviation of 3.903 and a minimum score of 0 and high of 20; the Theorist Learning style yielded a mean of 10.75 with a standard deviation of 3.549 and a minimum score of 0 and high of 20; and the Pragmatist Learning style yielded a mean of 11.08 with a standard deviation of 3.176 and a minimum score of 0 and high of 19.

Four teachers only taught one section of seventh grade mathematics. Each of these teachers taught one treatment group each, using the GeoLeg manipulative tool with their students. Three teachers taught multiple sections of seventh grade mathematics. Each teacher who taught multiple sections of seventh grade mathematics had at least one section of students who were a control group. The control and treatment group sections were assigned arbitrarily to each teacher by using a random number generator (Haahr, 2009). All teachers used the same instructional materials doing the same activities with both the control and treatment groups. The only difference between the two groups was the use of the GeoLeg manipulative tool.

Research Questions

1. Do seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on a posttest than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum?
2. Does the use of the GeoLeg manipulative tool have the same effectiveness for seventh grade males and females when learning geometry concepts?
3. Is there a relationship between the student learning style of “Reflector” as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

4. Is there a relationship between the student learning style of "Theorist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

5. Is there a relationship between the student learning style of "Pragmatist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

6. Is there a relationship between the student learning style of "Activist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

7. Does the "Reflector" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg manipulative tool intervention on the geometry posttest scores?

8. Does the "Theorist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

9. Does the "Pragmatist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

10. Does the "Activist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

Quantitative Findings

Analysis of Findings: Question 1. Do seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on a posttest than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum?

It was expected that the treatment group students would outperform the control group upon the completion of the geometry unit. The null form of the hypothesis tested is there is no difference between the treatment group and the control group on the posttest.

Figure 8 is an error bar chart which shows the average (95% confidence interval) posttest scores, separately for the control and treatment groups. The Levene's Test of Equality of Variances was statistically significant, $F(1,315) = 8.08$: $p = 0.005$, indicating that the equal variances assumption of the two-sample t-test was violated, so the unequal variances two-sample t-test was used to test the hypotheses. Table 1 shows that the treatment group had a statistically significantly larger average posttest score than the control group. The average (SD) posttest score was 60.5 (22.6) versus 77.1 (19.3) for the control and treatment groups respectively, $t(146.4) = 6.16$; $p < 0.001$; $d=0.79$. Therefore, the null hypothesis was rejected and it was concluded that the GeoLeg teaching method produced better academic performance than the standard teaching method. The size of the difference in the average posttest scores between the two groups represents a large effect size.

Figure 8. Control and treatment group error bar chart

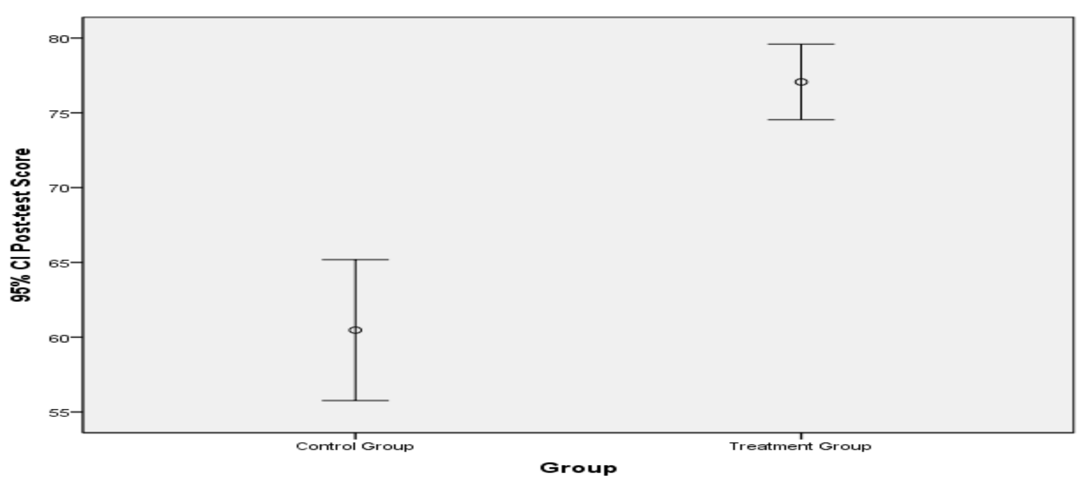


Table 1. Control and Treatment Group Statistics for Posttest Scores

	Group	N	Mean	Std. Deviation	Std. Error Mean
Posttest Score	Control Group	91	60.47	22.603	2.369
	Treatment Group	226	77.07	19.275	1.282

Analysis of Findings: Question 2. Does the use of the GeoLeg manipulative tool have the same effectiveness for seventh grade males and females when learning geometry concepts? The null hypothesis is there is no significant difference in the effectiveness of the GeoLeg manipulative tool between males and females.

Figure 9 is an error bar chart which shows the average (95% confidence interval) posttest scores, separately for males and females among students in the treatment group. The Levene's Test of Equality of Variances was statistically significant, F(1,224)=5.33: p = 0.022, indicating that the equal variances assumption of the two-sample t-test was violated, so the unequal variances two-sample t-test was used to test the hypotheses. There was not a statistically significant difference in the average posttest score between males and females. The average (SD) posttest score was 77.8 (17.8) versus 76.3 (20.7) for males and females respectively, t(215.1) = 0.59; p = 0.56. Therefore, the null hypothesis was not rejected and it was concluded that there is no difference in the average posttest score between males and females among students who were taught using the GeoLeg teaching method.

Figure 9. Gender Error Bar Graph

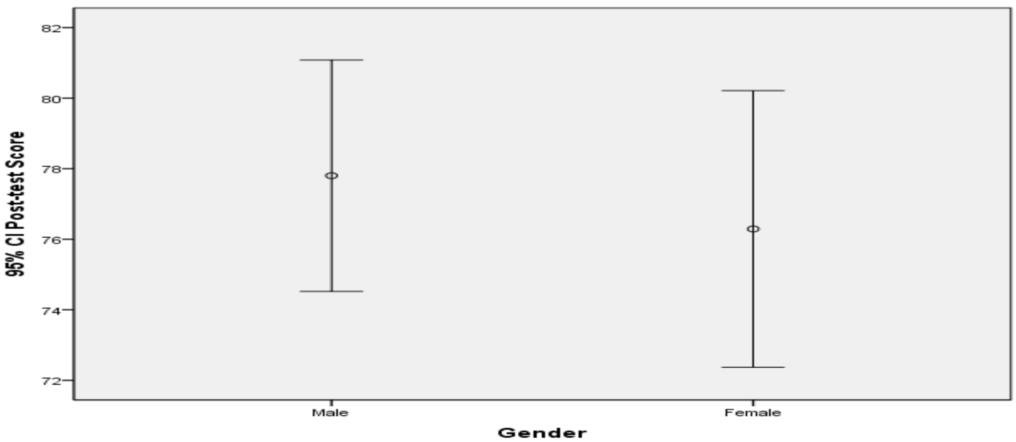


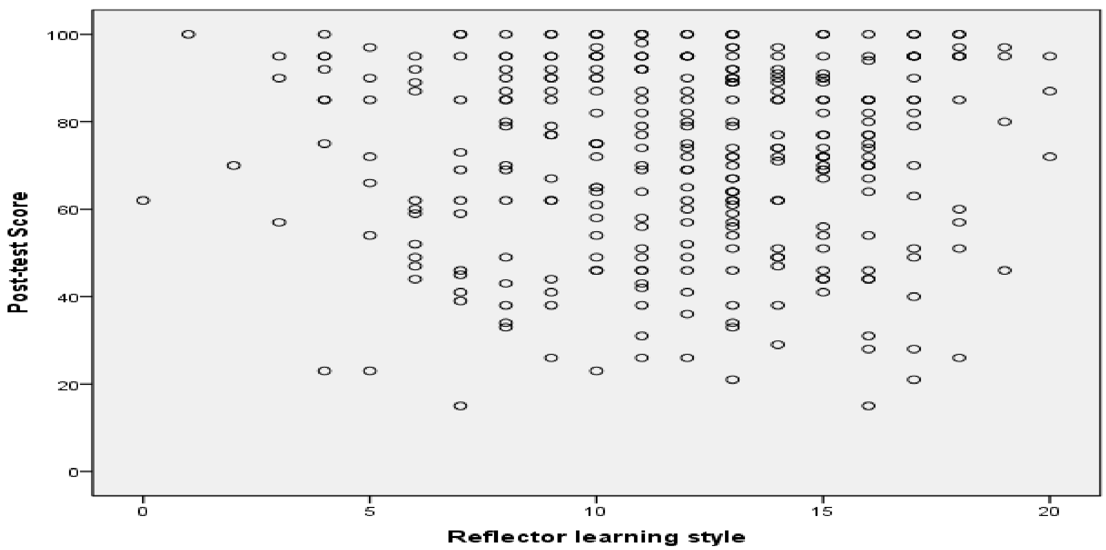
Table 2. Group Statistics for Gender

	Gender	N	Mean	Std. Deviation	Std. Error Mean
Posttest Score	Male	116	77.80	17.829	1.655
	Female	110	76.29	20.744	1.978

Analysis of Findings: Question 3. Is there a relationship between the student learning style of "Reflector," as identified by the Honey and Mumford Learning Style Questionnaire (LSQ), and student achievement on a geometry unit posttest? The hypothesis is there will be a relationship between the "Reflector" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. The null hypothesis is there will not be a relationship between the "Reflector" learning style as identified on the Honey and Mumford student survey and student achievement on a geometry posttest.

Figure 10 graphically depicts the relationship between the posttest score and the Reflector learning style score on a scatterplot. There was not a statistically significant correlation between the posttest score and the Reflector learning style score, r(317)= 0.018; p = 0.75. Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence of a correlation between posttest scores and a Reflector learning style.

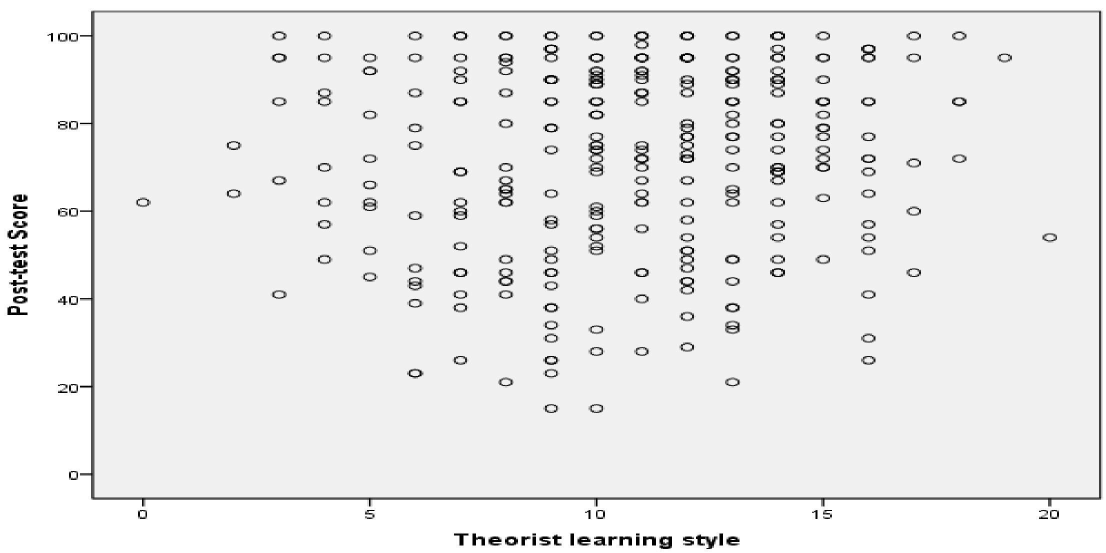
Figure 10. Reflector Learning Style Scatter Plot



Analysis of Findings: Question 4. Is there a relationship between the student learning style of "Theorist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest? The hypothesis is there will be a relationship between the "Theorist" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. The null hypothesis is there will not be relationship between the "Theorist" learning style, as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest.

Figure 11 graphically depicts the relationship between the posttest score and the Theorist learning style score. There was not a statistically significant correlation between the posttest score and the Theorist learning style score, $r(317)= 0.10$; $p = 0.071$. Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence of a correlation between posttest scores and a Theorist learning style.

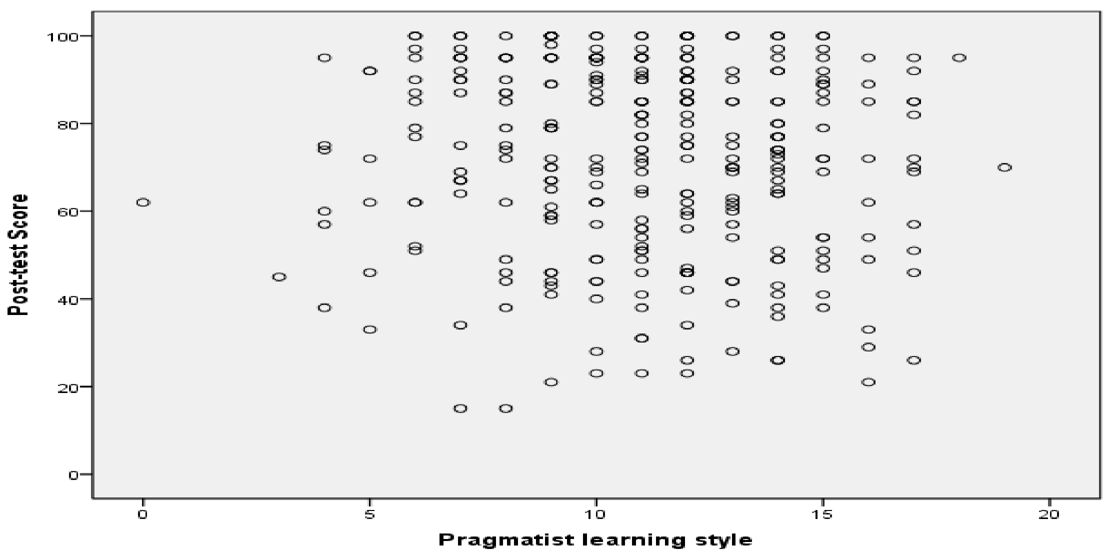
Figure 11. Theorist Learning Style Scatter Plot



Analysis of Findings: Question 5. Is there a relationship between the student learning style of "Pragmatist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest? The hypothesis is there will be a relationship between the "Pragmatist" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. The null hypothesis s there will not be relationship between the "Pragmatist" learning style, as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest.

Figure 12 graphically depicts the relationship between the posttest score and the Pragmatist learning style score. There was not a statistically significant correlation between the posttest score and the Pragmatist learning style score, $r(317)= -0.066$; $p = 0.25$. Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence of a correlation between posttest scores and a Pragmatist learning style.

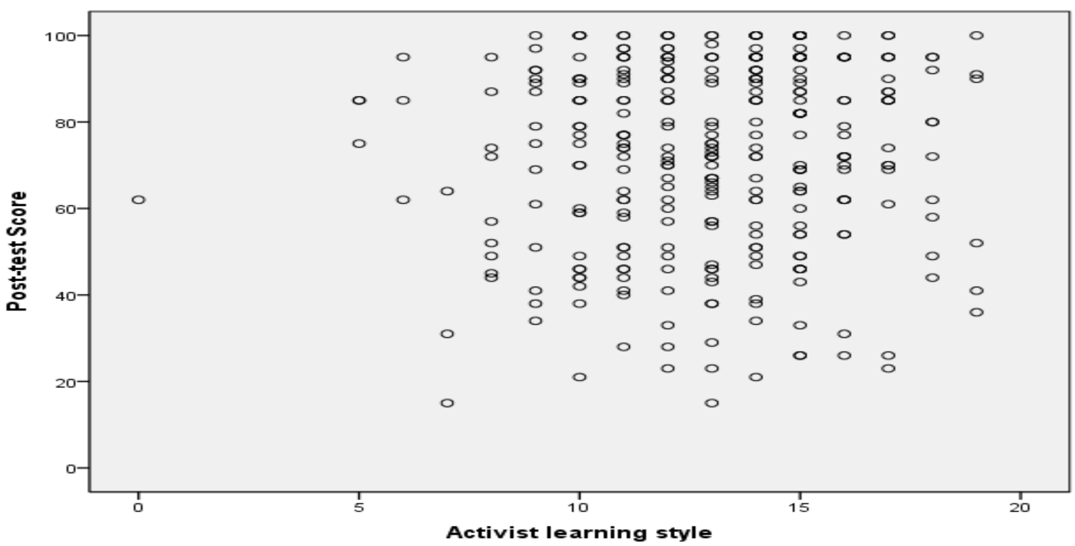
Figure 12. Pragmatist Learning Style Scatter Plot



Analysis of Findings: Question 6. Is there a relationship between the student learning style of "Activist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest? The hypothesis is there will be a relationship between the "Activist" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest. The null hypothesis is there will not be relationship between the "Activist" learning style as identified on the Honey and Mumford student survey, and student achievement on a geometry posttest.

Figure 13 graphically depicts the relationship between the posttest score and the Activist learning style score. There was not a statistically significant correlation between the posttest score and the Activist learning style score, r(317)= 0.073; p = 0.19. Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence of a correlation between posttest scores and a Activist learning style.

Figure 13. Activist Learning Style Scatter Plot



Analysis of Findings: Question 7. Does the “Reflector” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg manipulative tool intervention on the geometry posttest scores? The hypothesis is that a “Reflector” learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores. The null hypothesis is that a “Reflector” learning style, as identified by the Honey and Mumford student survey, does not moderate the effect of the intervention of the GeoLeg manipulative tool on the posttest scores.

The results show that collectively, study group (control versus treatment), Reflector learning style, and the interaction between study group and Reflector learning style explained a statistically significant percentage of the total variance in the posttest scores, $R^2 = 0.13$; $F(3, 313) = 15.2$; $p < 0.001$. However, Table 3 shows that only study group was statistically significant ($p < 0.001$). In particular, the interaction between study group and the Reflector learning style score was not statistically significant ($p = 0.21$). Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence to suggest a Reflector learning style moderates the effect of the treatment on the posttest score.

Table 3. Reflector Learning Style Coefficients

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	p-value
1	(Constant)	60.653	2.139		28.351	<.001
	Group	16.479	2.530	.346	6.515	<.001
	Reflector_Centered	-.462	.620	-.084	-.745	.457
	ReflectorXGroup	.882	.704	.140	1.254	.211

a. Dependent Variable: Posttest Score

Analysis of Findings: Question 8. Does the "Theorist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores? The hypothesis is that a "Theorist" learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores. The null hypothesis is that a “Theorist” learning style, as identified by the Honey and Mumford student survey, does not moderate the effect of the intervention of the GeoLeg manipulative tool on the posttest scores.

The results show that collectively, study group (control versus treatment), Theorist learning style, and the interaction between study group and Theorist learning style explained a statistically significant percentage of the total variance in the posttest scores, $R^2 = 0.13$; $F(3, 313) = 15.8$; $p < 0.001$. However, Table 4 shows that only study group was statistically significant ($p < 0.001$). In particular, the interaction between study group and the Theorist learning style score was not statistically significant ($p = 0.59$). Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence to suggest a Theorist learning style moderates the effect of the treatment on the posttest score.

Table 4. Theorist Learning Style Coefficients

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	p-value
1	(Constant)	60.505	2.120		28.540	<.001
	Group	16.554	2.511	.347	6.593	<.001
	Theorist_Centered	.887	.612	.146	1.448	.148
	TheoristXGroup	-.390	.719	-.055	-.542	.588

a. Dependent Variable: Posttest Score

Analysis of Findings: Question 9. Does the “Pragmatist” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores? The hypothesis is that a “Pragmatist” learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores. The null hypothesis is that a “Pragmatist” learning style, as identified by the Honey and Mumford student survey, does not moderate the effect of the intervention of the Geoleg manipulative tool on the posttest scores.

The results show that collectively, study group (control versus treatment), Pragmatist learning style, and the interaction between study group and Pragmatist learning style explained a statistically significant percentage of the total variance in the posttest scores, $R^2 = 0.12$; $F(3, 313) = 14.8$; $p < 0.001$. However, Table 5 shows that only study group was statistically significant ($p < 0.001$). In particular, the interaction between study group and the Pragmatist learning style score was not statistically significant ($p = 0.62$). Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence to suggest a Pragmatist learning style moderates the effect of the treatment on the posttest score.

Table 5. Pragmatist Learning Style Coefficients

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	p-value
1	(Constant)	60.488	2.134		28.348	<.001
	Group	16.537	2.526	.347	6.547	<.001
	Pragmatist_Centered	-.072	.651	-.011	-.110	.912
	PragmatistXGroup	-.389	.781	-.048	-.498	.619

a. Dependent Variable: Posttest Score

Analysis of Findings: Question 10. Does the “Activist” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores? The hypothesis is that an “Activist” learning style, as identified by the Honey and Mumford student survey, will moderate the effect of the GeoLeg manipulative tool intervention on the posttest scores. The null hypothesis is that an “Activist” learning style, as identified by the Honey and Mumford student survey, does not moderate the effect of the intervention of the Geoleg manipulative tool on the posttest scores.

The results show that collectively, study group (control versus treatment), Activist learning style, and the interaction between study group and Activist learning style explained a statistically significant percentage of the total variance in the posttest scores, $R^2 = 0.13$; $F(3, 313) = 15.7$; $p < 0.001$. However, Table 6 shows that only study group was statistically significant ($p < 0.001$). In particular, the interaction between study group and the Activist learning style score was not statistically significant ($p = 0.32$). Therefore, the null hypothesis was not rejected and it was concluded that there is no evidence to suggest an Activist learning style moderates the effect of the treatment on the posttest score.

Table 6. Activist Learning Style Coefficients

Coefficients[a]

Model	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	t	p-value
1 (Constant)	60.307	2.123		28.407	<.001
Group	16.779	2.514	.352	6.675	<.001
Activist_Centered	1.120	.634	.151	1.767	.078
ActivistXGroup	-.797	.805	-.085	-.990	.323

a. Dependent Variable: Posttest Score

Qualitative Findings

Teacher Interviews. In this research study, all seven participating teachers were interviewed after the students had completed the posttest evaluation. The teacher names used in this portion of the analysis are pseudonyms in order to maintain the confidentiality of the study. The researcher decided upon the pseudonyms by using alternating gender names that appear in alphabetical order. These names are not linked to the gender of the teacher. The pseudonyms chosen were: Albert, Brenda, Charles, Deborah, Eddie, Francis, and George. The description of each teacher's years of teaching experience is accurate.

The interview protocol and questions were developed by the researcher and approved in the IRB prior to the commencement of the study. The questions developed in the protocol were designed to elicit responses that would address student conceptual understanding, student engagement, teacher engagement, teacher efficacy, and student efficacy. The following analysis contains quoted excerpts from those interviews in an attempt to elucidate the qualitative dimension of the research.

The interviews were scheduled at a time that was chosen by each teacher. All interviews were conducted in the Principal's conference room and were conducted as scheduled. Two dominant themes emerged from the interviews – student engagement and student achievement.

Teacher Responses Regarding Student Engagement and Achievement.

Albert. Teaching middle school for almost 20 years has not dulled Albert's enthusiasm for teaching. Albert participates regularly in the school district professional development and is willing to integrate new ideas into classroom instruction. He teaches primarily seventh and eighth grade mathematics. He taught one treatment group section.

Interviewer: "Can you tell me about your experiences when teaching this geometry unit?" Albert stated: "As we proceeded through the lessons and activities, the children became more adept and more curious about the GeoLeg. For them to adapt to this new tool was pretty easy. They were very eager to try the new GeoLeg and to do the activities."

Interviewer: "How did your students respond to your teaching during this unit?" Albert responded: "They were happy to participate. They were curious. Once they saw that they could learn and shine with a new tool they were really into it. Learning and adjusting to how to use the GeoLeg properly, and helping each other was a really beautiful thing to see. With this unit I could see the students immediately jump in there and participate."

Brenda. Brenda has more than 30 years experience in the classroom teaching at the middle school level. Additionally, she has experience teaching special education. She teaches seventh and eighth grade mathematics. Her classroom is neat and organized in double rows. Her room configuration appears to be set up for students to work in partners. She taught one treatment group section.

Interviewer: "Can you tell me about your experiences when teaching this geometry unit?" Brenda, stated: "You know what? I think the kids really enjoyed this manipulative. I think it was a great tool. I think it was very versatile-part of it you could use as a ruler, other part for a protractor and compass . . .and they really liked using it and I think that it made it easier for them to grasp the concepts. I think they were all excited to use it. Whether they were low, middle or high learners... I think . . .that they looked at it as a fun way of learning. They had something in their hand to work with, I think. It wasn't just a worksheet or just the textbook. I think it really helped because they actually worked with the tool."

Interviewer: "Is this student response any different from the way that these students have responded to instructional material in the past?" Brenda stated: "I just think they were more motivated because of the tool. I think it just kind of enlightened them and they wanted to do the lesson."

Interviewer: "Can you expand on what you mean by enlightened?"
Brenda responded, "When we were going through some of the geometry sheets and we were measuring the angles and they saw how you can piece things together to get the whole. I think that they saw better because of using the tool and actually participating hands-on. I just remember some of the light bulbs going on. I could see it in their eyes.

They got it. They got it and I saw that everyday from them. It was amazing. I know that they have seen or heard these concepts in the past, but it seemed that they actually understood for the first time."

Interviewer: "Reflecting on the classroom experiences you have had with these students, do you think that your students were more or less engaged during this geometry unit than on other units you have taught them?" Brenda responded, "I think that they were more engaged. Kids that are normally off task and have their head down, stop participating and will not do the writing. . . I have a bunch of kids in there that are like that. I saw them participate more with the GeoLeg. These kids were doing, asking questions and learning. I think they were more on task."

Charles. Charles has been teaching for 23 years. His teaching experience includes middle school mathematics, social studies, and English. Currently, he is teaching sixth and seventh grade mathematics. He frequently takes district professional development courses offered by the school district and enthusiastically implements new strategies and methods. He holds a Master's degree in Curriculum and Instruction and is classified as a "highly qualified" teacher in middle grades mathematics, social studies, and English. He taught one treatment group section.

Interviewer: "Can you tell me about your experiences when teaching this geometry unit?" Charles replied, "They liked seeing and measuring the angles and triangles rather than just hearing about them. They liked seeing the finished product in the GeoLeg. It helped them see the relationship in their angles and triangles."

Interviewer: "Reflecting on the classroom experiences you have had with these students, do you think that your students were more or less engaged during this geometry unit than on other units you have taught them?" Charles articulate, " I have seen a lot of engagement in this. They loved, for instance, when we talked about different triangles. They loved putting the GeoLegs together to come up with that triangle. They liked seeing the finished product in the GeoLeg. They like constructing them themselves without a pencil."

Interviewer: "Is this student response any different from the way that these students have responded to instructional material in the past?" Charles stated, "very different I think. The GeoLeg gave them more freedom. They weren't constantly having to switch from one tool to another. Everything was right there and it gave them the freedom to explore because they did not have to keep switching between a protractor, compass, and ruler. They were really excited about learning."

Deborah. Deborah has been teaching 20 years. She currently teaches 6th and 7th grade mathematics. She is certified in varying exceptionalities K through 12, middle grades mathematics, and middle grades integrated curriculum. She attends all professional development offered at her school site. She taught one treatment group section.

Interviewer: "Can you tell me about your experiences when teaching this geometry unit?" Deborah expressed, "I felt it was very, very valuable especially the questions about irregular figures because they always have they those type of figures on standardized tests. Now the students have a strategy work with them."

Interviewer: "Reflecting on the classroom experiences you have had with these students, do you think that your students were more or less engaged during this geometry unit than on other units you have taught them?" Deborah responded, "The students were definitely engaged, including some of the more disruptive ones that are usually disruptive. They were engaged with the GeoLeg. They were doing the work. They were not disruptive at all. They really got into it. They were all doing their work and engaged. Very positive."

Interviewer: "Is this student response any different from the way that these students have responded to instructional material in the past?" Deborah's opinion was, "No, not really. They all worked and went along with the lessons. But I think they learned more with the GeoLeg. We have used a protractor in the past and they don't really understand that any better than they understood the GeoLeg at first. But then they got the GeoLeg. I don't think they ever really got the protractor activities. The GeoLeg is a more effective tool."

Eddie. Eddie is an energetic and enthusiastic first-year teacher assigned to seventh grade mathematics and eighth grade pre-algebra. He is certified in middle grades mathematics and middle grades social studies. He taught one control group section and two treatment group sections.

Interviewer: "Can you tell me about your experiences when teaching this geometry unit?" Eddie responded, "The GeoLegs.. the kids *loved* it. They liked it so much. They liked it better than the protractor." Interviewer: "Why?" Eddie said, "because the protractor was hard to read the numbers. And the GeoLegs, they got a kick out of it. They had a blast especially when they had to get the measurements for the angles and when they put them together to shapes. It was much easier to understand the measurements of the angles."

Interviewer: “Can you describe any differences you noticed between the control and treatment groups?” Eddie responded, “The main difference was that when they were using the protractor it was harder for them to read the numbers. For example, when it was a supplementary angle and it need to add up to 180, the kids with the protractor their angles didn’t match up to 180. And when they used the GeoLeg it matched up to exactly 180. So the kids with the GeoLegs answers were more precise… I noticed using the protractor I had to keep on re-teaching the same thing over and over again with them because it was harder to get the measures from the angles. The GeoLeg, I taught it maybe 10 or 15 minutes and kids already picked up on it. Really fast. And I had a lot more engagement with the GeoLeg than I did with the protractor. The GeoLegs kids understood what I was teaching.”

Interviewer: “Reflecting on the classroom experiences you have had with these students, do you think that your students were more or less engaged during this geometry unit than on other units you have taught them?” Eddie said, “With the Geoleg? Forget about it! Everyone was participating. I was getting more correct answers from other students. I was even calling on students that don’t normally answer and they got the correct answers. The protractor it was pretty much the same students wanted raised their hand. Umm, I really didn’t get that much participation with the protractor group. Using the protractor.. it was more boring. They really didn’t get a kick out of it. So, it was pulling legs with them to actually get their measurements of angles. They really didn’t want to do it and I had to struggle to get them to find their measurements. Using the GeoLeg, I had no problem with the group. It was easier, much easier for them to understand the geometry using this tool.”

Francis. Francis is an energetic and enthusiastic first-year teacher who is assigned seventh grade mathematics and eighth grade pre-algebra. She is certified in middle grades mathematics. She taught two treatment groups and one control group.

Interviewer: “Can you tell me about your experiences when teaching this geometry unit?” Francis said, “I think it was easier to measure with the GeoLeg compared to the protractor. I still think that with the protractor some of the kids, not at this point because I think they got it now, but at first I think that with the protractor it took them longer to determine what measurement to get. The GeoLeg was pretty much if you set it down right and you were opening it the right way the measure was right there in front of you. You lined up the lines and you were good to go. They had a difficult time lining up the degree measure because of that space on the protractor.”

Interviewer: “Can you describe any differences you noticed between the control and treatment groups?” Francis responded, “I think that with the GeoLegs the level of excitement might have been higher for them. The protractor took a lot more time for the students because the tool is harder for them to use. The GeoLeg allowed them to move along faster and get the right answer more often. I had to re-teach many times the skills with the control groups.”

Interviewer: “Reflecting on the classroom experiences you have had with these students, do you think that your students were more or less engaged during this geometry unit than on other units you have taught them?” Francis said, “Well, there were definitely different levels of engagement with the GeoLegs compared to the protractor. I think that the treatment groups were more into the GeoLeg than the protractor groups. That is not to say that the control group were not engaged. They were still engaged and doing the work but I think with the GeoLeg it was just more involved and engaged."

George. George has been teaching for 20 years. He teaches seventh and eighth grade mathematics. He is certified in special education and middle grades mathematics. He regularly attends professional development workshops offered by the district. He taught two treatment group sections and two control group sections.

Interviewer: "Can you tell me about your experiences when teaching this geometry unit?" George said, "Everything I experienced with the protractor group took more time, more work, more questions, more difficulty manipulating the tool. Umm, every part of the unit was great. The kids loved it. They loved both the GeoLeg and protractor lessons. They loved both because they loved doing something. But the comparisons pretty much stopped there. Everything that the children did that used the GeoLeg tool was more exciting, more fun, more positive experiences getting the right answer. There was less frustration. The GeoLeg all around was much easier, which then translated into the kids feeling successful."

Interviewer: "Can you describe any differences you noticed between the control and treatment groups?" George responded, " all around the activities with the protractor were much more challenging in every way, from reading the numbers, to getting the exact measurements. If I gave the kids a warm-up of one of the angles, you know, the GeoLeg kids in five seconds got the measurement but with the protractor you could time it. It would be a minute or two. It was very obvious. And more varied answers always with the protractor."

Interviewer: “Reflecting on the classroom experiences you have had with these students, do you think that your students were more or less engaged during this geometry unit than on other units you have taught them?” George stated, “if there were 80% of the kids on task, you know, then 75% of them get the right answer with the GeoLeg. If there were 80% of the kids on task with the protractor, meaning not flinging or playing with it with other kids, 50% of them might have gotten the right answer.”

Analysis of Teacher Responses. Two dominant themes emerged from the interviews – student engagement and student achievement. Teachers who taught only treatment group students and teachers who taught both control group and treatment group students had positive statements regarding student engagement when using the GeoLeg manipulative tool in the geometry unit. The teachers of only treatment group of students had the following responses: “they (the students) were happy to participate” (Albert); “they were all excited to use it (GeoLeg tool)” (Brenda); “they were more motivated because of the tool” (Brenda); “I have seen a lot of engagement in this” (Charles); and “the students were definitely engaged” (Deborah).

Positive responses regarding student engagement from the teachers that taught both a treatment group and a control group included: “I had a lot more engagement with the GeoLeg that I did with the protractor” (Eddie); “I really didn’t get that much participation with the protractor group. Using the protractor … it was boring”; “I think that the treatment groups were more into the GeoLeg that the protractor groups. That is not to say that the control group was not engaged. They were still engaged and doing the work but I think with the GeoLeg it was just more involved and engaged” (Francis); and

“Everything the children did that used the GeoLeg tool was more exciting, more fun, more positive experiences getting the right answer” (George).

The theme of positive student achievement also emerged through the teacher responses. The teachers of only treatment group of students had the following responses: “Learning and adjusting to how to use the GeoLeg properly, and helping each other was a really beautiful thing to see” (Albert); “They got it and I saw that everyday from them. It was amazing. I know that they have seen or heard these concepts in the past, but it seemed that they actually understood for the first time” (Brenda); “My students finally understood the concepts, some for the first time” (Charles); and “I think that they learned more with the GeoLeg. We have used a protractor in the past and they really don’t understand that any better than they understood the GeoLeg at first. But then they got the GeoLeg. I don’t think they ever really got the protractor activities. The GeoLeg is a more effective tool” (Deborah).

The theme of positive student achievement emerged through the teacher responses given by the teachers who taught both a treatment group and a control group as well. The teachers had the following responses: “I noticed using the protractor I had to keep re-teaching the same thing over and over again with them because it was harder to get the measures from the angles. The GeoLeg, I taught it maybe 10 or 15 minutes and the kids really picked up on it. The GeoLegs kids understood what I was teaching” (Eddie); “the protractor took a lot more time for the students because the tool was harder for them to use. The GeoLeg allowed them to move along faster and get the right answer more often. I had to re-teach many times the skills with the control groups” (Francis); and “if there were 80% of the kids on task, you know, then 75% of them got the right answer with the

GeoLeg. If there were 80% of the kids on task with the protractor, 50% of them might have gotten the right answer" (George).

Student Responses Regarding Achievement. Student interviews were also conducted following the posttest. This researcher used a Random Number Generator (Haahr, 2009) to select students from each participating section (pseudonyms ST1, ST2, ST3, ST4, ST5, ST6, ST7, ST8, ST9, ST10, ST11, ST12, ST13, and ST14). Interview protocols were developed and used in these interviews (Appendix E). These interviews add depth to the quantitative data and clarify students' perceptions of the geometry unit.

The interview protocol and questions were developed by the researcher and approved in the IRB prior to the commencement of the study. The questions developed in the protocol were designed to elicit responses that would address student conceptual understanding and student engagement. The following analysis contains quoted excerpts from those interviews in an attempt to elucidate the qualitative dimension of the research.

The interviews were scheduled at a time that was chosen by the administration to minimize academic disruption. All interviews were conducted in the Principal's conference room and were conducted as scheduled. Two dominant themes emerged from the interviews – student engagement and student achievement. Reporting will be divided into two groups, those students participating in the control group and then those students participating in the treatment group.

Student Interviews - Control Group Responses. Interviewer: "Tell me about your experiences in this geometry unit." **ST1** said, "Working with the circles was kind of difficult. Not to bad or hard but difficult. I had a hard time remembering which one was the radius and which one was the diameter."

When probed further as to the experience using the protractor in the geometry unit, **ST1** said, “It was a little bit confusing because I had a hard time telling exactly which number I was supposed to use on the protractor. It was hard to read to get the measurements. I couldn’t tell exactly what number it was on when doing the angle measurements. I got a lot wrong and it was frustrating… My teacher gave me extra help… I wasn’t sure if you had to estimate or guess or try to line it up and get it exactly right. I finally got more right. But if it was 60 sometimes I would get 65. If it was a triangle, I would just change it because I knew that they all had to add up to 180 degrees.

Interviewer: “When you completed this unit, did you feel confident that you understood the material?” **ST1** sated, “Well the trapezoid was confusing me and I never really got it. I wasn’t comfortable with that. But I feel that I did learn a lot. My teacher worked hard on explaining to us.”

Interviewer: “Tell me about your experiences in this geometry unit.” **ST2** stated, “It was better than the other geometry stuff I have done before. Well, like sometimes you can tell it was like 75 degrees. So I would just use the protractor and estimate to get close… The stuff we did in this unit helped me remember more stuff than I normally do because I actually interacted with the stuff. We did fun activities. The activities stayed in my mind and then I know how to do it.”

Interviewer: “What about when you were required to get an exact answer?” **ST2** said, “I was right most of the time. I would double check and get real close.” Interviewer: “Do you think by doing these activities that more students in your class participated and did the activities?” **ST2** said, “Yeah, pretty much everybody did everything. More than other times when we do math. People did more of their work even the kids who usually put their head down."

Interviewer: "Tell me about your experiences in this geometry unit." **ST3** stated, "They (protractors) were easy to use. I liked that you did not have to actually multiply all of the problems. You just put the protractor down and check it. It's like you don't have to look at the whole problem to figure out the angle. It was just a fairly easy unit. I like doing the triangles, drawing and ½ base times height. I liked it because I was always one the first to get it right."

Interviewer: "Tell me about your experiences in this geometry unit." **ST7** "It was really good because it got you thinking. Like if you take geometry in the future you have experience using the protractor and stuff. It got us to know about it. I had used the protractor before but this helped me a lot because I think I am better now."

Interviewer: "Tell me about your experiences in this geometry unit." **ST10** "Well, the unit we just finished, I thought it was pretty interesting. I really liked it. I thought it was pretty cool how we could all of the materials like the protractor and compass to help us learn."

Interviewer: "Tell me about your experiences in this geometry unit." **ST11** "Well we used a protractor and it was pretty hard. The protractor was really hard with all of the lines to measure the angles. You can't really line it up and it is really hard to tell which degree it is. My teacher helped me but it was really hard." Interviewer: "Did you ever use a protractor before?" **ST11** "Yes and it was hard."

Student Interviews - Treatment Group Responses. Interviewer: "Tell me about your experiences in this geometry unit." **ST4** "It was cool. It was cool how you could take so many tools and mix it into one thing that you have never seen before. Then you can use it for the same basic things that you have been using other tools. It was interesting".

Interviewer: "Could you compare your experiences using a traditional protractor with the GeoLeg?" **ST4** "Well, the regular protractor – it takes a lot longer to get the angles mainly because the protractor doesn't have the holes in it to line up with the angle and the top part of the angle. The regular protractor has lots of lines that make it confusing to read. The GeoLeg has it labeled all around so if the red and blue lines are lined up to whatever angle you want no matter which way it is pointed, you can just look at it and get the measure of the angle. It is easy… I think the GeoLeg makes it much more accurate because it is a lot harder to make the angles with the protractor…because unless you use a ruler or a straightedge to make sure it is lined up with the angle you kind of have to guess to make the dot where the line is. Also not all of them have a hole or dot where zero is so that you can get it lined up. You can't really draw when you have the protractor up to it."

Interviewer: "Tell me about your experiences in this geometry unit." **ST15** "Measuring the angles were a lot easier because the GeoLeg helps you get an exact measure. It is so much easier to use. You just have to choose the angle you want. You put the GeoLeg on the angle and you can get the measure right away. It helps when you want to draw the angle. I can do it much faster and better with the GeoLeg. You can do a lot with it, like multiple shapes. It helps you with different questions. Like if you are going for formulas on shapes, like triangles and rectangles. The triangles were my favorite because when you connected the three GeoLegs you can make the triangle. It is better than the one that is just on your paper. It made it a lot easier for me to understand. The angles are easier to draw because with the protractor you have to umm set it down and then try to get the right angle. But the Geoleg you just set it at the angle make the dots and you are done. The GeoLeg takes a little less time than the protractor."

Interviewer: "Tell me about your experiences in this geometry unit." **ST6** "Well, I never used or seen one of these before. Well… I think that the GeoLeg is awesome. It made it 10 times easier than using a protractor. It is easier to find the measurements. It is just easier."

Interviewer: "How would you compare the protractor, compass and GeoLeg?" **ST6** "Well, the protractors are sometimes colored and you can't really see the numbers. It is really hard for you to place it at the right spot so that you can get the measurement. With the GeoLeg it is easier because it is clear and you can see what you are doing. All you have to do is line up the rays with the red and blue line and you have the measurement. There is no guessing. The GeoLeg makes the shapes for you. It is just like a protractor but you don't always have to move it around to try to get the measurements. It is always on the rays. With protractor it doesn't always fit and you have to make the lines longer. Sometimes, you just have to guess with the protractor but with the GeoLeg I got the exact answer every time. I was more accurate with the GeoLeg. With the protractor I can never keep it steady and it always moves. My fingers get in the way and my protractor was a solid color so I couldn't really see the numbers through it. Using the GeoLeg was just easy and simple. Once I learned how to use it, it was great. I was able to do things 10 times faster with the GeoLeg than the protractor."

Interviewer: "Tell me about your experiences in this geometry unit." **ST8** "It made a lot of things easier because it wasn't just a protractor. It also has like a compass and a ruler and you can put them together. I guess it was because we don't have to have so many things to work with and just the way it is."

Interviewer: "How would you compare the protractor, compass and GeoLeg?" **ST8** "The protractor had the whole thing and it was hard to figure out. This just has two sides of the angle. The protractor has just the stick and it is confusing because it has too many numbers and lines to figure out where to put the angle. The GeoLeg makes it easier because it just has the two sides of the angle that you need. All you have to do is line up the blue and the red lines. This makes it easier because you can see it better. And it is clear to see through. Sometimes protractors aren't and it makes it hard to read the numbers or tell which line the angle is on. The GeoLeg is easier because you can just see everything. I think I was more accurate with the GeoLeg."

Interviewer: "Tell me about your experiences in this geometry unit." **ST9** "It was really easy to use with all the angles and stuff. It was easy because you already have the angle in front of you. It is so much easier than a protractor because you get confused with the sides and the lines and numbers. I always pick the wrong numbers with the protractor. This is the first time I used a GeoLeg. I could use this better. It was easy."

Interviewer: "Tell me about your experiences in this geometry unit." **ST12** "Well the GeoLeg helped me understand and measure better. It also helped me draw circles better. It was better than using the old techniques of the protractor and compass. It is easier to measure angles with and it has a ruler so I can also measure in centimeters. It really helped me learn a lot."

Interviewer: "How would you compare the protractor, compass and GeoLeg?" **ST12** "It was much easier. With a compass you have to put it on the paper and draw a circle and it is hard to use. Sometimes the lines go all over and it is really hard to use. The GeoLeg is easy to use and I can make a perfect circle every time. I can also make different sizes really easily. The GeoLeg was easier to use than a protractor. My teacher explained to us step-by-step how to use it. It was kind of confusing at first because we never used them before, but once I practiced I got it. No problem. It was fun to use. We also put them together to make triangles and squares and looked at the angle degrees. It was really fun. I had never seen anything like this before. I didn't know you could actually do that."

Interviewer: "Tell me about your experiences in this geometry unit." **ST13** "I never used this tool before. When I first started using it, it was kind of confusing because I would try to measure and I would accidentally made it backwards. But then after a couple of different tries.. it would be ahh easy and I would know how to work it."

Interviewer: How would you compare the protractor, compass and GeoLeg? **ST13** "This one is easier and better because it is a ruler and you can make angles and you can measure angles all with just one tool. It is better because you don't have to get out the protractor and the ruler and a bunch of different things. You can just have one thing to use. It was easier to use than other tools. It is one tool. It has the measures on it and it is very easy to get angle measures and get them right. It is much easier to understand. You only have to have one tool on your desk instead of having a protractor, ruler and compass. You can just use this to do everything."

Interviewer: "Tell me about your experiences in this geometry unit." **ST14** "Well, the GeoLeg at first was a bit confusing since it has a protractor with 360 degrees. But once I got used to it, I was pretty good. I could measure angles with no problem. It was good for measuring and stuff. My favorites were the ones that we did the drawings with the GeoLeg. I also liked the angle measurements and circles. They were fun. I liked putting the GeoLegs together to make triangles. I think it was pretty cool that you could make shapes with these. I think that I understood the angles better when I saw them put together like this. Using the GeoLeg was easier. With a regular protractor you can't make shapes and it is harder to read than the GeoLeg. The GeoLeg compass is not that much different than other compasses but it is easier to use. The GeoLeg is more efficient and easier to use. It makes learning easier because you can concentrate on learning the geometry instead of trying to figure out how to use the tools."

Summary of Qualitative Findings

Teachers were very positive about the study and their experiences using the GeoLeg manipulative tool. The teacher experience ranged from first-year teachers to 20-year veteran teachers. They repeatedly commented on the ease of use of the GeoLeg tool as compared to the traditional tools; repeated frustrations when using the protractor; difficulties students had using a protractor; and an increased level of student engagement when using the GeoLeg tool.

Students in the control group confirmed the teacher comments regarding their frustration using the protractor. Comments from the students included: "it was a little bit confusing" (ST1) and "you can't really line it (protractor) up and it is really hard to tell which degree it is" (ST11). The students reiterated confusion in measurement of angles difficulty with the general use of the protractor, and estimation compensation strategies they used in order to solve problems when asked to get exact measurements. For example, "I had a hard time telling exactly which number I was supposed to use on the protractor" (ST1) " I double checked to get real close" (ST2) and "the protractor was really hard with all of the lines to measure the angles" (ST11). Finally, the students, as with the teachers, also stated that they enjoyed the unit despite their difficulties. For example, "my teacher worked hard to explaining to us" (ST1); "it was better that the geometry stuff I have done before" (ST2); "we did fun activities" (ST2); "I liked doing the triangles" (ST3); "it was really good because it got you thinking" (ST7); and "I really liked it. I thought it was pretty cool how we could use all of the materials like the protractor and compass to help us learn" (ST10).

The treatment group students also confirmed the teacher comments as they described their experiences during the geometry unit. The students repeatedly commented that the GeoLeg manipulative tool was easy to use. The following are examples of student comments: "it (GeoLeg) is easier. I think the GeoLeg makes it much more accurate because it is a lot harder to make the angles with the protractor" (ST4); "measuring the angles were a lot easier because the GeoLeg helps you get the exact measure" (ST15); "I think that the GeoLeg is awesome. It made it 10 times easier than using a protractor. It is easier to find measurement" (ST6); "the GeoLeg is easy to use and I can make a perfect circle every time" (ST12); "it (GeoLeg) was easy because you have the angle right in front of you" (ST9); and "the GeoLeg was easier to use than a protractor" (ST12).

The students liked the fact they only had to work with one tool. For example, ST13 stated, "It (GeoLeg) is one tool. It has the measures on it and it is very easy to get angle measures and get them right. You only have one tool on your desk instead of having a protractor, ruler and a compass. You can use this to do everything."

The students reported that they found the GeoLeg tool easy to read as they worked with angle measurement, triangles, and circles. Examples of student responses include: “triangles were my favorite because when you connected three GeoLegs you can make a triangle… It is better than the one just on your paper… It made it a lot easier for me to understand” (ST15); and “All you have to do is line up the rays with the red and blue line and you have the measurement. There is no guessing” (ST6).

The students expressed that they understood the geometry better. Some examples of student responses are: “sometimes, you have to guess with the protractor but with the GeoLeg I got the exact answer every time” (ST6); and “the GeoLeg helped me understand and measure better. It helped me draw circles better. It is easier to measure angles. It really helped me learn a lot” (ST12).

Finally, several students compared the use of the GeoLeg tool to their prior experiences with the protractor and confirmed the same difficulties that the teachers and the students in control had with that tool. The students said, “This is easier and better because it is a ruler and you can make angles and you can measure angles all with just one tool” (ST13); and “the GeoLeg compass is not that much different than other compasses but it is easier to use…The GeoLeg is more efficient and easier to use... It makes learning easier because you can concentrate on learning the geometry instead of trying to figure out how to use the tools” (ST14).

Summary of the Chapter

The researcher addressed a total of 10 research questions in Chapter 4. The results demonstrated that the students in the treatment group had a statistically significantly larger average posttest score than the students in the control group. The null hypothesis was rejected and it was concluded that the teaching method using the GeoLeg teaching method produced better academic performance compared to the standard teaching method using a protractor, ruler and compass.

The results from research questions 2 through 10 were that they did not reject the null hypothesis and therefore concluded the following: There was no difference in the average posttest scores between males and females among students who were taught using the GeoLeg manipulative tool; there was no difference between posttest scores and learning styles; and learning styles do not moderate the effect of the treatment on the posttest scores.

Chapter 5: Discussions, Recommendations, and Conclusions

This chapter will discuss the results of the study presented in Chapter 4. This study sought to a) determine the effects of the use of the GeoLeg mathematical manipulative tool with seventh grade students in learning geometric mathematical concepts; b) determine if the use of the GeoLeg manipulative tool has the same effectiveness for seventh grade males and females when learning geometry concepts; c) determine if there is a relationship between the student learning style of “Reflector” as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest; d) determine if there is a relationship between the student learning style of “Theorist” as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest; e) determine if there is a relationship between the student learning style of “Pragmatist” as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest; f) determine if there is a relationship between the student learning style of “Activist” as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest; g) determine if the “Reflector” learning style, as identified by the Honey and Mumford student survey, moderates the effect of the GeoLeg manipulative tool intervention on the geometry posttest scores; h) determine if the “Theorist” learning style, as identified by the Honey and Mumford student survey, moderates the effect of the

GeoLeg intervention on the geometry posttest scores; i) determine if the "Pragmatist" learning style, as identified by the Honey and Mumford student survey, moderates the effect of the GeoLeg intervention on the geometry posttest scores; and j) determine if the "Activist" learning style, as identified by the Honey and Mumford student survey, moderates the effect of the GeoLeg intervention on the geometry posttest scores.

Discussion of Findings

Research Question 1. Do seventh grade students who use the GeoLeg manipulative tool while learning geometry concepts score significantly higher on a posttest than those students who do not use the GeoLeg manipulative tool while learning geometry concepts using the same curriculum?

There is very strong evidence to show that the use of the GeoLeg manipulative tool produces better student outcomes on a Geometry posttest. The control group contained 91 students (28.7%) and the treatment group contained 226 students (71.3%). Tables 2 and 3 reveal that the treatment group had a statistically significantly larger average posttest score than the control group. The average (standard deviation) posttest score was 77.1 (19.3) versus 60.5 (22.6) for the treatment and control groups respectively. The effect size was $d=0.79$. According to Cohen (1988), small, medium, and large effect sizes for a two-sample t-test are $d=0.2$; $d=0.5$; and $d= 0.8$ respectively. Thus, the difference in the posttest scores between the treatment and control group reflects a large effect size.

When reflecting upon the methodology and controls implemented in this research, I cannot think of any alternative explanation for the difference in the results other than the effectiveness of the GeoLeg manipulative tool intervention. I am confident that the use of the GeoLeg manipulative tool produced the better student performance compared to the traditional teaching method of using a protractor, ruler, and compass in this particular school setting.

However, it cannot be ascertained what population these results would generalize to. This research was conducted at only one school site. The school site used in this study is unique, although it is likely there are many other schools with the same unique features. While it is not possible to say with certainty which other schools these study results generalize to, it is plausible that these results could generalize to other schools that are similar to this setting in terms of teacher quality, teacher qualifications, teacher experience, administrative support, the socio-demographic characteristics of the community, student demographics, and all of the characteristics that makes this school site a unique learning community.

The findings of question one are consistent with the GeoLeg manipulative tool student achievement findings in the Oklahoma Shawnee School District (Paul, 2006). Both of these studies reported a statistically significant improvement in student achievement. These findings are also consistent with research indicating that concrete manipulatives have a positive effect on student achievement (Chester, Davis, & Reglin 1991; Howley & Bowen, 1993; Olkun, 2003; Raymond & Leinenbach, 2000; Witzer, Mercer, & Miller, 2003).

The findings for question one were supported by teacher observations regarding student engagement with the GeoLeg manipulative tool. For example, Brenda stated, “ I think it (GeoLeg) really helped because they (the students) actually worked with the tool.” Consistent with this observation, Charles said, “I have seen a lot of engagement with this (GeoLeg).” The teacher comments were especially poignant in regards to engaging students who are normally disruptive or more difficult to engage. Deborah specifically spoke to this issue, stating that these particularly challenging students were engaged with the GeoLeg manipulative tool, doing their work, engaged in the activities, not disruptive, and displaying understanding.

The comparisons that the teachers made between the control and treatment groups were also consistent with the findings. All three teachers who had both control and treatment groups found that the students who used the GeoLeg were able to understand the skills and concepts easier than those students who were using the protractors, rulers and compasses. For example, Eddie expressed frustration as he explained that he had to re-teach the measurement skills multiple times because the students had a more difficult time using the traditional protractor, compass, and ruler, as compared to the ease of instruction he experienced when teaching the same skills with the GeoLeg manipulative tool. This was supported by ST1, a student in the control group, who stated, “I couldn’t tell exactly what number it was when doing the angle measurements. I got a lot wrong and it was frustrating. My teacher gave me a lot of extra help.”

Eddie also compared student engagement. With the GeoLeg manipulative tool group, “everybody was participating. I was getting more correct answers from other students. I was even calling on students that don’t normally answer and they got the correct answers.” However, Eddie did not notice any difference in the behavior of the students in the control group. “It was pretty much the same kids raising their hands.”

Research Question 2. Does the use of the GeoLeg manipulative tool have the same effectiveness for seventh grade males and females when learning geometry concepts?

The findings revealed that there was no statistically significant difference between male and female students with respect to the posttest scores within the treatment group. With a large sample size of 226 students in the treatment group, if there was a large difference between male and female performance it most likely would have been detected. This finding is consistent with the very large and compelling national study NAEP (2009). It can be concluded that there is no difference in the effectiveness of the GeoLeg manipulative tool teaching method between the two genders. Therefore, the GeoLeg manipulative tool appears to work equally well with both genders. The findings of this question are consistent with several studies that found no gender differences in mathematical performance (Hyde, Lindberg, Ellis, & Williams, 2008; LaChance & Mazzocco, 2005, Marshall & Smith, 1987).

Research Question 3. Is there a relationship between the student learning style of "Reflector" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest?

This question was designed to examine whether or not the Reflector learning style was associated with the posttest scores when examining all of the students within the study. The Pearson correlation statistic (Table 7), reveals that the r-value is very close to 0 (.018) with a very large p-value (.749), demonstrating that there is not a statistically significant correlation between the posttest score and the Reflector learning style. It can be concluded from the findings that whether a student has a strong tendency toward a Reflector learning style or a weak tendency toward a Reflector learning style, the student posttest scores were not likely to be different. Regardless of where the student fell on the Reflector continuum, from low to high, the student was equally likely to perform well on the posttest geometry assessment. Therefore, with this large sample size, it can be concluded with confidence that there is no correlation between the extent to which a person has a Reflector Learning style and their performance on the posttest.

Research Question 4. Is there a relationship between the student learning style of "Theorist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest? The Pearson correlation statistic (Table 8), reveals that the r-value is very close to 0 (.102) with a p-value of .071, demonstrating that there is not a statistically significant correlation between the posttest score and the Theorist learning style. It can be concluded from the findings that whether a student has a strong tendency toward a Theorist learning style or a weak tendency toward a Theorist learning style, the student posttest scores were not likely to be different. Regardless of where the student fell on the Theorist continuum, from low to high, the student was equally likely to perform well on the posttest geometry assessment. Therefore, with this large sample size, it can be concluded with confidence that there is no correlation between the extent to which a person has a Theorist Learning style and their performance on the posttest.

Research Question 5. Is there a relationship between the student learning style of "Pragmatist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest? The Pearson correlation statistic (Table 9), reveals that the r-value is very close to 0 (.066) with a p-value of .245, demonstrating that there is not a statistically significant correlation between the posttest score and the Pragmatist learning style. It can be concluded from the findings that whether a student has a strong tendency toward a Pragmatist learning style or a weak tendency toward a Pragmatist learning style, the student posttest scores were not likely to be different. Regardless of where the student fell on the Pragmatist continuum, from low to high, the student was equally likely to perform well on the posttest geometry assessment. Therefore, with this large sample size, it can be concluded with confidence that there is no correlation between the extent to which a person has a Pragmatist Learning style and their performance on the posttest.

Research Question 6. Is there a relationship between the student learning style of "Activist" as identified by the Honey and Mumford Learning Style Questionnaire (LSQ) and student achievement on a geometry unit posttest? The Pearson correlation statistic (Table 10), reveals that the r-value is very close to 0 (.073) with a p-value of .194, demonstrating that there is not a statistically significant correlation between the posttest score and the Activist learning style. It can be concluded from the findings that whether a student has a strong tendency toward an Activist learning style or a weak tendency toward an Activist learning style, the student posttest scores were not likely to be different. Regardless of where the student fell on the Reflector continuum, from low to high, the student was equally likely to perform well on the posttest geometry assessment. Therefore, with this large sample size, it can be concluded with confidence that there is no correlation between the extent to which a person has an Activist Learning style and their performance on the posttest.

The research findings for questions three through six are consistent with other research findings (Garton, Spain, Lamberson, & Spiers, 1999; Hillis, 2009; Yushau, 2005). However, the methods of identifying learning styles and comparing learning styles to student achievement used in these studies are varied. Additionally, the populations ofthese studies are not matched to this research.

On the other hand, the findings of this research are not consistent with the findings of Kopsovich (2001). However, the Kopsovich study used the Dunn and Dunn instrument and standardized test scores. Although the population is closer to that of this research study, the methodology is too different to compare.

Research Question 7. Does the "Reflector" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg manipulative tool intervention on the geometry posttest scores?

The null hypothesis was not rejected and therefore there was no evidence to suggest that a Reflector learning style moderates the effect of the teaching method on posttest outcomes. These results are consistent with the conclusion that the GeoLeg manipulative tool is equally effective regardless of the extent to which a student possesses a Reflector learning style.

Research Question 8. Does the "Theorist" learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

The null hypothesis was not rejected and therefore there was no evidence to suggest that a Theorist learning style moderates the effect of the teaching method on posttest outcomes. These results are consistent with the conclusion that the GeoLeg manipulative tool is equally effective regardless of the extent to which a student possesses a Theorist learning style.

Research Question 9. Does the “Pragmatist” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

The null hypothesis was not rejected and therefore there was no evidence to suggest that a Pragmatist learning style moderates the effect of the teaching method on posttest outcomes. These results are consistent with the conclusion that the GeoLeg manipulative tool is equally effective regardless of the extent to which a student possesses a Pragmatist learning style.

Research Question 10. Does the “Activist” learning style, as identified by the Honey and Mumford student survey, moderate the effect of the GeoLeg intervention on the geometry posttest scores?

The null hypothesis was not rejected and therefore there was no evidence to suggest that an Activist learning style moderates the effect of the teaching method on posttest outcomes. These results are consistent with the conclusion that the GeoLeg manipulative tool is equally effective regardless of the extent to which a student possesses an Activist learning style.

Recommendations

The results of this research lead to the consideration of the use of the GeoLeg manipulative geometry tool with seventh grade students in similar settings to this study. For example, schools may consider implementing this manipulative tool in a slow, “phase-in” model in which they can monitor achievement and progress of students. This tool may yield similar results to this study in any school site with similar demographics.

Recommendations for Further Study

The following recommendations are made:

1. A similar research study should be conducted with seventh grade students' to examine if the GeoLeg manipulative tool affects student's achievement in student populations similar to the educational setting of this study. Replication of this research will add validity to the findings.

2. A similar research study should be conducted with seventh grade students to examine if the GeoLeg manipulative tool affects student achievement in a student population that is particularly different to that of this study. For example, a school site with a different demographic signature may produce varying results in student achievement.

3. A similar research study should be conducted to examine if the GeoLeg manipulative tool affects student achievement at different grade levels in a student population that is similar to that of this study. A study of this type may produce information regarding the degree of effectiveness of the GeoLeg manipulative tool at specific grade levels.

4. A similar research study should be conducted to examine if the GeoLeg manipulative tool affects student achievement at different grade levels in a student population that is particularly different to that of this study. A study of this nature may shed light not only on the degree of effectiveness of the GeoLeg manipulative tool at specific grade levels, but may also present insight into student achievement of varying subgroups.

5. A further research study should be conducted to examine if the GeoLeg manipulative tool affects student achievement for males and females of different ethnic groups or different socio-economic status. For example, do students within a given ethnic group achieve equally well using the GeoLeg manipulative tool? The examination of subgroup achievement may shed light on particular achievement gaps.

6. A further research study should be conducted to examine the international student achievement in countries such as Canada and Korea using the GeoLeg manipulative tool, and then compare these results to those of students in the United States.

7. A further research study should be conducted using an alternative learning styles assessment in order to add validity to the findings of this study.

Summary and Conclusions

In conclusion, this mixed method research study sought to determine if the use of the GeoLeg manipulative tool had an effect on seventh grade student achievement. Additionally, the research examined whether or not the GeoLeg manipulative tool had the same effectiveness for males and females when learning geometry. Finally, the research sought to determine whether or not learning styles impacted the geometry posttest scores.

The research findings revealed that students using the GeoLeg manipulative tool produced significantly better student performance on a posttest in this particular school setting. Although these results cannot be generalized to other school sites, it is plausible that these results could generalize to school sites whose demographics are similar. The demographics of the school in this study could be similar to hundreds of schools in the United States.

The research findings revealed that there was no statistically significant difference between male and female students within the treatment group. The significant finding is that the GeoLeg manipulative tool appears to work equally well with both genders.

None of the learning styles, as identified by the Honey and Mumford Learning Styles Questionnaire, were correlated with student posttest score achievement on the tested geometry unit. In addition, there was no evidence to suggest a student's learning style moderates the effectiveness of the use of the GeoLeg manipulative tool. Based upon the results of this study, the use GeoLeg manipulative tool produces better student performance than the traditional teaching method using rulers, protractors, and compasses.

This is a significant study that researched a manipulative tool that had no prior research. With over one million students currently using this tool in the United States, Canada, and Korea, the findings of this study will add to the body of knowledge in regard to the use of manipulatives in mathematics. These findings will aid administrators and teachers as they make instructional material choices for their students.

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APPENDIX A

PARENTAL CONSENT FORM

1) Title of Research Study: A STUDY OF SEVENTH GRADE GEOMETRY POSTTEST SCORES AFTER USING THE GEOLEG MANIPULATIVE TOOL.

2) Investigator(s): Joseph M. Furner, Ph.D. and Phyllis T. Pacilli, (doctoral student)

3) Purpose: The purpose of this research study is to assess how the use of a hands-on geometry tool affects seventh grade student achievement.

4) Procedures: As part of your child's participation in this research study he/she will take a fifteen-minute on-line survey, which will be administered by computer at your child's school. Each student will also participate in a two-week geometry unit. The material will include district-approved curriculum and correlate to the state mathematics standards. Your child's class will be randomly selected to use either the standard geometry tools (protractor, compass and ruler) or new geometry tool. At the end of the chapter, all students will take a test on the material that was covered in class. This testing procedure will be similar to the testing procedure that is currently used during the school year. After the researcher receives the results of the test, your child may be asked to participate in a 30-minute audiotaped interview with the researcher. Approximately 6 to 10 will be asked to interview.

Alternative Procedures: If you do not want your child to participate, then she/he will be allowed to work in the class as they normally would and no data will be collected from your child.

5) Risks: The risks involved with participation in this study are no more than your child would experience in their regular daily activities. It is unlikely your child will experience any harm or discomfort. There is a very small risk that the on-line data could be compromised and others could know your child's responses. However, on-line security measures including encrypting data will be taken to protect confidentiality.

6) Benefits:
Your child may gain a greater knowledge of mathematics, his/her learning style and the satisfaction of knowing that they have contributed to a better understanding of mathematical knowledge in middle school students in South Florida.

7) Data Collection & Storage: Any information collected about your child will be kept confidential and secure and only the people working with the study will see your child's data, unless required by law. Students will take an on-line survey that will be managed electronically through Survey Monkey. Through this service, there will be multiple layers of security to try to make certain that that the account and the data remain private and secure. The researcher chose Survey Monkey because this service employs a third party firm that conducts daily audits of the system security. All collected data resides behind

the latest firewall and intrusion prevention technology. The researcher will also pay for an additional service that will add SSL to the account. This extra measure will keep all collected data in an encrypted environment. Although extra security measures will be taken, there stills remains a slight measure of risk with any on-line data collection that the data could be compromised. Once the on-line data is collected and the reports have been generated, all data will be removed from the website and destroyed. The researcher will make every effort to complete these tasks in an expeditious manner.
Although extra security measures will be taken in order to keep all data confidential, there still remains a slight measure of risk that the online data could be comprised. All data, including on-line results, posttest results and interview information will be kept locked and secured until it has been processed and then destroyed.

8) Contact Information: *For questions or problems regarding your child's rights as a research subject, you can contact the Florida Atlantic University Division of Research at (561) 297-0777. For other questions about the study, you should call the principal investigator(s), Dr. Joseph M. Furner at (XXX) XXX-XXXX or Phyllis T. Pacilli at (XXX) XXX-XXXX.

9) Consent Statement: *I have read, or had read to me, the information describing this study. All of my questions have been answered to my satisfaction. I will allow my child to take part in this study. My child can stop participating at any time without giving any reason and without penalty. I can ask to have the information related to my child returned to me, removed from the research records, or destroyed. I have received a copy of this consent form.

I agree ____ I do not agree ___ to be audiotaped.

Signature of Parent or Guardian:____________________________ Date: ____________

Signature of Investigator: _________________________________ Date: ____________

APPENDIX B

TEACHER CONSENT FORM

1) Title of Research Study: A STUDY OF SEVENTH GRADE GEOMETRY POSTTEST SCORES AFTER USING THE GEOLEG MANIPULATIVE TOOL.

2) Investigator: Joseph M. Furner, Ph.D. and Phyllis T. Pacilli, (doctoral student)

3) Purpose: The purpose of this research study is to assess how the use of hands on materials affects seventh grade student achievement.

4) Procedures:
Participation in this study will require you to 1) participate in a three hour professional development, 2) teach an agreed upon geometry unit using the same methods with both your treatment group (those using the GeoLeg tool) and your control group (those using standard tools of a protractor, compass and ruler), 3) allow the researcher to observe classroom instruction for two full class periods and record field notes, 4) provide researcher with a copy of all instructional material during classroom observations, 5) participate in a 30 minute post-treatment audiotaped interview with the researcher and 5) provide a copy of student posttests to the researcher.

5) Risks:
The risks involved with participation in this study are no more than you would experience in your regular daily activities. Every effort will be made to keep all collected data from interviews and field notes secure and confidential. Research will use a pseudo name when identifying all participants in order to protect confidentiality.

6) Benefits:
Potential benefits that subjects may attain from participation in this research study include a greater knowledge of how students learn mathematics and the satisfaction of knowing that you have contributed to a better understanding of mathematical knowledge in middle school students in South Florida.

7) Data Collection & Storage:
All of the results will be kept confidential and secure and only the people working with the study will see your data, unless required by law. The data will be kept in a locked cabinet and computer files will be password protected until data has been processed, approximately six months, and then destroyed. Research will use a pseudo name when identifying all participants in order to protect confidentiality.

8) Contact Information:
*For related problems or questions regarding your rights as a research subject, contact the Florida Atlantic University Division of Research at (561) 297-0777. For other questions about the study, you should call the principal investigators, Dr. Joseph M. Furner at (XXX) XXX-XXXX or Phyllis T. Pacilli at (XXX) XXX-XXXX.

9) <u>Consent Statement:</u>
*I have read or had read to me the preceding information describing this study. All my questions have been answered to my satisfaction. I am 18 years of age or older and freely consent to participate. I understand that I am free to withdraw from the study at any time without penalty. I have received a copy of this consent form.

I agree ____ I do not agree ___ to be audiotaped.

Signature of Subject: ______________________________ Date: ________________

Signature of Investigator: __________________________ Date: _________________

APPENDIX C

CHILD ASSENT

Title of Research: A STUDY OF SEVENTH GRADE GEOMETRY POSTTEST SCORES AFTER USING THE GEOLEG MANIPULATIVE TOOL.

Researchers from Florida Atlantic University in the College of Education are trying to learn about the effectiveness of a hands-on geometry tool with seventh grade students. You have been asked to participate because the seventh grade students in your school have been selected to participate in this study. If you decide to participate in this study, you will be asked to complete an on-line survey (about 15 minutes) and to participate in regular instruction by your teacher in a geometry chapter. Your class may be randomly selected to use either the standard geometry tools (protractor, compass and ruler) or new geometry tool. At the end of the chapter, you will take a chapter test. Some students may be asked to participate in an audiotaped interview with the researcher. You will be asked about your thoughts on the geometry chapter and the activities you participated in. Some things may make you slightly uncomfortable such as describing how you best learn in the on-line survey. This study will take place in your regular math classroom at Logger's Run Middle School and should take about ten classes.

The researchers hope this study will help explain whether or not specific geometry tools help students learn geometry better.

You do not have to be in this study if you don't want to and you can quit the study at any time. If you don't like a question, you don't have to answer it and, if you ask, your answers will not be used in the study. No one will get mad at you if you decide you don't want to participate.

Other than the researchers, no one will know your answers, including strangers, your friends, other students or other teachers. If you have any questions, just ask the researcher, Miss Phyllis T. Pacilli.

I agree ____ I do not agree ___ to be audiotaped.

This research study has been explained to me and I agree to be in this study.

_______________________________________ ____________________

Subject's Signature for Assent **Date**

APPENDIX D

STUDENT LEARNING STYLE QUESTIONNAIRE

The following is a document representing the on-line survey. All choices were buttons from which student chose.

How Students Think and Learn

The following questions are designed to look at how people think and learn.

All of your answers will be kept confidential and secure and only the people working with this study will see the information collected. There are no names attached to your survey.

Please answer your questions as honestly as possible.

Thank you for taking the time to answer the following questions.

About Me

Check the answer that BEST describes you.

I am a Male Female

I am in the 6th Grade 7th Grade 8th Grade

Please complete the following information. After the survey is complete, □a report will be given to you from the researcher about how you think and learn. □□THE ONLY PERSON WHO HAS ACCESS TO YOUR INFORMATION IS THE□RESEARCHER. YOUR INFORMATION WILL STAY ANONYMOUS AND NOT SHARED WITH OTHERS.

First Name :
Last Name :
School :
Math Teacher :
Math Period :

Questions

There is no time limit for the survey. The whole survey will probably take between 10-15 minutes.

The accuracy of the survey depends on your honesty when answering the questions. There are no right or wrong answers. Check the box that you MOST AGREE WITH.

1. I have strong beliefs about what is right and wrong. Yes No

2. I often act without considering the possible consequences. Yes No

3. I tend to solve problems using a step-by-step approach. Yes No

4. I believe that rules, procedures and policies restrict people. Yes No

5. I am known for saying what I think, straight forward and direct. Yes No

6. I think that things that are done based on feelings are as good as those that are done after being carefully thought out. Yes No

7. I like jobs that give me enough time to thoroughly prepare for what I need to do as well as enough time to completely finish the job. Yes No

8. I regularly question people about their ideas. Yes No

9. What matters most is whether something works in real-life. Yes No

10. I like to look for new experiences. Yes No

11. When I hear a new idea or concept, I immediately try to find ways to make it work in real-life. Yes No

12. I am self-disciplined. For example, I watch my diet, regularly exercise, and have a regular routine. Yes No

13. I take pride in doing a carefully detailed job. Yes No

14. I get along best with people who are able to think sensibly and come to conclusions based on facts RATHER THAN people who think with their emotions and are impulsive and DO NOT think things out logically. Yes No

15. I carefully look at the facts available to me and I avoid jumping to conclusions. Yes No

16. I like to reach my decisions carefully after thinking about the options and choices. Yes No

17. I like new, unique and creative ideas rather than ideas that others might call sensible. Yes No

18. I DO NOT like things that are unorganized. I prefer that things are orderly.

Yes No

19. I accept and stick to rules and procedures as long as I think they are the best way to get things done. Yes No

20. I like to relate the things that I do to a general rules and principles.

Yes No

21. When talking to others, I like to get right to the point.

Yes No

22. I usually have distant and formal relationships at school (people I say hi to but don't hang out with or talk to outside school). Yes No

23. I love the challenge of working on something that is somewhat difficult, new and different. Yes No

24. I like spontaneous and fun-loving people. Yes No

25. I pay very close attention to details before I come to conclusions.

Yes No

26. It is hard for me to come up with ideas on the spur of the moment.

Yes No

27. I believe in getting to the point right away.

Yes No

28. I am careful when I hear something NOT to jump to conclusions too quickly.

Yes No

29. I prefer to have lots of information and facts available to me … the more the better for me to think over. Yes No

30. People who DO NOT take things seriously when they should usually irritate me.

Yes No

31. I listen to other people's ideas and thoughts before I tell my own ideas and thoughts.

Yes No

32. I am usually open about how I feel. Yes No

33. When talking to others, I like to watch how others act. Yes No

34. I prefer to be spontaneous and flexible rather than plan out everything in advance.

Yes No

35. I like things that are carefully planned out. Yes No

36. It bothers me when I rush through my work to get it in on time.

Yes No

37. I tend to judge ideas that I hear based on whether it is valuable in everyday life.

Yes No

38. People who are quiet and thoughtful make me uncomfortable.

Yes No

39. People who want to rush things annoy me. Yes No

40. It is more important to enjoy the present than to think about the past or future.

Yes No

41. I think that decisions that are based facts are better than ones based on instincts, feelings and emotions. Yes No

42. I tend to be a perfectionist. Yes No

43. When talking to other people, I am usually spontaneous, upfront, and tell them whatever ideas are on my mind. Yes No

44. When talking to others, I usually have ideas that are practical, full of common sense and based on facts. Yes No

45. More often than not, rules are made to be broken. Yes No

46. I prefer to stand back, watch things and take it all in. Yes No

47. When talking to others, I can often see weaknesses and contradictions in their ideas, reasoning and arguments. Yes No

48. I talk more than I listen. Yes No

49. I usually see better and more practical ways to do things. Yes No

50. I think that written reports should be short and to the point. Yes No

51. I believe that practical and logical thinking is best. Yes No

52. My discussions with others are usually on specific things and topics rather than unimportant, meaningless and trivial topics. Yes No

53. I like be with people who are practical, realistic and look at the facts of the real world

rather than people who are hypothetical (things that might happen). Yes No

54. When talking to others, I get impatient when they talk about things that I think are not important. Yes No

55. When I have an assignment to write, I write lots of drafts before the final version.

Yes No

56. I like to try new things to see if they really work. Yes No

57. I prefer to find answers that are well organized, logical and based on facts.

Yes No

58. I enjoy being the one who talks a lot. Yes No

59. When I talk to others, I am the one who sticks to the facts and does not guess or speculate about things. Yes No

60. I like to think about many different ways and alternatives that something can be done before I make up my mind. Yes No

61. When I talk to others, I am objective and base my ideas on the facts and NOT on feelings, opinions or emotions. Yes No

62. When talking to others, I am more likely to "lay low" rather than talking to most.

Yes No

63. I like to relate the things I am doing now to my future and "big picture" of things.

Yes No

64. When things do not go well, I am happy to blow it off and "chalk it up to experience."

Yes No

65. I look at wild and spontaneous ideas and actions as impractical and unrealistic.

Yes No

66. It is best to think carefully before acting. Yes No

67. I usually do the listening rather than the talking. Yes No

68. I tend to be tough on people who have a hard time making logical choices.

Yes No

69. Most of the time I believe that the actions I take are acceptable because of the specific results I want to achieve. Yes No

70. I don't mind hurting peoples' feelings as long as what I need to do gets done.

Yes No

71. I DO NOT like to have a specific step-by-step laid out plan and set of directions when I have something to do. Yes No

72. I am usually one of the people who you would call "the life of the party."

Yes No

73. I do what ever is the most useful in a situation to get the job done.

Yes No

74. I quickly get bored with methodical, exact, planned, structured and detailed work.

Yes No

75. I like to explore the meaning behind why things work and happen.

Yes No

76. I am always interested in finding out what people think. Yes No

77. I like classes that are methodical, planned, straightforward and stick to a laid out plan.

Yes No

78. I DO NOT like topics that are based on somebody's opinions and/or feelings. I prefer topics based on facts or have more than one possible meaning or interpretation.

Yes No

79. I enjoy the drama and the excitement of a crisis situation. Yes No

80. People often find me NOT sensitive to their feelings. Yes No

Thank you for participating in this survey.

APPENDIX E

POST-TREATMENT TEACHER INTERVIEW PROTOCOL GUIDE

Title of Study: A STUDY OF SEVENTH GRADE GEOMETRY POSTTEST SCORES AFTER USING THE GEOLEG MANIPULATIVE TOOL.

Name of Interviewer: Phyllis T. Pacilli
Name of Interviewee and position:

Place:

Date:

Starting Time:

Ending Time:

Introduction:

Opening statements: Thank you for taking the time to meet with me. I am conducting this interview with each teacher involved in this study in order to gather information about each teacher after teaching the research geometry unit.

Overview of study: I will be looking at data from multiple sources to determine the extent to which the treatment of the GeoLeg tool is effective when teaching geometry. I am interviewing each teacher involved in this study before and after the teaching of the geometry unit. I have observed you teaching the research geometry unit. Additionally, I will be interviewing selected students to further explore their reactions to the unit.

Promise confidentiality: I promise to keep your name and any identifying information completely confidential. I will identify you with a fake name.

Ask for permission to audiotape: Do you mind if I tape this interview?

Confirm time interview will end: This interview will take 30/45 minutes.

APPENDIX F

POST-TREATMENT STUDENT INTERVIEW PROTOCOL GUIDE

Title of Study: A STUDY OF SEVENTH GRADE GEOMETRY POSTTEST SCORES AFTER USING THE GEOLEG MANIPULATIVE TOOL.

Name of Interviewer: Phyllis T. Pacilli
Name of Interviewee and position:

Place:

Date:

Starting Time:

Ending Time:

Introduction:

Opening statements: Thank you for taking the time to meet with me. I am conducting this interview with a randomly selected group of students involved in this study in order to gather information about students feel after experiencing this geometry unit.

Overview of study: I will be looking at data from multiple sources to determine the extent to which the treatment of the GeoLeg tool is effective when teaching geometry. I am interviewing students involved in this study after the teaching of the geometry unit. I have observed the instruction in your class two times. Additionally, I will be interviewing all of the teachers involved with this study to further explore their reactions to the unit.

Promise confidentiality: I promise to keep your name and any identifying information completely confidential. I will identify you with a fake name.

Ask for permission to audiotape: Do you mind if I tape this interview?

Student background

1. Name

Student Responses to GeoLeg Manipulative Usage

(Have a GeoLeg tool available in case student wishes to demonstrate a point)

1. How do you feel about learning mathematics? (probe) Why?

2. Tell me about your experiences using the geometry tools while learning geometry unit? (probe)

   Hard to use? Why?
   Easy to Use? Why?
   Fun? Why?
   Prefer to use alone? Partner? Groups?

3. **(Only for students who were in treatment group)** How was learning geometry with the GeoLeg tool different from any other way that you have learned geometry in the past? (probe)

4. Would you recommend that teachers use geometry tools when teaching geometry to other students?

5. Is there anything else about your experience you would like to tell me?

APPENDIX G

STUDENT ACTIVITY 1A

Activity 1A
Materials: Protractor, Calculator, Colored Pencils Name ____________________ Date: __________

MEASURE ANGLES

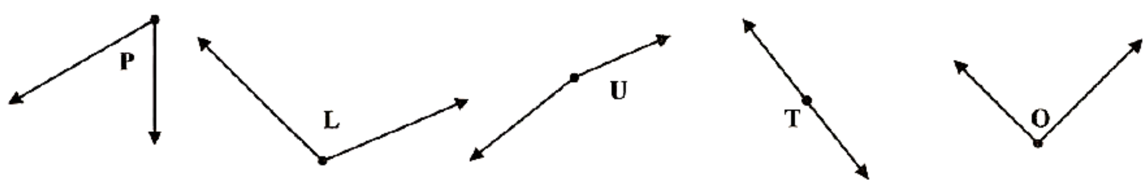
An angle is a part of the circle and gets its measure from the circle. A 90° angle is ¼ of a circle.

Use your protractor to draw a 75° angle (use a different color for each line). The center is the vertex. The two lines are rays. One line is permanently set at 0°. The other gives the angle measure.

1. Look at the angles below.

   a) Estimate their measure (m) to be equal (=) to 90°, less than (<) 90° or greater that (>) 90°. (Ex: m∠X=90°). Use a GeoLeg right angle to compare.

m∠P ________ m∠L ____________ m∠U ____________ m∠T ____________ m∠O ________



b) Classify the angles as right, acute, obtuse, or straight.

∠P ________ ∠L ____________ ∠U ____________ ∠T ____________ ∠O ________

c) Measure the above angles to the nearest 5°.

m∠P ________ m∠L ____________ m∠U ____________ m∠T ____________ m∠O ________

2. Angles are easy to draw using a Protractor.

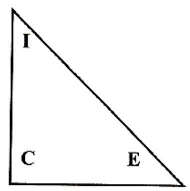
   a) Draw a 90° with your protractor. Mark a point at the vertex and at the 3cm point on each ray. Connect the points to form the angle.

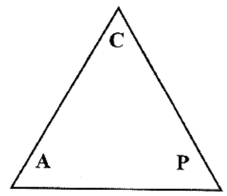
b) Draw the following angles below:

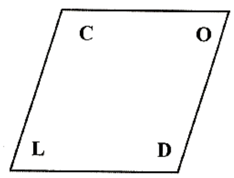
Use your protractor	Use your protractor	Use your protractor
Draw a 150°angle	Draw a 90° angle.	Draw a 45° angle.

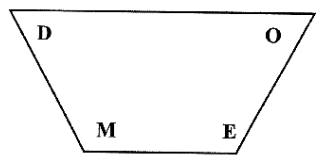
Congruent Angles: Angles are congruent (≅) if they have the same angle measures.

3. Look at the following polygons. Measure their angles and record them inside the angles.





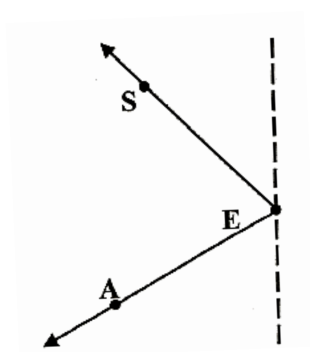




b. Under each polygon, name the congruent angles within that polygon. (ex: ∠A≅ ∠B)

c. In a triangle, there is a relationship between the number of angles and number of sides that are congruent. Measure the lengths in ΔICE and ΔCAP. Write a statement about that relationship.

An angle can be drawn and reflected over a line to show its mirror, congruent image. Measure ∠SEA. Redraw it over the line so that its mirror image is drawn.



APPENDIX H

STUDENT ACTIVITY 1AG

Activity 1AG
Materials: GeoLeg, Calculator, Colored Pencils   Name ______________________   Date: _______

MEASURE ANGLES

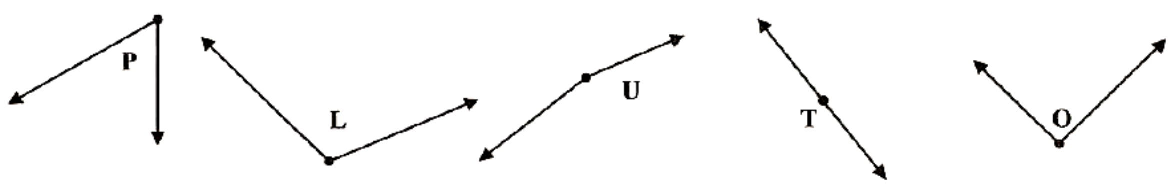
An angle is a part of the circle and gets its measure from the circle. A 90° angle is ¼ of a circle.

Open your **GeoLeg** to see a 75° angle. The center is the vertex. The red and blue lines are rays. The blue line is permanently set at 0°. The red line gives the angle measure.

1. Look at the angles below.

   a) Estimate their measure (m) to be equal (=) to 90°, less than (<) 90° or greater that (>) 90°. (Ex: m∠X=90°). Use a GeoLeg right angle to compare.

m∠P ________ m∠L ____________ m∠U ____________ m∠T ______________ m∠O ______



b) Classify the angles as right, acute, obtuse, or straight.

∠P ________ ∠L ____________ ∠U ____________ ∠T ______________ ∠O ______

c) Measure the above angles to the nearest 5°.

m∠P ________ m∠L ____________ m∠U ____________ m∠T ______________ m∠O ______

2. Angles are easy to draw using a GeoLeg.

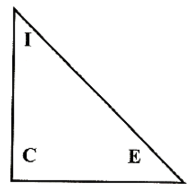
   a. Open a GeoLeg to 90°. Place it on the paper. Mark a point at the vertex and at the 3cm point on each ray. Connect the points to form the angle.

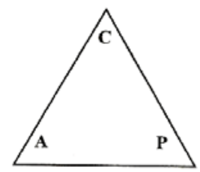
   b. Use your GeoLeg to draw the following angles below:

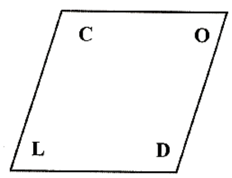
Draw a 150°angle   Draw a 90° angle.   Draw a 45° angle.

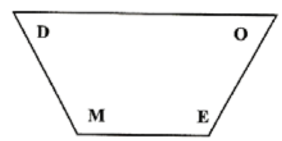
Congruent Angles: Angles are congruent (≅) if they have the same angle measures.

3. Look at the following polygons. Measure their angles and record them inside the angles.



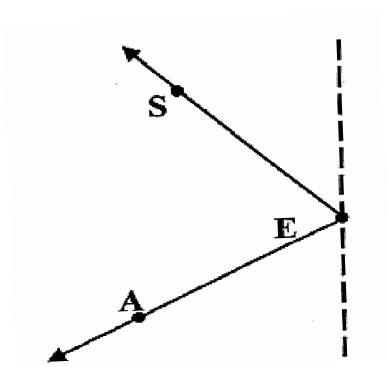






b. Under each polygon, name the congruent angles within that polygon. (ex: ∠A≅ ∠B)
c. In a triangle, there is a relationship between the number of angles and number of sides that are congruent. Measure the lengths in ΔICE and ΔCAP. Write a statement about that relationship.

An angle can be drawn and reflected over a line to show its mirror, congruent image. Measure ∠SEA. Redraw it over the line so that its mirror image is drawn.



APPENDIX I

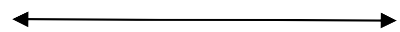
STUDENT ACTIVITY 2A

Activity 2A
Materials: Protractor, Calculator, Colored Pencils Name ____________________ Date: ______

ANGLES – ADJACENT, VERTICAL, COMPLEMENTRAY AND SUPPLEMENTARY

Adjacent Angles are a pair of angles that share a common side and vertex.
Vertical angles are a pair of congruent angles formed by intersecting lines.
Complementary angles are two angles whose sum is 90°.
Supplementary angles are two angles whose sum is 180°.

1. Using your protractor, draw a 75° angle on line *m*.
   b. Place points on your angle to label it angle ABC

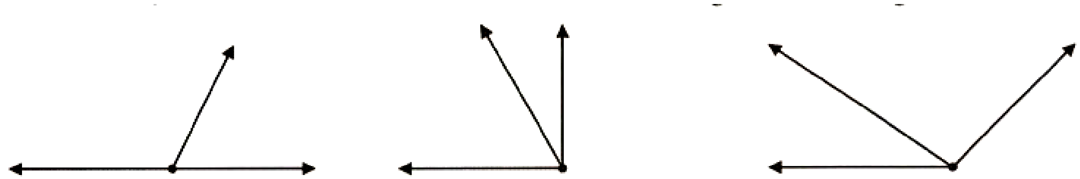


   c. Measure the adjacent angle.
   d. Label the adjacent angle DBA
   e. Use a colored pencil to highlight the common side. What the common side? ______
   f. What is the measure of the straight angle? ______________________________
   g. Write an equation for the sum of these two adjacent angles ____________________

2. Look at the adjacent angles below. Outline the shared side and mark the vertex in red.

b. Measure and record each angle.

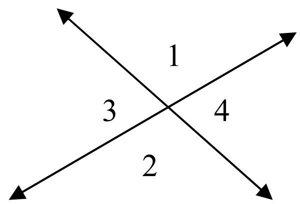
c. Under each write a number sentence to find the sum of the larger combined angle.



____________________ ____________________ ____________________

3. Look at the drawing at the right of intersecting lines.

b. Shade the vertical angles 1 and 2 red; and shade the vertical angles 3 and 4 blue.



c. Measure and record the angles.

d. Compare the measures of angles 1 and 2. ______________________________________

e. Compare the measures of angles 3 and 4. ______________________________________

f. Describe the relationship between the vertical angles ____________________________

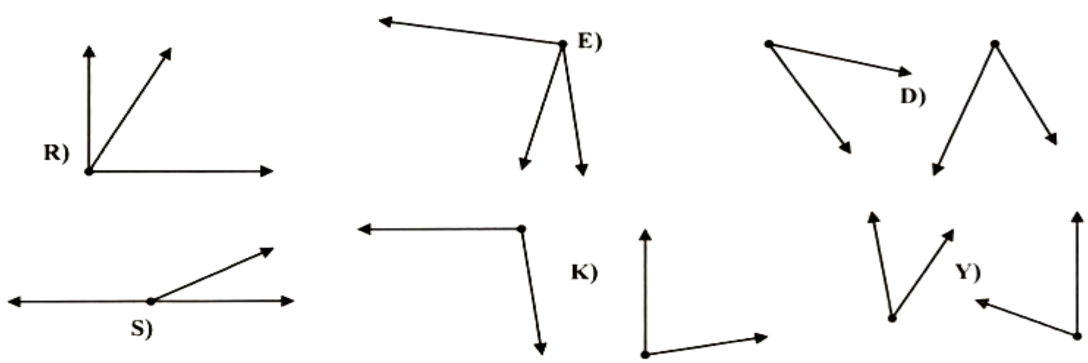
g. What is the sum of ∠1 and ∠3? _________________________

h. What is the sum of ∠1 and ∠4? _________________________

i. What is the sum of ∠2 and ∠3? _________________________

j. What is the sum of ∠2 and ∠4? _________________________

4. Complementary and supplementary angles can be **adjacent or not adjacent angles**. Look at the following pairs of angles. Measure and record each angle measurement.



b. Write a number sentence to show the sum of the angle pairs. State if the pair is complementary, supplementary or neither.

R) ___________________________________ S) ___________________________________

E) ___________________________________ K) ___________________________________

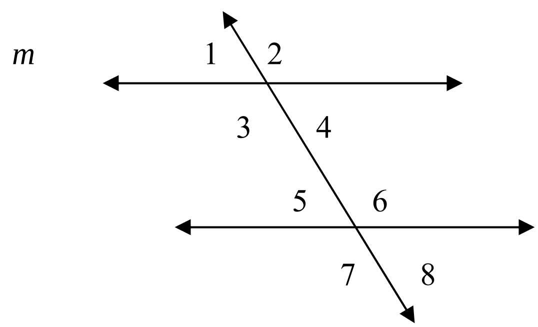
D) ___________________________________ Y) __________________________________

5. If two angles are complements and one of the measurements is 32°.

Write and equation to find the other angle ______________________________

b. Write an equation to find the supplement of a 70° angle. ____________________

c. One angle is 18.5°. Find its complement. ____________________________



6. Look at the parallel lines *m* and *n* that are intersected by transversal *r* at the right.

a. Outline the parallel lines in red and the transversal in blue.
b. Measure and record each angle.

c. What is the relationship between ∠1 and ∠2? ____________________

d. What is the relationship between ∠2 and ∠3? ____________________

e. What is the relationship between ∠1 and ∠5? ____________________

f. What is the relationship between ∠2 and ∠8? ____________________

g. What is the relationship between ∠3 and ∠7? ____________________

h. What is the relationship between ∠3 and ∠5? ____________________

i. What is the relationship between ∠4 and ∠5? ____________________

Describe what you have learned about a pair of parallel lines that are intersected by a transversal.

____

____

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APPENDIX J

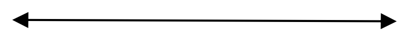
STUDENT ACTIVITY 2AG

Activity 2AG
Materials: GeoLeg, Calculator, Colored Pencils Name ______________________ Date: ______

ANGLES – ADJACENT, VERTICAL, COMPLEMENTRAY AND SUPPLEMENTARY

Adjacent Angles are a pair of angles that share a common side and vertex.
Vertical angles are a pair of congruent angles formed by intersecting lines.
Complementary angles are two angles whose sum is 90°.
Supplementary angles are two angles whose sum is 180°.

1. Using your GeoLeg tool, draw a 75° angle on line *m*.
   b. Place points on your angle to label it angle ABC

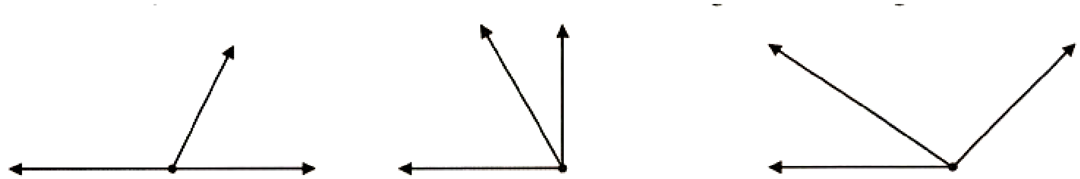


   c. Measure the adjacent angle.
   d. Label the adjacent angle DBA
   e. Use a colored pencil to highlight the common side. What the common side? ______
   f. What is the measure of the straight angle? ____________________________________
   g. Write an equation for the sum of these two adjacent angles ____________________

2. Look at the adjacent angles below. Outline the shared side and mark the vertex in red.

b. Measure and record each angle.

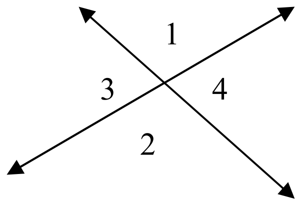
c. Under each write a number sentence to find the sum of the larger combined angle.



____________________ ____________________ ____________________

3. Look at the drawing at the right of intersecting lines.

b. Shade the vertical angles 1 and 2 red; and shade the vertical angles 3 and 4 blue.



c. Measure and record the angles.

d. Compare the measures of angles 1 and 2. ______________________________________

e. Compare the measures of angles 3 and 4. ______________________________________

f. Describe the relationship between the vertical angles ____________________________

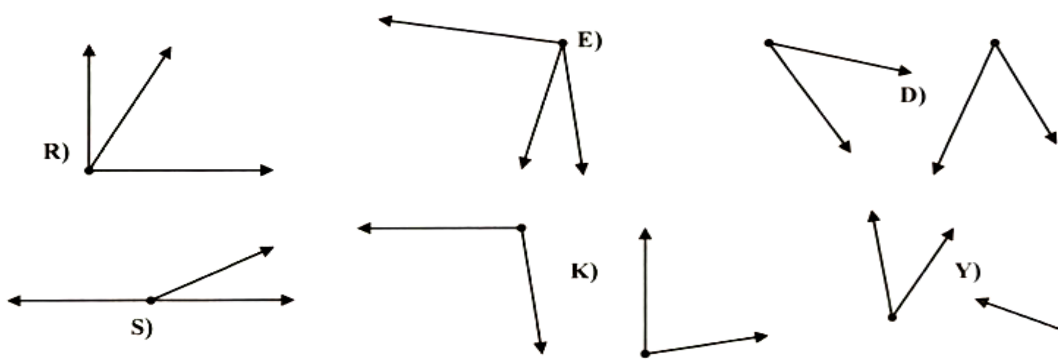
g. What is the sum of ∠1 and ∠3? _________________________

h. What is the sum of ∠1 and ∠4? _________________________

i. What is the sum of ∠2 and ∠3? _________________________

j. What is the sum of ∠2 and ∠4? _________________________

4. Complementary and supplementary angles can be **adjacent or not adjacent angles**. Look at the following pairs of angles. Measure and record each angle measurement.



b. Write a number sentence to show the sum of the angle pairs. State if the pair is complementary, supplementary or neither.

R) ___________________________________ S) ___________________________________

E) ___________________________________ K) ___________________________________

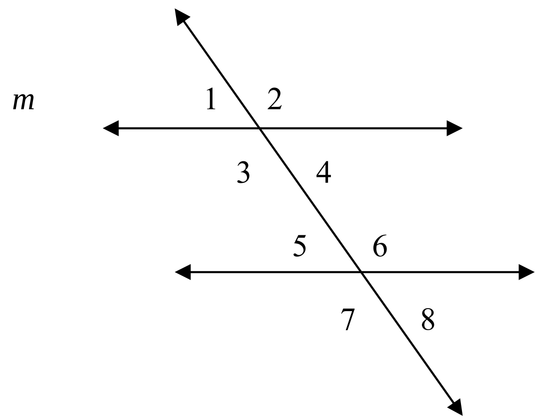
D) ___________________________________ Y) ___________________________________

5. If two angles are complements and one of the measurements is 32°.

Write and equation to find the other angle ______________________________________

b. Write an equation to find the supplement of a 70° angle. _________________________

c. One angle is 18.5°. Find its complement. ______________________________________



6. Look at the parallel lines *m* and *n* that are intersected by transversal *r* at the right.

a. Outline the parallel lines in red and the transversal in blue.
b. Measure and record each angle.

c. What is the relationship between ∠1 and ∠2? _________________________

d. What is the relationship between ∠2 and ∠3? _________________________

e. What is the relationship between ∠1 and ∠5? _________________________

f. What is the relationship between ∠2 and ∠8? _________________________

g. What is the relationship between ∠3 and ∠7? _________________________

h. What is the relationship between ∠3 and ∠5? _________________________

i. What is the relationship between ∠4 and ∠5? _________________________

Describe what you have learned about a pair of parallel lines that are intersected by a transversal.

____

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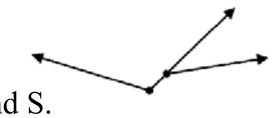
APPENDIX K

STUDENT ACTIVITY SKILLS PRACTICE 2A

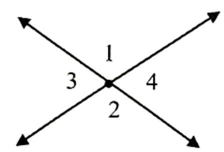
Activity 2A Name ___________________________ Date: _________

SKILLS PRACTICE – ANGLES



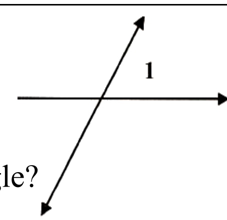
1. Look at the two angles to the right. Name the angles R and S. Are they adjacent? Explain. ___________________________________

2. Look at the drawn intersecting lines.
   a. Shade vertical angles 1 and 2 red; shade vertical angles 3 and 4 blue.
   b. Measure and record the angles.
   c. Compare the measures of ∠1 to ∠2.________________________________
   d. Compare the measures of ∠3 to ∠4. ________________________________
   e. What is the measure relationship of vertical angles? ____________________



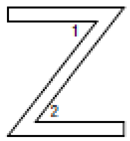
3. Look at angles 1 and 3. Write an equation to show the angle sum. _____________
   a. If a = sum of ∠1 and ∠3, find a. ________________________________
   b. If b = sum of ∠1 and ∠4, find b. ________________________________
   c. If c = sum of ∠2 and ∠3, find c. ________________________________
   d. If d = sum of ∠2 and ∠4, find d. ________________________________

4. Write an equation to find the sum of ∠1, ∠2, ∠3, and ∠4.

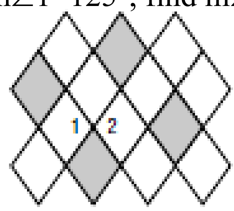


5. Look at the angles forms by the intersecting lines.
   a. Measure and record ∠1. Do you see its vertical angle?
   b. What is the measure relationship? __________________
   c. Shade ∠1 and its vertical angle red.
   d. Look at the two angles that are adjacent ∠1.
   e. What are the measured? ________________
   f. Write a number sentence to show the sum of the measures of ∠1 and an adjacent angle.
   g. What is the relationship of ∠1 to an adjacent angle?
   h. Shade the angles supplementary to ∠1 blue.
   i. What is the relationship of the blue angles to each other?
   j. If x is the sum of the four angles formed by the intersecting lines, write an equation to x. ____

6. The top and bottom segments of the letter Z are parallel as shown below. If $m\angle 1=43^{o}$, find the $m\angle 2$. Explain your reasoning.



7. Garrett is designed a floor with diamond-shaped tiles as shown below. If $m\angle 1=125^{o}$, find $m\angle 2$. Explain your reasoning.



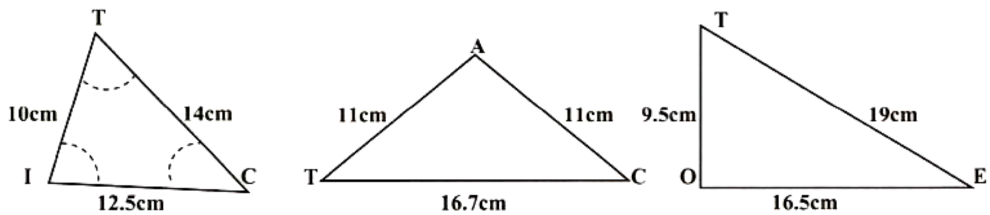
APPENDIX L

STUDENT ACTIVITY 3A

Activity 3A
Materials: Protractor, Calculator, Colored Pencils Name ______________________ Date: _______

TRIANGLE RELATIONSHIPS

1. Look at the triangles below. Measure and record the interior angles (nearest 5°)



b. To verify your angle measures, have a classmate check your measurements for accuracy.

c. Write an equation to find the sum of the three interior angles:

Δ TIC ____

Δ TAC ____

Δ TOE ____

d. What is the sum of the angles of each triangle? ________________________

e. Outline the perimeter with a colored pencil. Write an equation to find the perimeter of each triangle.

Δ TIC ____

Δ TAC ____

Δ TOE ____

2. If the lengths of the above triangles were increased by a scale factor of 2 (measurements of sides doubled), what would be the new perimeters? Write an equation to show each of the new perimeters.

Δ TIC ____

Δ TAC ____

Δ TOE ____

b. How did the perimeters change with a scale factor of 2 (as the lengths are doubled)?

____

c. A ratio is a fraction that shows measurement relationships. Next to the doubled perimeters above, write a ratio showing the length of the doubled perimeters to the original perimeters. ____

3. If the lengths of the above triangles were increased by a scale factor of 3 (measurements of sides tripled), what would be the new perimeters? Write an equation to show each of the new perimeters.

Δ TIC ____

Δ TAC ____

Δ TOE ____

b. How did the perimeters change with a scale factor of 3 (as the lengths are tripled)?

____

c. A ratio is a fraction that shows measurement relationships. Next to the new perimeters above, write ratios showing the length of the tripled perimeters to the original perimeters.

4. Did you know that in a triangle the length of a side controls the measure of its opposite angle? Look at the three triangles on the previous page.

b. In each triangle draw a line in a different color from the largest angle to the longest side. Where is the longest side in relationship to the largest angle?

____

c. In each triangle, draw a line in a different color from the shortest side to the smallest angle. Where is the shortest side in relationship to the smallest angle?

____

5. You have investigated several basic principles about triangles in this activity. Write a summary about what you have learned.

____

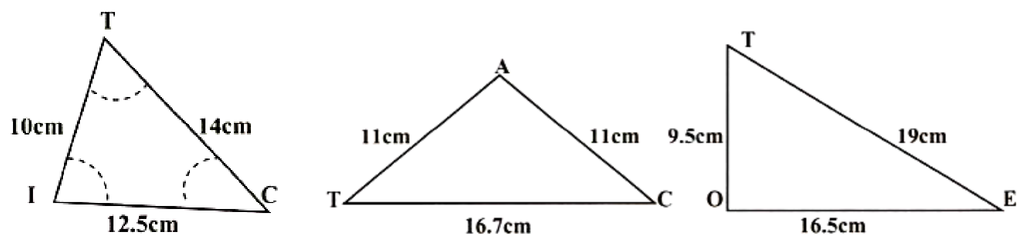
APPENDIX M

STUDENT ACTIVITY 3AG

Activity 3AG
Materials: (3) GeoLegs, Calculator, Colored Pencils Name ____________________ Date: _______

TRIANGLE RELATIONSHIPS

1. Look at the triangles below. Measure and record the interior angles (nearest 5°)



b. To verify your angle measures, use 3 GeoLegs to build each triangle using the given lengths. *For accurate measures in a GeoLeg triangle, keel the red and blue legs extremely straight.*

c. Write an equation to find the sum of the three interior angles:

Δ TIC ____

Δ TAC ____

Δ TOE ____

d. What is the sum of the angles of each triangle? ________________________

e. Outline the perimeter with a colored pencil. Write an equation to find the perimeter of each triangle.

Δ TIC ____

Δ TAC ____

Δ TOE ____

2. If the lengths of the above triangles were increased by a scale factor of 2 (measurements of sides doubled), what would be the new perimeters? Write an equation to show each of the new perimeters.

Δ TIC ____

Δ TAC ____

Δ TOE ____

b. How did the perimeters change with a scale factor of 2 (as the lengths are doubled)?

____

c. A ratio is a fraction that shows measurement relationships. Next to the doubled perimeters above, write a ratio showing the length of the doubled perimeters to the original perimeters.

3. If the lengths of the above triangles were increased by a scale factor of 3 (measurements of sides tripled), what would be the new perimeters? Write an equation to show each of the new perimeters.

Δ TIC ____

Δ TAC ____

Δ TOE ____

b. How did the perimeters change with a scale factor of 3 (as the lengths are tripled)?

____

c. A ratio is a fraction that shows measurement relationships. Next to the new perimeters above, write ratios showing the length of the tripled perimeters to the original perimeters.

4. Did you know that in a triangle the length of a side controls the measure of its opposite angle? Look at the three triangles on the previous page.

b. In each triangle draw a line in a different color from the largest angle to the longest side. Where is the longest side in relationship to the largest angle? ___________________

c. In each triangle, draw a line in a different color from the shortest side to the smallest angle. Where is the shortest side in relationship to the smallest angle? ________________

5. You have investigated several basic principles about triangles in this activity. Write a summary about what you have learned.

____

____

____

APPENDIX N

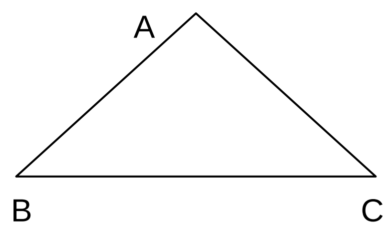
STUDENT ACTIVITY SKILLS PRACTICE 3A

Activity 3A Name ______________________________ Date: __________

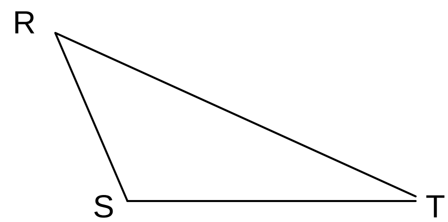
SKILLS PRACTICE - TRIANGLE RELATIONSHIPS

Measure the interior angles of each of the following triangles to the nearest 5º, measure the lengths of the sides to the nearest millimeter and then determine the perimeter of each triangle.

1.



2.



m∠ABC ________________

m∠BCA ________________

m∠CAB ________________

Side AB ________________

Side BC ________________

Side AC ________________

Perimeter ____________________

m∠RST ________________

m∠STR ________________

m∠TRS ________________

Side RS _________________

Side ST _________________

Side TR _________________

Perimeter ___________________

3. Use a scale factor of 2 for each of the above triangles. Determine the length of each new side and the new perimeter.

Side AB ________________

Side BC ________________

Side AC ________________

Perimeter ________________

Side RS ___________________

Side ST ___________________

Side TR ___________________

Perimeter ___________________

4. Use a scale factor of 3 for each of the above triangles. Determine the length of each new side and the new perimeter.

Side AB ________________

Side BC ________________

Side AC ________________

Perimeter __________________

Side RS __________________

Side ST __________________

Side TR __________________

Perimeter ___________________

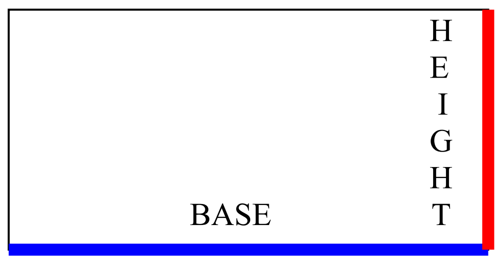
APPENDIX 0

STUDENT ACTIVITY 4A

Activity 4A
Materials: Protractor, Calculator, Colored Pencils Name ______________________ Date: ________
Index cards, Scissors, Ruler

AREA OF A TRIANGLE ~ RELATIONSHIPS

1. Let's explore how the formula for finding the area of a triangle was developed. Take an index card and place it on your desk with the longest side on the bottom and the shortest side on the right. Draw a blue line on the bottom and write the word BASE. On the right side, draw a red line and write the word HEIGHT.



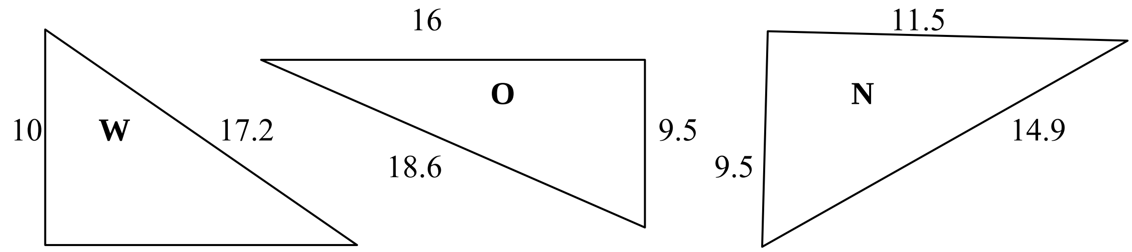
a. Measure the sides and write the measurement next to each word. To find the AREA, multiply the BASE times the HEIGHT. Write the formula in the center of the card. Draw a DIAGONAL line. Cut the card along the DIAGONAL line.

b. Write a statement describing the relationship between the area of the rectangle and the area of the two triangles.
____.

c. Based on your observations, write the formula for finding the area of a triangle.
____.

The perpendicular sides of the right angle form the height and the base of the triangle.

2. Look at the right triangles below. Use your ruler and a colored pencil to *make a square* then identify the base of the triangle with a **B** and the height of the triangle with an **H**.



Write a number sentence to find the area of each triangle to the nearest tenth. (area = ½ (base · height).

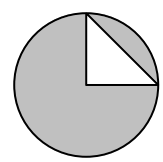
△ W ____

△ O ____

△ N ____

3. Explain how to find the area of a triangle ____________________________________

4. A right triangle is inscribed in the circle to the right. The length of the radius is 15cm.



Two sides of the triangle are the radii of the circle.

a. Find the area of the circle (area = Πr^2). ________________________________

b. Find the area of the triangle. ____

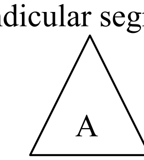
c. Explain how to find the area of the shaded portion of the circle.

____

d. Find the area of the shaded portion of the circle. _____________________________

5. Look at equilateral △ A. Does it have a perpendicular height? _________________

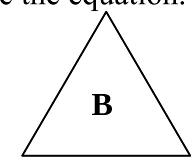
A perpendicular line must be drawn from the base to form the height and a right angle. Use your protractor and ruler to make a right angle and draw a perpendicular segment from the base to its opposite angle. Label the segment the height.



a. If the base of △A is 6cm and the height is 5.2cm. Write and solve an equation to find the area.

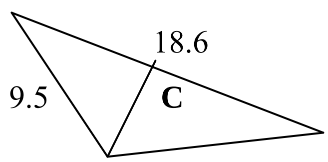
---

b. Equilateral △B has lengths of 8am and a height of 6.9cm. Draw the height and record the lengths on the triangle. Write an equation for finding the area and solve the equation.



c. Look at scalene triangle C. Look that the height that has been drawn to form a height of 8cm.

Write an equation for finding the area and solve the equation.



6. You have looked at relationships between rectangles and triangles as well as finding the area of a triangle. Write a summary of what you have learned from these activities.

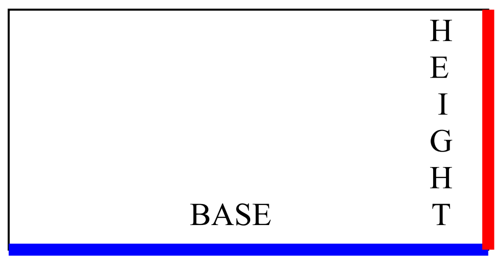
APPENDIX P

STUDENT ACTIVITY 4AG

Activity 4AG
Materials: (4) GeoLeg, Calculator, Colored Pencils Name ______________________ Date: ________
Index cards, Scissors, Ruler

AREA OF A TRIANGLE ~ RELATIONSHIPS

1. Let's explore how the formula for finding the area of a triangle was developed. Take an index card and place it on your desk with the longest side on the bottom and the shortest side on the right. Draw a blue line on the bottom and write the word BASE. On the right side, draw a red line and write the word HEIGHT.



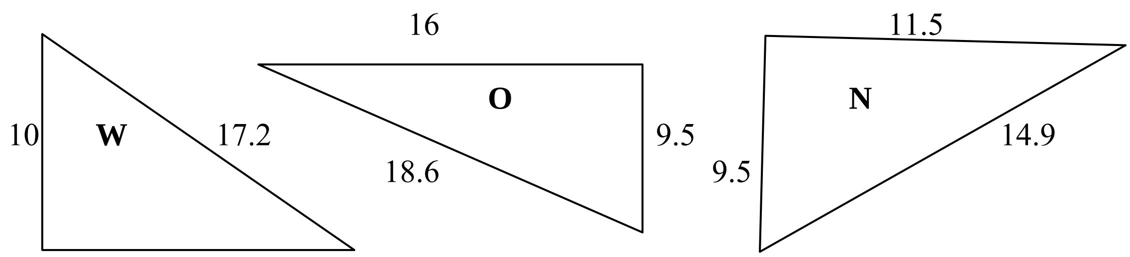
d. Measure the sides and write the measurement next to each word. To find the AREA, multiply the BASE times the HEIGHT. Write the formula in the center of the card. Draw a DIAGONAL line. Cut the card along the DIAGONAL line.

e. Write a statement describing the relationship between the area of the rectangle and the area of the two triangles.
____.

f. Based on your observations, write the formula for finding the area of a triangle.
____.

The perpendicular sides of the right angle form the height and the base of the triangle.

2. Look at the right triangles below. Use your ruler and a colored pencil to *make a square* then identify the base of the triangle with a **B** and the height of the triangle with an **H**.



Write a number sentence to find the area of each triangle to the nearest tenth. (area = ½ (base · height).

△ W ____

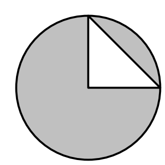
△ O ____

△ N ____

3. Explain how to find the area of a triangle ________________________________

4. A right triangle is inscribed in the circle to the right. The length of the radius is 15cm.

Two sides of the triangle are the radii of the circle.



e. Find the area of the circle (area = ∏ r^2). ________________________________

f. Find the area of the triangle. ____

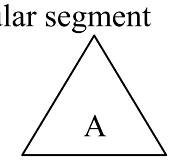
g. Explain how to find the area of the shaded portion of the circle.

____

h. Find the area of the shaded portion of the circle. ___________________________

5. Look at equilateral △ A. Does it have a perpendicular height? ________________

A perpendicular line must be drawn from the base to form the height and a right angle. Use your GeoLeg and ruler to make a right angle and draw a perpendicular segment from the base to it's opposite angle. Label the segment the height.

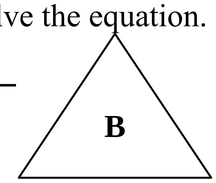


a. If the base of △A is 6cm and the height is 5.2cm. Write and solve an equation to find the area.

____

b. Equilateral △B has lengths of 8am and a height of 6.9cm. Draw the height and record the lengths on the triangle. Write an equation for finding the area and solve the equation.

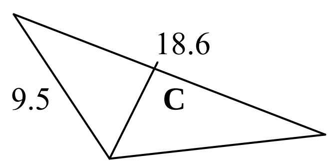
____



c. Look at scalene triangle C. Look that the height that has been drawn to form a height of 8cm.

Write an equation for finding the area and solve the equation.

____



6. You have looked at relationships between rectangles and triangles as well as finding the area of a triangle. Write a summary of what you have learned from these activities.

____

____

____

____

APPENDIX Q

STUDENT ACTIVITY SKILLS PRACTICE 4A

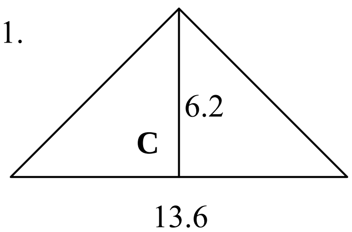
Activity 4A

Name ____________________________ Date: __________

SKILLS PRACTICE – AREA OF A TRIANGLE

For each of the following triangles, write the equation for the area and solve the equation.

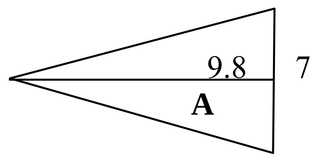
1.



Equation _______________

Area ____________________

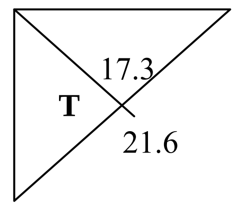
2.



Equation ________________

Area ____________________

3.

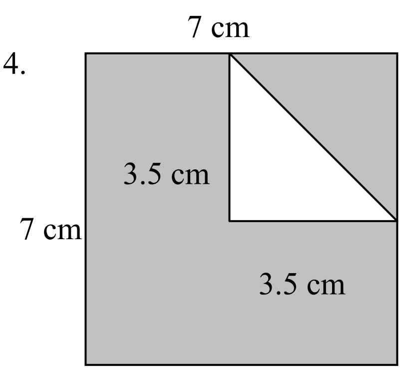


Equation _______________

Area __________________

Find the area of the shaded region.

4.



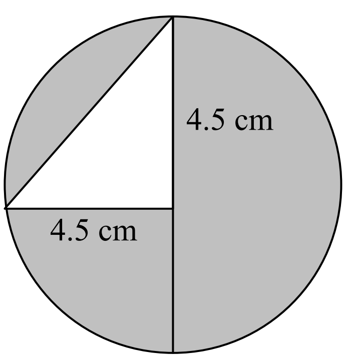
a. Describe how to find the area of the shaded region. _____________________________________

____

b. Find the area of the shaded region

____

5.



a. Describe how to find the area of the shaded region. ___________________________________

b. Find the area of the shaded region

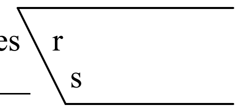
____

APPENDIX R

STUDENT ACTIVITY 5A

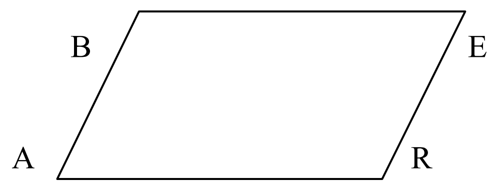
Activity 5A
Materials: ruler, protractor, Calculator, Colored Pencils Name ____________________ Date:________
Index cards, Scissors

QUADRILATERALS ~ RELATIONSHIPS



1. What do you know about angles r and s formed between parallel lines by a transversal? ____

2. Let's explore the relationships in parallelogram **BEAR.** Measure and label the angles.



m∠BER ________________________

m∠ERA ________________________

m∠RAB ________________________

m∠ABE ________________________

b. Write an equation showing the relationship between the consecutive angles.

____

c. Write an equation showing the sum of the four angles.

____

d. Which angles are supplementary? -

____

e. Use a colored pencil to connect the opposite angles with a diagonal. What is the relationship of the opposite angles?

____

f. Write two equations showing the relationships between the opposite angles.

____

g. Measure the lengths of the sides of BEAR to the nearest 0.5 cm. What is the relationship between the opposite sides?

____

h. If **P** = perimeter, write an equation to find **P**. ______________________________________

2. Explore: Use your protractor and ruler to draw a figure. Let the opposite sides measure 10cm and 12cm. Let the angles equal 90°. Let's call this drawing #1. What is the name of this parallelogram? ____________.

b. Draw a second figure. Change the angles to 60° and 120°. Let's call this #2. Are both drawings parallelograms? _________________

Explain.____

Which one would you predict has the larger area? _______________________________

Find the area of #1 _______________________ Find the area of #2 ________________

Explain and describe your findings. ____

____

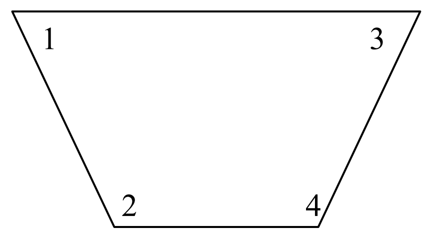
3. Let's look at a trapezoid. Use your protractor to measure each angle to the nearest 5°.

a. m∠1 _____________________

m∠2 _____________________

m∠3 _____________________

m∠4 _____________________



b. What relationship do you notice between ∠1 and ∠2? ________________________

c. Write an equation to describe the sum of ∠1 and ∠2? _________________________

d. What relationship do you notice between ∠3 and ∠4? ________________________

e. Write an equation to describe the sum of ∠3 and ∠4? _________________________

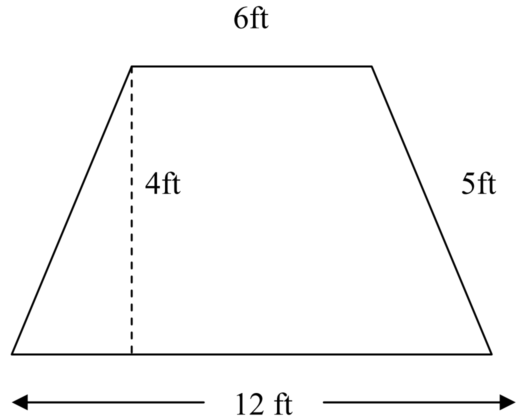
f. Write an equation to find the sum of the angles of the trapezoid. __________________

4. In a trapezoid K, three of the angles are 30, 90 and 150. Draw a trapezoid with your protractor and ruler. Write an equation to find the missing angle, find the missing angle and then draw a picture of the trapezoid.

Equation ______________________________ Drawing

Missing angle measurement ______________

5. The area of a trapezoid is easy to find if you draw heights and turn it into a rectangle and two triangles.



a. What are the dimensions of the rectangle? (Label all sides)

b. What is the area of the rectangle? (area = base · height) _______________

c. What are the dimensions of each triangle? (Label all sides)

d. What is the area of each triangle? [½ (base · height)] _______________

e. Describe how you would find the area of the trapezoid. ___________

____

f. If A = the area of a trapezoid, find A. ___________________________

6. Describe a trapezoid. Talk about the relationships between the parallel lines and supplementary angles. Use words and sketches to show different types of trapezoids.

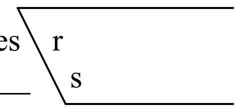
APPENDIX S

STUDENT ACTIVITY 5AG

Activity 5AG
Materials: ruler, GeoLeg, Calculator, Colored Pencils   Name ____________________   Date:_______
Index cards, Scissors

QUADRILATERALS ~ RELATIONSHIPS

1. What do you know about angles r and s formed between parallel lines by a transversal? ____



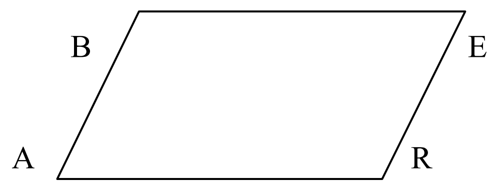
2. Let’s explore the relationships in parallelogram **BEAR.** Measure and label the angles.

m∠BER ______________________

m∠ERA ______________________

m∠RAB ______________________

m∠ABE ______________________



b. Write an equation showing the relationship between the consecutive angles.

____

c. Write an equation showing the sum of the four angles.

____

d. Which angles are supplementary? -

____

e. Use a colored pencil to connect the opposite angles with a diagonal. What is the relationship of the opposite angles?

____

f. Write two equations showing the relationships between the opposite angles.

____

g. Measure the lengths of the sides of BEAR to the nearest 0.5 cm. What is the relationship between the opposite sides?

____

h. If **P** = perimeter, write an equation to find **P**. ___________________________________

2. Explore: Use 4 GeoLegs to form a figure. Let the opposite sides measure 10cm and 12cm. Let the angles equal 90°. Let's call this figure #1. What is the name of this figure? ______________________.

b. Draw a second figure. Change the angles to 60° and 120°. Let's call this #2. Are both drawings parallelograms? ________________

Explain.____

Which one would you predict has the larger area? ______________________________

Find the area of #1 ______________________ Find the area of #2 ________________

Explain and describe your findings. ____

____

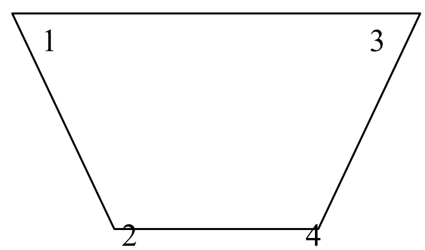
3 Let's look at a trapezoid. Use your GeoLeg to measure each angle to the nearest 5°.

a. m∠1 ____________________

m∠2 ____________________

m∠3 ____________________

m∠4 ____________________



b. What relationship do you notice between ∠1 and ∠2? _______________________

c. Write an equation to describe the sum of ∠1 and ∠2? ________________________

d. What relationship do you notice between ∠3 and ∠4? _______________________

e. Write an equation to describe the sum of ∠3 and ∠4? ________________________

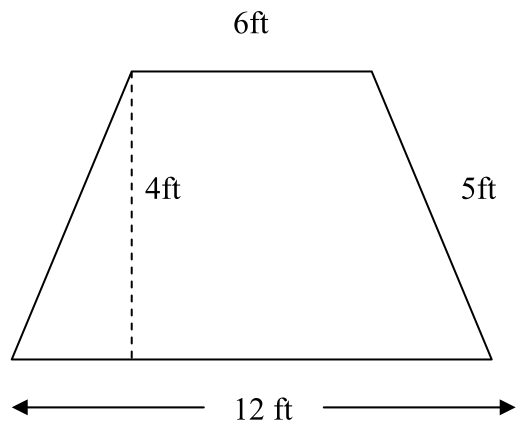
f. Write an equation to find the sum of the angles of the trapezoid. _________________

4. In a trapezoid K, three of the angles are 30, 90 and 150. Make the trapezoid with your GeoLeg tools. Write an equation to find the missing angle, find the missing angle and then draw a picture of the trapezoid.

Equation ______________________________ Drawing

Missing angle measurement ______________

5. The area of a trapezoid is easy to find if you draw heights and turn it into a rectangle and two triangles.



a. What are the dimensions of the rectangle? (Label all sides)

b. What is the area of the rectangle? (area = base · height) _______________

c. What are the dimensions of each triangle? (Label all sides)

d. What is the area of each triangle? [½ (base · height)] _______________

e. Describe how you would find the area of the trapezoid. ___________

____

f. If A = the area of a trapezoid, find A. ____________________________

6. Describe a trapezoid. Talk about the relationships between the parallel lines and supplementary angles. Use words and sketches to show different types of trapezoids. Use your GeoLeg tools to explore different trapezoids to help you describe the relationships.

APPENDIX T

STUDENT ACTIVITY SKILLS PRACTICE 5A

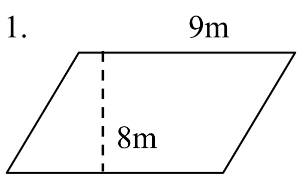
Skills Activity 5A

Name ____________________________ Date: __________

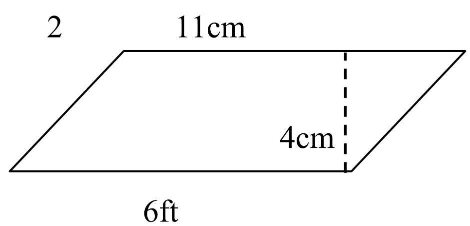
SKILLS PRACTICE – AREA OF A TRIANGLES AND QUADRILATERALS

For each of the following, write the equation for the area and solve then the equation. Show your work.

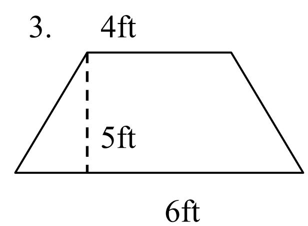
1.



2



3.



Equation ___________ Equation _____________ Equation ___________

Area _______________ Area ________________ Area ________________

Draw a sketch of each. Then write the equation for the area and solve then the equation. Show your work.

4. parallelogram: base,7.8 in.;height,7.5 in.

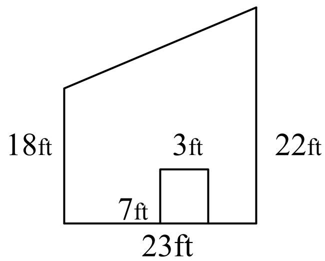
5. triangle: base,7.2 cm;height,3.5 cm

6. trapezoid: height,2.8 mi bases,3.4 mi and 7.6 mi

Equation _____________ Equation _____________ Equation _____________

Area ________________ Area _________________ Area ________________

7. CONSTRUCTION: A 7-foot by 3-foot doorway is to be cut into the trapezoid-shaped wall shown. What is the area of the wall, without the doorway?



8. CONSTRUCTION: The wall in Exercise 7 is to be painted. If one can of paint covers 110 square feet, how many cans of paint will be needed if only one coat of paint is applied?

Describe your strategy for solving #7 and #8.

____

____

____

____

____

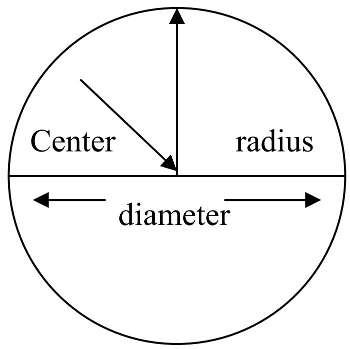
____

APPENDIX U

STUDENT ACTIVITY 6A

Activity 6A
Materials: Protractor, Colored Pencils, Ruler Pencil, Compass   Name ____________________ Date: __________

DRAWING CIRCLES ~ RELATIONSHIPS



A circle is divided into 360 measures called **degrees** (°).

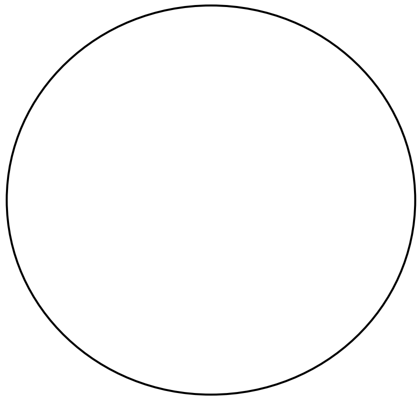
The **circumference** is the perimeter of the circle.
The **center point** is equidistant to all points on the circumference.
An **arc** is a part of the circle itself.
A **radius** is drawn from the center to outside edge of the circle.
A **diameter** goes through the center of the circle and its endpoints lie on the circle.
A chord connects points on the circle that do not go through the center.
chord

1. Practice drawing circles with your compass. Use a blank sheet of paper.
   a. Mark a center point on your paper.
   b. Place the compass center over you center point. Carefully hold it on the center point.
   c. Open your compass to 5cm. Make a circle with a 5cm radius.

2. Continue practicing making circles with 7cm, 9cm and 11cm. radii.

3. A circle is drawn for you. Mark a point on the circle at 0°

0°



b. Draw a radius from the center of the circle to 0°

c. Extend the radius into a diameter by drawing it to the other side.
Measure the length of the diameter. __________.

d. Mark the point where the diameter intersects the circle and number it 180 ° (the diameter divides the circle into 2 semicircles)

e. If a circle is 360°, the measure of a semicircle is ___________________.

4. Using a blank sheet of paper, mark a center point and make a 7cm circle.

a. Starting a 0° working in a *clockwise* direction using your protractor to place a mark on the circle at 30°, 60°, 90°, 120° and 150°. Write the degree measurements on the outside of the circle.
b. **Repeat step f** in a *counterclockwise* direction.
c. Starting a 0°, use a colored pencil to make a triangle by working clockwise direction. Connect points 0° to 120°, 120° to 60°, and 60° to 0°.
d. Make another triangle using a different colored pencil. Connect 60° to 180 °, 180 ° to 120°, and 120° to 60°.
e. Use a different colored pencil to make as many diameters as you can.
f. Using a different colored pencil, connect points 0° to 60°, 60° to 120°, 120° to 180 °, 180 ° to 60°, 60° to 120°, and 120° to 0°.
g. Finally, beginning at 0°, label each point A, B, C, D, E, F, G, H, I, J, K, L

5. From the circle drawing you made in #4, name as many geometry figures as you are able. Use the letters as end point labels.

____

____

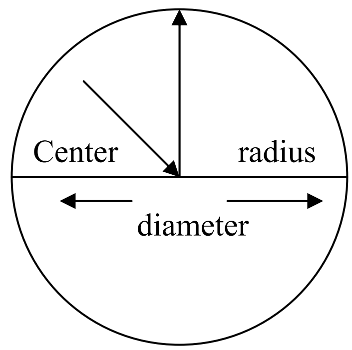
____

APPENDIX V

STUDENT ACTIVITY 6AG

Activity 6AG
Materials: (1) GeoLeg, Colored Pencils, Ruler Pencil, Compass Name ________________________ Date: __________

DRAWING CIRCLES ~ RELATIONSHIPS



A circle is divided into 360 measures called **degrees** (°).

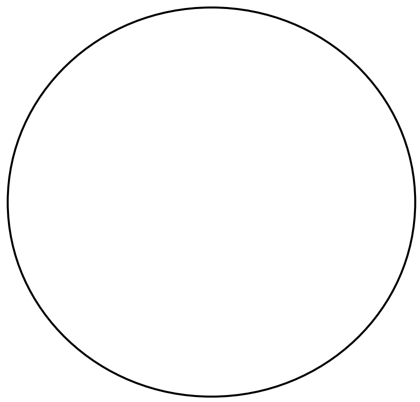
The **circumference** is the perimeter of the circle.
The **center point** is equidistant to all points on the circumference.
An **arc** is a part of the circle itself.
A **radius** is drawn from the center to outside edge of the circle.
A **diameter** goes through the center of the circle and its endpoints lie on the circle.
A chord connects points on the circle that do not go through the center.
chord

1. Practice drawing circles with your GeoLeg. Use a blank sheet of paper.
   a. Mark a center point on your paper.
   b. Place the GeoLeg center over you center point. Hold it there with a sharp regular pencil.
   c. Place a second pencil in the 5cm hole on your GeoLeg. Trace around the GeoLeg to make a circle with a 5cm radius.

2. Continue practicing making circles with 7cm, 9cm and 11cm. radii.

3. A circle is drawn for you. Mark a point on the circle at 0°

0°



b. Draw a radius from the center of the circle to 0°

c. Extend the radius into a diameter by drawing it to the other side.
Measure the length of the diameter. __________.

d. Mark the point where the diameter intersects the circle and number it 180 ° (the diameter divides the circle into 2 semicircles)

e. If a circle is 360°, the measure of a semicircle is ___________________.

4. Using a blank sheet of paper, mark a center point and make a 7cm circle.

a. Starting a 0° working in a *clockwise* direction using your GeoLeg to place a mark on the circle at 30°, 60°, 90°, 120° and 150°. Write the degree measurements on the outside of the circle.
b. **Repeat step f** in a *counterclockwise* direction. Place the blue line of the GeoLeg tool at 0°
c. Starting a 0°, use a colored pencil to make a triangle by working clockwise direction. Connect points 0° to 120°, 120° to 60°, and 60° to 0°.
d. Make another triangle using a different colored pencil. Connect 60° to 180 °, 180 ° to 120°, and 120° to 60°.
e. Use a different colored pencil to make as many diameters as you can.
f. Using a different colored pencil, connect points 0° to 60°, 60° to 120°, 120° to 180 °, 180 ° to 60°, 60° to 120°, and 120° to 0°.
g. Finally, beginning at 0°, label each point A, B, C, D, E, F, G, H, I, J, K, L

5. From the circle drawing you made in #4, name as many geometry figures as you are able. Use the letters as end point labels.

____

____

____

____

APPENDIX W

STUDENT ACTIVITY 7A

Activity 7A
Materials: Protractor, Calculator, Colored Pencils Name ______________________ Date: __________
Cardstock, Scissors, Ruler

AREA OF A CIRCLE

1. Let's explore how the formula for finding the area of a CIRCLE was developed.
   - Take a piece of cardstock paper and draw a circle with a radius of 9cm.
   - Shade half of the circumference with a colored pencil.
   - Fold the paper in four times in order to divide it into 16 equal parts.
   - Cut along the folds.
   - Reassemble the pieces to form a parallelogram-shaped figure.

a. Based on what you know about the area of a parallelogram, how can you find the area of a circle?
____.

b. Write a statement describing the relationship between the area of the parallelogram and the area of a circle.
____.

c. Based on your observations, write the formula for finding the area of a circle.
____.

2. Use your calculator to find the following. Round to the nearest tenth.
   a. If the radius is 14, estimate its area using 3 for pi.

      ____

   b. Find a more exact area using

      Π.______________________________________

   c. If the diameter is 18, find the area using Π.

      ____

   d. If the area is 300, find the $radius^2$ using Π.

      ____

e. If the area is 154, find the radius using Π.

____

f. If the area is 201, the diameter is

____

3. To find the area of circle that is within the area of another circle, find the areas of both circles. Then subtract the smaller area from the larger.

   a. An artist is painting a large yellow circle that has a diameter of 5 ft. A scale model is drawn. Draw the diameter. Write an equation to find the area of the yellow circle to the nearest tenth.

   ____

   ____

   b. Around his yellow circle, she will paint a concentric circle with a diameter of 8ft. The area outside the yellow circle is orange.
   Write an equation to find the orange area.

   ____

   c. Explain how you found the orange area.

   ____

   ____

   ____

   ____

APPENDIX X

STUDENT ACTIVITY 7AG

Activity 7AG
Materials: GeoLeg, Calculator, Colored Pencils Name ______________________ Date: __________
Cardstock, Scissors, Ruler

AREA OF A CIRCLE

4. Let's explore how the formula for finding the area of a CIRCLE was developed.
   - Take a piece of cardstock paper and draw a circle with a radius of 9cm.
   - Shade half of the circumference with a colored pencil.
   - Fold the paper in four times in order to divide it into 16 equal parts.
   - Cut along the folds.
   - Reassemble the pieces to form a parallelogram-shaped figure.

a. Based on what you know about the area of a parallelogram, how can you find the area of a circle?
____.

d. Write a statement describing the relationship between the area of the parallelogram and the area of a circle.
____.

e. Based on your observations, write the formula for finding the area of a circle.
____.

5. Use your calculator to find the following. Round to the nearest tenth.
   a. If the radius is 14, estimate its area using 3 for pi.

      ____

   b. Find a more exact area using
      Π.____
   c. If the diameter is 18, find the area using Π.

      ____

   d. If the area is 300, find the radius^2 using Π.

      ____

   e. If the area is 154, find the radius using Π.

      ____

f. If the area is 201, the diameter is

6. To find the area of circle that is within the area of another circle, find the areas of both circles. Then subtract the smaller area from the larger.

   a. An artist is painting a large yellow circle that has a diameter of 5 ft. A scale model is drawn. Draw the diameter. Write an equation to find the area of the yellow circle to the nearest tenth.

   b. Around his yellow circle, she will paint a concentric circle with a diameter of 8ft. The area outside the yellow circle is orange.
   Write an equation to find the orange area.

   c. Explain how you found the orange area.

APPENDIX Y

STUDENT ACTIVITY SKILLS PRACTICE 7A

Skills Activity 7

Name ____________________________ Date: __________

SKILLS PRACTICE FOR AREA OF A CIRCLE

1. What happens to the area of a circle as you increase its radius? Let's investigate. Fill in the following chart. Round to the nearest tenth:

Radius	**Radius^2**	**Estimated Area** R^2 **x 3 = A**	**Area using Pi** R^2 **x 3.14 = A**
4			
	64		
		Y x 3 = 432	

b. How did the area change as the radius double from 4 to 8?

____

c. By what factor did the area increase as the radius length tripled from 4 to 12?

____

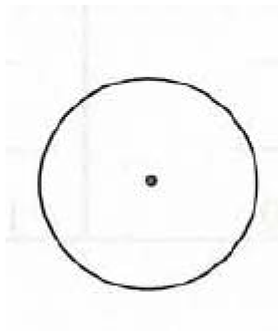
2. Let's explore to see how the area increases as the radius increases. Below are two circles. One has a radius of 1cm; the other has a radius that is doubled to 2 cm. Draw a radius in each. Write the equation to find the areas:

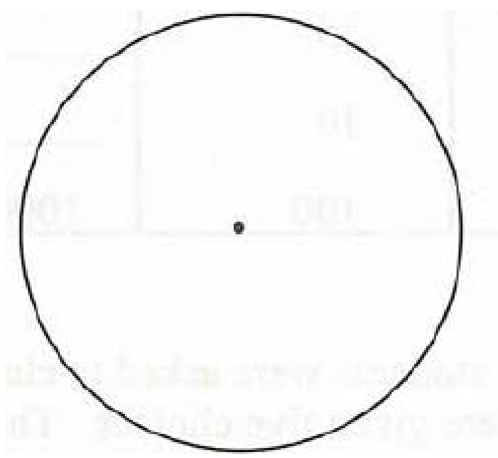
1cm___________________________

2cm _________________________________

b. As the radius increased by a factor 2, how did the area change?

c. If the area increased by factor of 4 you should be able to draw 4 small circles in the larger one. Estimate the size of the smaller circle and draw 4 small circles inside the larger one. (If you have a penny, trace around it because it is about the size of the smaller circle.) Allow for overlapping.





d. Can 4 small circles fit inside the larger circle? ___________________

3. Round mirrors sell for $.29 per square inch. What is the difference in price for a mirror with a diameter of 8 inches and one with a diameter of 16 inches? Write an equation to solve.

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APPENDIX Z

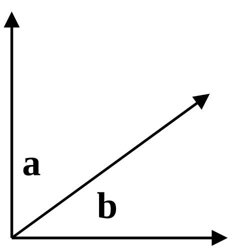
STUDENT POST TEST ASSESSMENT

Assessment **Name** ___________________ **Per** _______ **Date** _________
Geometry Unit

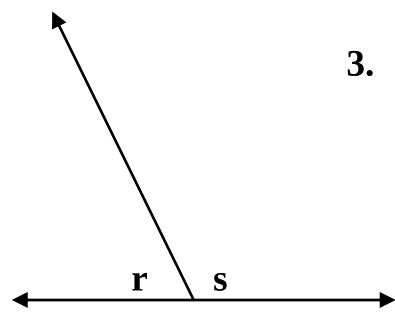
Solve each of the following problems. Show all of your work.

For questions 1-3, measure each angle, identify them as complementary, supplementary or neither.

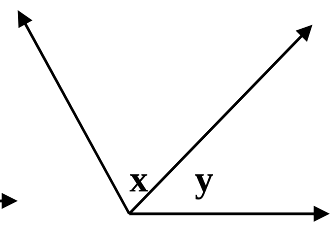
1.



2.



3.

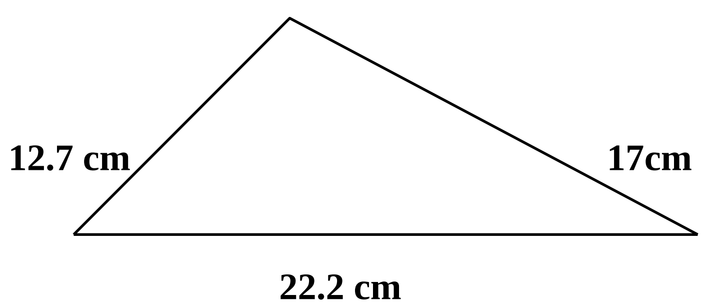


m∠a = _____________ **m∠r =** ______________ **m∠x =** ______________

m∠b = _____________ **m∠s =** ______________ **m∠y =** ______________

____________________ _____________________ ______________________

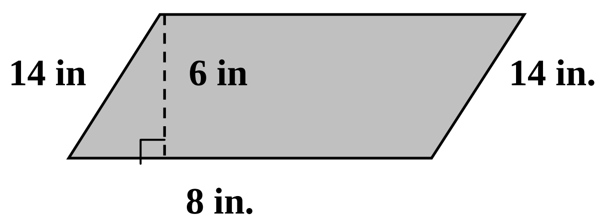
4. Apply a scale factor of 3 to the following triangle. Then find the perimeter.



4. _________________________

For questions 5 – 11, find the area of each figure. Round to the nearest tenth.

5.

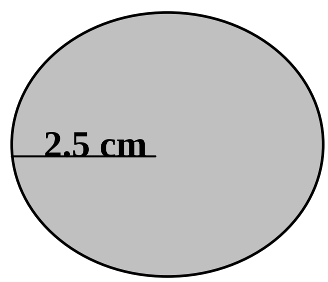


5. ______________________

6. Trapezoid: height 1.5 meters, bases 2.2 meters and 5.8 meters.

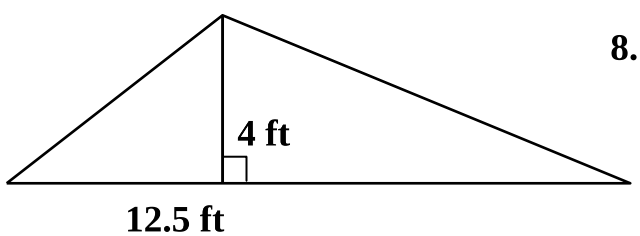
6. ______________________

7.



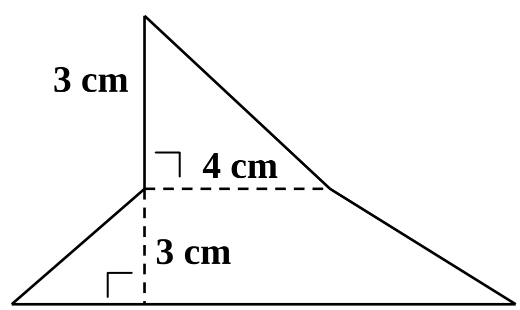
7. ______________________

8.



8. ______________________

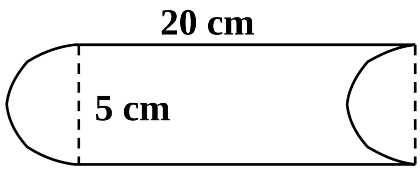
9.



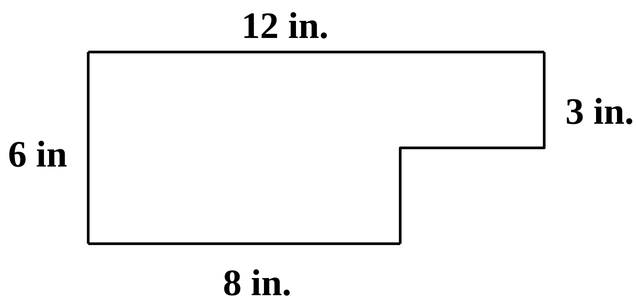
9. ______________________

10 cm

10. **10.** ______________________

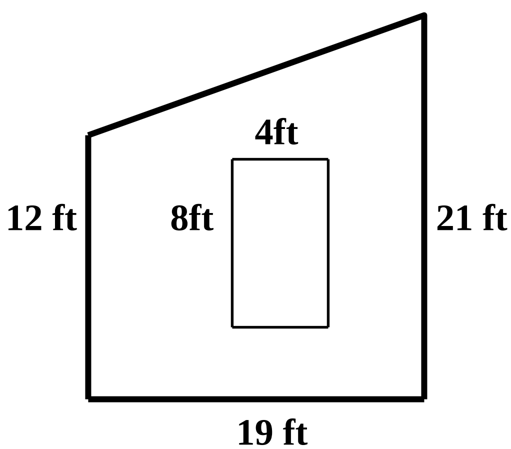


11. **11.** ______________________



12. A deck is being build around a garden that is 8 ft by 4 ft. What is the area of the deck?

12. ______________________



13. The deck in problem #12 is to going to be painted.
If each gallon of paint covers 250 square feet, how many gallons of paint should be purchased in order to paint the deck with one coat of paint?

13. ______________________

14. If a circle with a radius of 4 cm is increased by a a scale factor of 2, how much greater is the area of the larger circle?

14. ______________________

15. You want to buy round stepping stones for the front of your house. They sell for $1.39 a square foot.
What is the difference in price between stones with a 9 inch Diameter and a 12 inch diameter?

15. _____________________